# Matrix Inflation

Amjad Ashoorioon (Uppsala University)
In collaboration with
Hassan Firouzjahi (IPM)
&
Shahin Sheikh-Jabbari (IPM)

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# Introduction

- WMAP7 strongly supports the idea of inflation as the theory of early universe and structure formation.
- Inflation is a paradigm and one can construct many inflationary models compatible with the current data.
- In its simplest form, inflation is driven by a scalar field minimally coupled to gravity.
- There has been lots of efforts to realize inflation in string theory, mainly using D-Dbar interactions in a warped throat.

KKLMMT (2003)

- On the other hand, general compactifications with fluxes naturally posses several branes to satisfy constraints like tadpole cancellation.
- In these theories, more than one scalar field is present.
- In multi-field inflationary models, one can usually perform a rotation in the field space, where the inflaton field evolves along the trajectory and the other fields are orthogonal to it.

Gordon, Wands, Bassett & Maartens (2001)

# Introduction

- In this talk, we promote the inflaton fields to general N X N hermitian matrices, and hence the name Matrix Inflation, or M-flation for brevity.

- Working with matrices, we are able to use their commutators in the potential, besides their simple products.

- In our class of Matrix inflation models, we consider three N X N matrices, $\Phi_i, \ i = 1, 2, 3$
We consider the potential which is quadratic in $\Phi_i$ and their commutators $[\Phi_i, \Phi_j]$.
Therefore, we have three types of terms in the potential: $\mathrm{Tr}[\Phi_i, \Phi_j]^2$, $\mathrm{Tr}\,\varepsilon_{ijk}\Phi_i[\Phi_j, \Phi_k]$, $\mathrm{Tr}\Phi_i^2$

- As we will see, the model is motivated from string theory and brane dynamics.

- Despite its simple form, Matrix inflation has a rich dynamics:

- It can solve the fine-tuning associated with standard chaotic inflationary models.

- Besides the adiabatic perturbations, we have isocurvature ones.

- The model has an embedded preheating mechanism that uses the isocurvature fields as preheat fields.

# Outline

- Matrix Inflation Setup
- Matrix Inflation from String Theory
- Truncation to the SU(2) Sector
- Consistency of the Truncation to the SU(2) Sector
- Analysis of the Matrix Inflation around the Single-Block Vacuum
- Mass and power spectra of isocurvature modes in Matrix Inflation
- Particle Creation and Preheat Scenario
- UV behavior of Matrix Inflation in presence of many species
- Analysis of the potential for n-block vacua
- Conclusion

# Matrix Inflation Setup

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P}{2}R - \frac{1}{2}\sum_i \mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) - V\left(\Phi_i,\left[\Phi_i,\Phi_j\right]\right)\right)$$

As we will show momentarily, the full potential can be motivated from dynamics of branes In string theory and takes the form:

$$V = \mathrm{Tr}\left(-\frac{\lambda}{4}\left[\Phi_i,\Phi_j\right]\left[\Phi_i,\Phi_j\right] + \frac{i\kappa}{3}\varepsilon_{jkl}\left[\Phi_k,\Phi_l\right]\Phi_j + \frac{m^2}{2}\Phi_i^2\right)$$

The potential is invariant under $U(N)$ group (acting on the matrices) and $SU(2)$ group acting on $i,\ j$ indices.

The EOMs are:

$$H^2 = \frac{1}{3M_P^2}\left(-\frac{1}{2}\mathrm{Tr}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i\right) + V\left(\Phi_i,\left[\Phi_i,\Phi_j\right]\right)\right)$$

$$\ddot{\Phi}_l + 3H\dot{\Phi}_l + \lambda\left[\Phi_j,\left[\Phi_l,\Phi_j\right]\right] + i\kappa\varepsilon_{ljk}\left[\Phi_j,\Phi_k\right] + m^2\Phi_l = 0$$

$$\dot{H} = -\frac{1}{2M_P^2}\sum_i \mathrm{Tr}\,\partial_\mu\Phi_i\partial^\mu\Phi_i$$

# Matrix Inflation from String Theory

In the context of string theory, it is known that the world-volume of *N* coincident 3-branes is described by supersymmetric *U(N)* gauge theory. In this system the transverse position of the branes, $\Phi_I, I = 4,\ldots,9$ are scalars in the adjoint representation of *U(N),* and hence *N X N* matrices*.* The DBI action for the system in the background of RR six form flux (sourced by distribution of D5 branes) is given by:

$$S = \frac{1}{(2\pi)^3 l_s^4 g_s} \int d^4x \, \mathrm{STr}\left( 1 - \sqrt{-|g_{ab}|}\sqrt{|Q^I_J|} + \frac{i g_s}{4\pi l_s^2}\left[X^I, X^J\right] C^{(6)}_{IJ0123} \right)$$

Myers (1999)

$$g_{ab} = G_{MN} \partial_a X^M \partial_b X^N \qquad M, N = 0, 1, \ldots, 9$$

$$I, J = 4,5,\ldots,9$$
$$a, b = 0, 1, 2, 3$$

$$Q^{IJ} = \delta^{IJ} + \frac{i}{2\pi l_s^2}\left[X^I, X^J\right]$$

We consider the the 10-d IIB supergravity background:

$$ds^2 = 2dx^+ dx^- - \hat{m}^2 \sum_{i=1}^{3} (x^i)^2 (dx^+)^2 + \sum_{K=1}^{8} dx_K dx_K$$

$$C_{+123\ ij} = \frac{2\hat{\kappa}}{3} \varepsilon_{ijk} \, x^k$$

$i, j = 1,2,3$ parameterize 3 out 6 dim $\perp$ to the D3-branes and $x^K$ denotes 3 spatial dim along and five transverse to D3-branes.

# Matrix Inflation from String Theory

With $\hat{m}^2 = \frac{4g_s^2\hat{\kappa}^2}{9}$ the above background with constant dilaton is solution to the SUGRA EOM. We compactify the transverse dimensions on a 6d CY manifold with two 3d cycles, one of which is very long and the other one is quite small. In the light-cone gauge on the D3-branes, expanding the action up to fourth order in $X^I$ yields

$$S = \frac{1}{(2\pi)^3 l_s^4 g_s} \int d^4x \, \mathrm{Tr}\left[ -\frac{1}{2}\partial_\mu X_i \partial^\mu X_i - V(X_i) \right]$$

$$V = -\frac{1}{4(2\pi l_s^2)^2}[X_i, X_j][X_i, X_j] + \frac{ig_s\hat{\kappa}}{3.2\pi l_s^2}\varepsilon^{ijk} X_i [X_j, X_k] + \frac{1}{2}\hat{m}^2 X_i^2$$

Upon the field redefinition $\Phi_i \equiv \frac{X_i}{\sqrt{(2\pi)^3 g_s l_s^2}}$

$$V = \mathrm{Tr}\left( -\frac{\lambda}{4}[\Phi_i, \Phi_j][\Phi_i, \Phi_j] + \frac{i\kappa}{3}\varepsilon_{jkl}[\Phi_k, \Phi_l]\Phi_j + \frac{m^2}{2}\Phi_i^2 \right)$$

$$\lambda = 2\pi g_s \qquad \kappa = \hat{\kappa}\, g_s . \sqrt{2\pi\, g_s} \qquad \hat{m}^2 = m^2$$

From the brane-theory perspective, it is necessary to choose $\hat{m}$ and $\hat{\kappa}$ such that $\hat{m}^2 = \frac{4g_s^2\hat{\kappa}^2}{9}$. However we may also relax this condition and take $\lambda, \kappa$ and $m^2$ as independent parameters.

# Truncation to the SU(2) Sector:

$\Phi_i$ are *N X N* matrices and therefore we have $3N^2$ scalars. It makes the analysis very difficult 

However from the specific form of the potential and since we have three $\Phi_i$, it is possible to show that one can consistently restrict the classical dynamics to a sector with single scalar field:

$$\Phi_i = \hat{\phi}(t) J_i, \qquad i = 1,2,3$$

$J_i$ are *N* dim. irreducible representation of the SU(2) algebra:

$$[J_i, J_j] = i\varepsilon_{ijk} J_k \qquad \mathrm{Tr}\left(J_i J_j\right) = \frac{N}{12}\left(N^2 - 1\right)\delta_{ij}$$

Plugging these to the action, we have:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P}{2}R + \mathrm{Tr}J^2\left(-\frac{1}{2}\partial_\mu\hat{\phi}\,\partial^\mu\hat{\phi} - \frac{\lambda}{2}\hat{\phi}^4 + \frac{2\kappa}{3}\hat{\phi}^3 - \frac{m^2}{2}\hat{\phi}^2\right)\right] \qquad \mathrm{Tr}\left(J^2\right) \equiv \sum_{i=1}^{3}\mathrm{Tr}\left(J_i^2\right)$$

Defining $\phi \equiv \left(\mathrm{Tr}\,J^2\right)^{1/2}\hat{\phi}$ to make the kinetic term canonical, the potential takes the form

$$V_0(\phi) = \frac{\lambda_{eff}}{4}\phi^4 - \frac{2\kappa_{eff}}{3}\phi^3 + \frac{m^2}{2}\phi^2 \qquad \lambda_{eff} \equiv \frac{2\lambda}{\mathrm{Tr}\boldsymbol{J}^2} = \frac{8\lambda}{N(N^2-1)}, \qquad \kappa_{eff} \equiv \frac{\kappa}{\sqrt{\mathrm{Tr}\boldsymbol{J}^2}} = \frac{2\kappa}{\sqrt{N(N^2-1)}},$$

# Consistency of the Truncation to the SU(2) Sector

- SU(2) sector is a sector in which the computations are tractable. But is it consistent?

To see that let us defines

$$\Psi_i = \Phi_i - \hat{\phi} J_i \qquad \hat{\phi} = \frac{4}{N(N^2-1)} \mathrm{Tr}(\Phi_i J_i) \qquad \mathrm{Tr}(\Psi_i J_i) = 0$$

$$V = V_0(\phi) + V_{(2)}(\hat{\phi}, \Psi_i) \qquad V_{(2)}(\hat{\phi}, \Psi_i = 0) = 0 \qquad \left(\frac{\delta V_{(2)}}{\delta \Psi_i}\right)_{\Psi_i = 0} = 0$$

If we start with the initial consitions $\Psi_i = \dot{\Psi}_i = 0$ and $\hat{\phi} \neq 0$, $\Psi_i$ will remain zero.

- What is the special role of SU(2) generators among other *N X N* matrices?

$$\Phi_i = \Gamma_i - \Xi_i \qquad \mathrm{Tr}(\Gamma_i \Xi_i) = 0$$

$$V = V_0(\Gamma_i) + V_{(1)}(\Gamma_i, \Xi_i)$$

$$V_{(1)} = \mathrm{Tr}\Big[\big(-\lambda[\Gamma_i,[\Gamma_i,\Gamma_k]] + i\varepsilon_{ijk}[\Gamma_i,\Gamma_j]\big)\Xi_k\Big] + O(\Gamma^2)$$

To have $\Gamma_i$ -sector decoupled ⟹ $[\Gamma_i, \Gamma_j] = f_{ijk}\Gamma_k$ ⟹ Three $\Gamma_i$ should form a Lie-Algebra

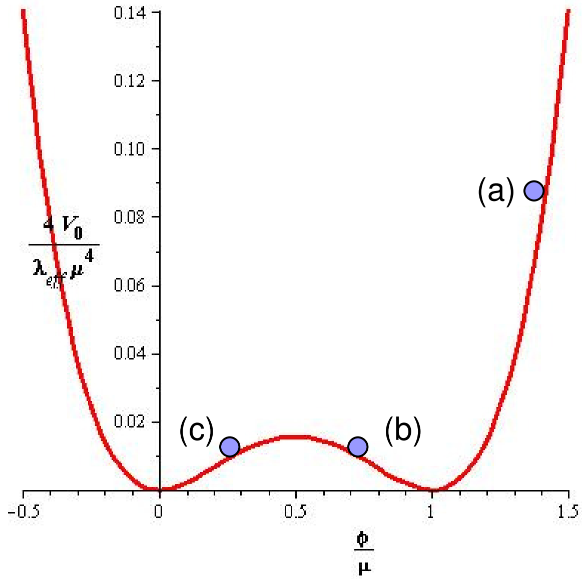
a) $f_{ijk} = i\varepsilon_{ijk}$ ⟹ $\Gamma_i$ are forming a $SU(2)$ algebra $\Phi_i = \sum_\alpha \phi_\alpha J_i^\alpha,$ $\quad i = 1,2,3 \quad N = \sum_\alpha N_\alpha$

b) $f_{ijk} = 0$ ⟹ $\Gamma_i$ are three Abelian subgroups of *U(N)* ⟹ No interesting inflationary dynamics

# Analysis of the Matrix Inflation around the Single-Block Vacuum

$$V(\phi) = \frac{\lambda_{\text{eff}}}{4}\phi^2(\phi-\mu)^2 \quad \phi_0 = \mu \equiv \frac{\sqrt{2}m}{\sqrt{\lambda_{\text{eff}}}}$$

Hill-top or Symmetry-Breaki
inflation, Linde (1992)
Lyth & Boubekeur (2005)




Note:This is exactly the same condition we had to satisfy to have our 10-d background be a supergravity solution.

In the stringy picture, we have *N* D3-branes that are blown up into a giant D5-brane under the influence of *RR* 6-form.

(a) $\phi_i > \mu$

$\phi_i \approx 43.57\, M_P$ $\quad \phi_f \approx 27.07\, M_P$ $\quad \mu \approx 26\, M_P$

$\lambda_{eff} \approx 4.91\times 10^{-14}$ $\quad m \approx 4.07\times 10^{-6}\, M_P$ $\quad \kappa_{eff} \approx 9.57\times 10^{-13}\, M_P$

$$\Delta\hat{\phi} \approx 10^{-7}\, M_P$$

↓

$$N \approx 10^5$$

(b) $\mu/2 < \phi_i < \mu$

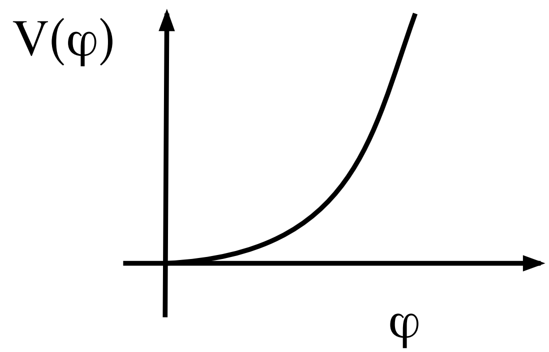
$\phi_i \approx 23.5\, M_P$ $\quad \phi_f \approx 35.03\, M_P$ $\quad \mu \approx 36 M_P$

$\lambda_{eff} \approx 7.18\times 10^{-14}$ $\quad m \approx 6.82\times 10^{-6}\, M_P$ $\quad \kappa_{eff} \approx 9.57\times 10^{-13}\, M_P$

(c) $0 < \phi_i < \mu/2$

Due to symmetry $\phi \to -\phi + \mu$ this inflationary region has the same properties as $\mu/2 < \phi_i < \mu$

$$\kappa^2 < 2m^2\lambda$$




Chaotic Inflationary scenarios with $\kappa = 0$ falls into this category:

a) $V = \frac{1}{2}m^2\phi^2$

To fit the WMAP data $\Delta\phi \approx 10\, M_P$

From EFT perspective super-Planckian excursions are problematic!

$\Delta\hat{\phi} \approx N^{-3/2}\Delta\phi$ ⟹ If $N >> 1$ ⟹ $\Delta\hat{\phi} << M_p^{-1}$

b) $V = \frac{1}{4}\lambda_{eff}\,\phi^4$

To fit the WMAP data $\lambda_{eff} \approx 10^{-14}$ and $\Delta\phi \approx 10\, M_P$

Assuming $\lambda \approx 1$ & $\lambda_{eff} = \frac{8\lambda}{N(N^2-1)}$, one needs $N \approx 10^5$

$$\Delta\hat{\phi} \approx 10^{-7} M_P$$

Such value of quartic coupling and field displacement are unnatural from EFT perspective!

# Mass Spectrum of $\Psi_\iota$ Modes in Matrix Inflation

The other $3N^2-1$ even though classically frozen, have quantum fluctuations. To compute these effects, let us calculate the mass spectrum of these modes. Expanding the action up to second order, we have:

$$V_{(2)} = \mathrm{Tr}\left[\frac{\lambda}{2}\hat{\phi}^2\Omega_i\Omega_i + \frac{m^2}{2}\Psi_i\Psi_i + \left(-\frac{\lambda}{2}\hat{\phi}^2 + \kappa\hat{\phi}\right)\Psi_i\Omega_i\right]$$

where

$$\Omega_k \equiv i\,\varepsilon_{ijk}\left[J_i, \Psi_j\right]$$

If we have the eigenvectors of the $\Omega_i$

$$\Omega_i\Psi_i = \omega\Psi_i$$

$$V_2 = \left(\frac{\lambda_{eff}}{4}\phi^2(\omega^2-\omega) + \kappa_{eff}\,\omega\phi + \frac{m^2}{2}\right)\mathrm{Tr}\,\Psi_i\Psi_i$$

It turns out that finding the eigenvectors of $\Omega_i$ is mathematically the same as finding the the vector spherical harmonics:

Dasgupta, Sheikh-Jabbari & Von Raamsdonk (2002)

# Mass Spectrum of $\Psi_\iota$ Modes in Matrix Inflation

(a) $N^2-1$ zero modes with $\omega=-1$

$$M^2=\lambda_{eff}\phi^2-2\kappa_{eff}\phi+m^2=\frac{V'}{\phi}$$

$\Lambda$ is an arbitrary traceless matrix

$$\Psi_i=[J_i,\Lambda]$$

(b) $(N-1)^2-1$ $\alpha$ -modes: $\omega=-(l+1),\quad l\in Z\quad 1\le l\le N-2$

Degeneracy of each $l$-mode is $2l+1$

$$M_l^2=\frac{1}{2}\lambda_{\text{eff}}(l+2)(l+3)\phi^2-2\kappa_{\text{eff}}(l+2)+m^2$$

(c) $(N+1)^2-1$ $\beta$-modes: $\omega=l,\quad l\in Z\quad 1\le l\le N$

Degeneracy of each $l$-mode is $2l+1$

$$M_l^2=\frac{1}{2}\lambda_{\text{eff}}(l-2)(l-1)\phi^2+2\kappa_{\text{eff}}(l-1)+m^2$$

$$M^2=\frac{\lambda_{eff}}{2}\phi^2(\omega^2-\omega)+2\kappa_{eff}\,\omega\phi+m^2$$

$$=V_0''(\omega+1)^2-\frac{V_0'}{\phi}(4\omega+3)(\omega+2)+\frac{V_0}{\phi^2}$$

# Power Spectra in the Presence of $\Psi_{r,lm}$ Modes

$$L = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}\partial_\mu\Psi^*{}_{r,lm}\,\partial^\mu\Psi_{r,lm} - V_0(\phi) - \frac{1}{2}M^2_{r,lm}(\phi)\Psi^*{}_{r,lm}\Psi_{r,lm} \qquad r = 0, \alpha, \beta$$

If you start from the initial condition $\Psi_{r,lm}=\dot{\Psi}_{r,lm}=0$ , they remain zero. Therefore the inflationary trajectory is a straight line in the field space and there is no cross-correlation between adiabatic and entropy spectra.

Mukhanov-Sasaki variable

$$\ddot{Q}_\phi + 3H\dot{Q}_\phi + \frac{k^2}{a^2}Q_\phi + \left(V_{0,\phi\phi} - \frac{1}{a^3M_P^2}\left(\frac{a^3}{H}\dot{\phi}^2\right)^{\cdot}\right)Q_\phi = 0; \qquad Q_\phi \equiv \delta\phi + \frac{\dot{\phi}}{H}\Phi$$

$$\delta\ddot{\Psi}_{r,lm} + 3H\,\delta\dot{\Psi}_{r,lm} + \left(\frac{k^2}{a^2} + M_{r,l}(\phi)^2\right)\delta\Psi_{r,lm} = 0 \qquad \Re = \frac{H}{\dot{\phi}}Q_\phi \qquad S_{r,lm} = \frac{H}{\dot{\phi}}\Psi_{r,lm}$$

$$\dot{\Re} = \frac{H}{\dot{H}}\frac{k^2}{a^2}\Phi$$

→ scalar metric perturbations in longitudinal gauge

$$P_{Q_\phi} = \frac{k^3}{2\pi^2}\delta^3(\mathbf{k}-\mathbf{k}')\left\langle Q^*{}_{\phi\mathbf{k}}Q_{\phi\mathbf{k}'}\right\rangle \qquad P_{\Psi_{r,lm}} = \frac{k^3}{2\pi^2}\delta^3(\mathbf{k}-\mathbf{k}')\left\langle \Psi^*{}_{r,lm\mathbf{k}}\ \Psi^*{}_{r,lm\mathbf{k}'}\right\rangle$$

$$C_{\psi^i Q_Q} = \frac{k^3}{2\pi^2}\delta^3(\mathbf{k}-\mathbf{k}')\left\langle Q^*{}_{\phi\mathbf{k}}\Psi_{r,lm}\right\rangle = 0$$

# Power Spectra in Symmetry-Breaking Inflation $\phi > \mu$

$\lambda_{eff} \approx 4.91\times10^{-14}$ $m \approx 4.07\times10^{-6} M_P$ $\kappa_{eff} \approx 9.57\times10^{-13} M_P$ $\Longrightarrow$ $n_{\Re} \approx 0.959$

| | | | | |
|---|---|---|---|---|
| Zero mode | $\lambda_{eff}\phi^2 - 2\kappa_{eff}\phi + m^2$ | $1.162 \times10^{-11}$ | $0.981$ | $N^2$ |
| $l = 1$ $\beta$ | $m^2$ | $1.131 \times10^{-12}$ | $0.978$ | $3$ |
| $l = 2$ $\beta$ | $2\kappa_{eff}\phi + m^2$ | $8.842 \times10^{-18}$ | $1.002$ | $7$ |

$N^2$ zero modes can be removed by gauge transformations

$P_T(k_{60}) \approx 4.84\times10^{-10}$ $\Longrightarrow$ $r \approx 0.2$

$n_T \approx -0.025$

Planck should be able to verify this model.

# Power Spectra in Symmetry-Breaking Inflation $\mu/2 < \phi < \mu$

$\lambda_{eff} \approx 7.187\times10^{-14}$ $m \approx 6.824\times10^{-6} M_P$ $\kappa_{eff} \approx 1.940\times10^{-12} M_P$ ⟹ $n_{\Re} \approx 0.961$

| | | | | |
|---|---|---|---|---|
| Zero mode | $\lambda_{eff}\phi^2 - 2\kappa_{eff}\phi + m^2$ | $1.46 \times10^{-11}$ | $0.987$ | $N^2$ |
| $l = 1$ $\beta$ | $2\kappa_{eff}\phi + m^2$ | $6.55 \times10^{-16}$ | $1.0545$ | $3$ |
| $l = 1$ $\alpha$ | $6\lambda_{eff}\phi^2 - 6\kappa_{eff}\phi + m^2$ | $4.69 \times10^{-19}$ | $1.007$ | $3$ |

$N^2$ zero modes can be removed by gauge transformations

$P_T(k_{60}) \approx 1.307\times10^{-11}$ ⟹ $r \approx 0.048$

$n_T \approx -0.006$

CMBPOL or QUIET should be able to verify this scenario.

# Power Spectra in Symmetry-Breaking Inflation $0 < \phi < \mu/2$

The potential is invariant under symmetry $\phi \to -\phi + \mu$

$\lambda_{eff} \approx 7.187\times10^{-14}$ $m \approx 6.824\times10^{-6} M_P$ $\kappa_{eff} \approx 1.940\times10^{-12} M_P$ ⟹ $n_{\mathfrak{R}} \approx 0.961$

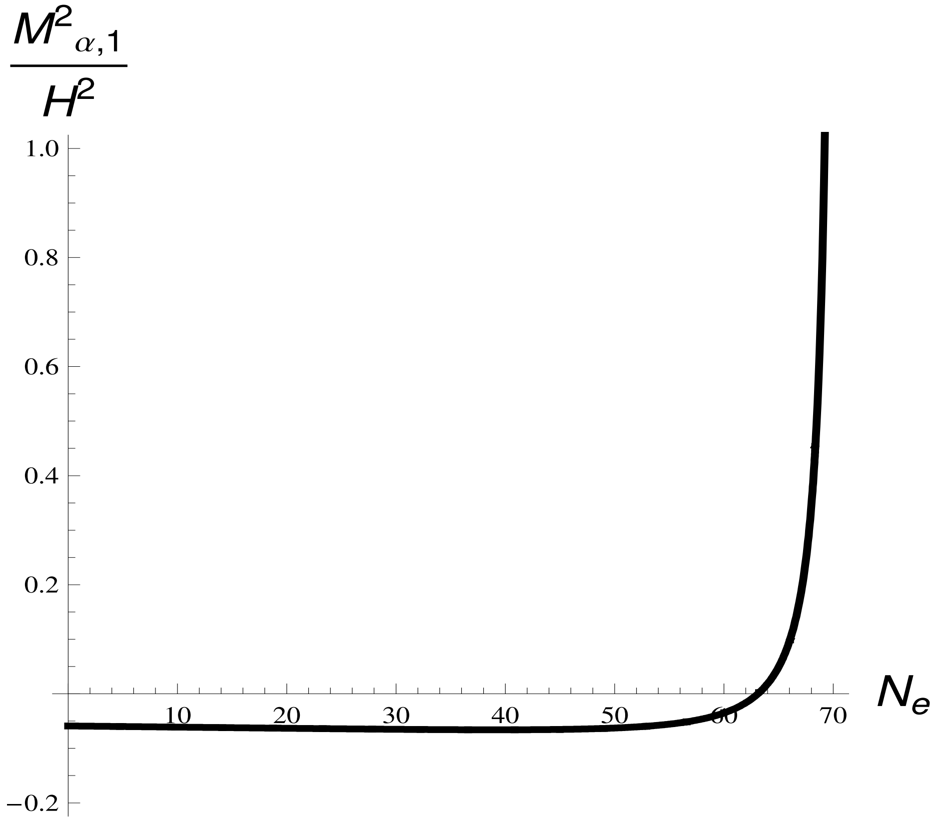
| | | | | |
|---|---|---|---|---|
| Zero mode | $\lambda_{eff}\phi^2 - 2\kappa_{eff}\phi + m^2$ | $3.84 \times10^{-14}$ | $1.006$ | $N^2$ |
| $l = 1$ $\alpha$ | $6\lambda_{eff}\phi^2 - 6\kappa_{eff}\phi + m^2$ | $1.23 \times10^{-11}$ | $0.953$ | $3$ |
| $l = 1$ $\beta$ | $2\kappa_{eff}\phi + m^2$ | $6.558 \times10^{-16}$ | $1.054$ | $3$ |

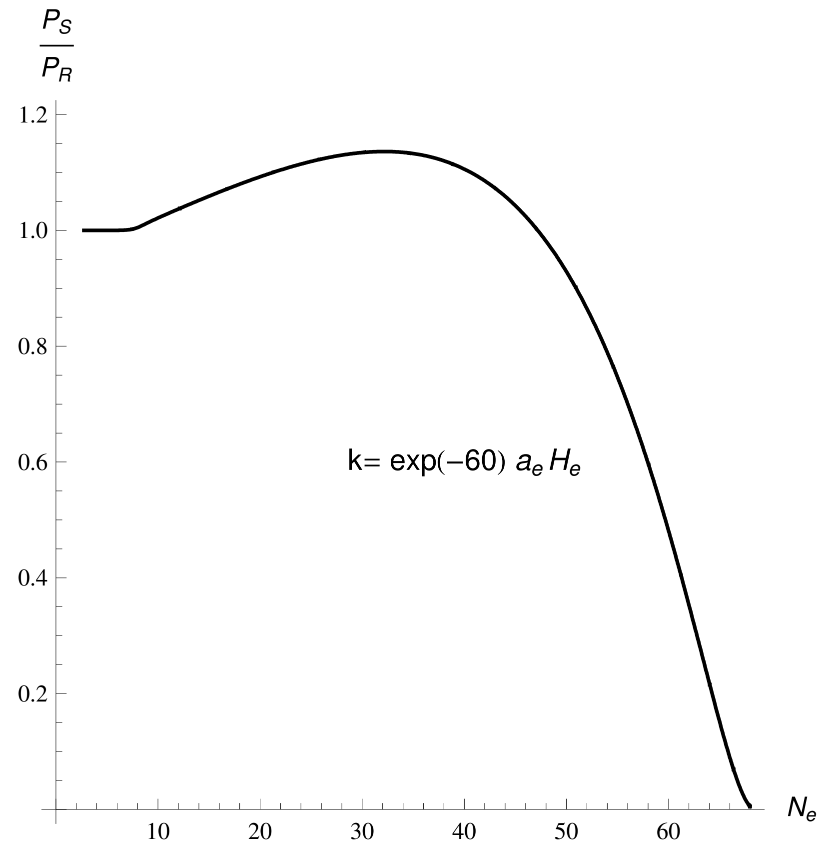
$N^2$ zero modes can b removed by gauge transformations

$P_T(k_{60}) \approx 1.307\times10^{-11}$ ⟹ $r \approx 0.048$

$n_T \approx -0.006$

CMBPOL or QUIET should be able to verify this scenario.


$\frac{M^2{}_{\alpha,1}}{H^2}$
1.0
0.8
0.6
0.4
0.2
−0.2
10
20
30
40
50
60
70
$N_e$


$\frac{P_S}{P_R}$
1.2
1.0
0.8
0.6
0.4
0.2
k= exp(−60) $a_e H_e$
10
20
30
40
50
60
$N_e$

# Particle Creation and Preheat Scenario

Even though one can show that backreaction of $\Psi_{r,lm}$ on the inflaton dynamics is not large,

It can become important when $\varepsilon, \eta \approx 1$

This could be the bonus of our model, as $\Psi_{r,lm}$ modes help to drain the energy of the inflaton, since their masses are changing very fast.

Preheating in the case of symmetry-breaking inflation is in progress! Ashoorioon & J.T. Giblin in preperation

We will focus on $\lambda\phi^4/4$ theory, as its preheating has already been worked out by Greene, Kofman, Linde & Starobinsky (1997)

$$V_{\text{eff}}(\phi,\chi) = \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$$

The structure of parametric resonance is completely determined by $g^2/\lambda$

For $g^2/\lambda = n(n+1)/2$ we have an enhancement in the parametric resonance leading to creation of $\chi$ particles

- $n \in 2Z+1$ particle creation is peaked around $k=0$ $\mu_k \propto \ln n_k$ ~0.15
- $n \in 2Z$ particle creation is peaked around $k^2 = \frac{3}{2}H_{\text{inf}}^2\varepsilon\sqrt{\frac{g^2}{2\lambda}}$ $\mu_k$ ~0.5

# Particle Creation and Preheat Scenario

$$V(\phi,\Psi_{r,lm}) = \frac{1}{4}\lambda_{eff}\phi^4 + \frac{1}{2}\lambda_{eff}\phi^2 \sum_{r,lm} \frac{1}{2}(\omega^2-\omega)\Psi^{\star}_{r,lm}\Psi_{r,lm}$$
$$= \frac{1}{4}\lambda_{eff}\phi^4 + \frac{1}{2}\lambda_{eff}\phi^2 \sum_{m=1}^{N^2} |\Psi_{0m}|^2$$
$$+ \frac{1}{2}\lambda_{eff}\phi^2 \left[ \sum_{l=0}^{N-1} \frac{l(l+1)}{2} \sum_{m=1}^{2l+1} |\Psi_{\alpha,\ lm}|^2 + \sum_{l=1}^{N-1} \frac{l(l-1)}{2} \sum_{m=1}^{2l+1} |\Psi_{\beta,\ lm}|^2 \right]$$

In this case $g^2/\lambda = n(n+1)/2$ with $n = 1, l, l-1$ respectively for zero, $\alpha$ and $\beta$ modes

- For zero mode, odd $l$ $\alpha$-mode and even $l$ $\beta$ – mode. $k=0$ $\mu_k$~0.15
- For even $l$ $\alpha$-mode and odd $l$ $\beta$ – mode. $k^2 = \frac{3}{4} H_{\inf}^2 \varepsilon \sqrt{l(l+1)}$ $\mu_k$~0.5

This means that large $l$ even $\alpha$-mode and odd $l$ $\beta$ –mode makes the biggest contribution to preheating. Also one should note that their preheating will be more effective here due to their *2l+1* degeneracy

$$N^2T^4 \sim 3H^2M_P^2 \xrightarrow[N\sim 10^5]{H\sim 10^{-5}M_P} T \sim 10^{13}\ \text{Gev}$$

# UV behavior of Matrix Inflation in presence of many species

In a theory with many particles the scale where quantum gravity effects become large is lowered to

$$\Lambda^2 = \frac{M_P^2}{N_{\rm cl}},$$

Dvali (2007)

$N_{\rm cl}$ : species with mass below the cutoff, $\Lambda$

In matrix inflation:

$$N_{\rm cl} = 3N^2$$

Let's compare the amount of excursions of the "physical" inflaton is less than $\Lambda$ when inflation happens in region $\phi > \mu$

$$\phi_i \approx 43.57\, M_P \qquad \phi_f \approx 27.07\, M_P \qquad \mu \approx 26\, M_P$$

$$\lambda_{eff} \approx 4.91\times 10^{-14} \qquad m \approx 4.07\times 10^{-6}\, M_P \qquad \kappa_{eff} \approx 9.57\times 10^{-13}\, M_P$$

To find the number of species in our case, we assume that $\lambda = 1$ .

$$\lambda_{eff} \equiv \frac{8\lambda}{N(N^2-1)} = 4.91\times 10^{-14} \Longrightarrow N = 54618$$

$$\Delta\hat{\phi} = \frac{\phi_f - \phi_i}{\sqrt{N(N^2-1)}} = 2.58\times 10^{-6}\, M_P < \Lambda = 1.05\times 10^{-5}\, M_P$$

Also, one should notice that the mass parameter of the inflaton turns out to be about 40% of the cutoff.

## Analysis of the potential for n-block vacua:

In this case, we have n-field inflationary, $\phi_\alpha$, $\alpha = 1..n$ that are only gravitationally coupled

$$H^2 = \frac{1}{3M_P^2}\sum_{\alpha=1}^{n}\left(\frac{1}{2}\dot{\phi}_\alpha^2 + V_\alpha(\phi_\alpha)\right) \qquad V(\phi_\alpha) = \sum_\alpha \frac{\lambda_\alpha}{4}\phi_\alpha^4 - \frac{2\kappa_\alpha}{3}\phi_\alpha^3 + \frac{m^2}{2}\phi_\alpha^2$$

$$\ddot{\phi}_\alpha + 3H\dot{\phi}_\alpha + \partial_{\phi_\alpha} V_\alpha = 0 \ , \qquad \lambda_\alpha = \frac{8\lambda}{N_\alpha(N_\alpha^2 - 1)} \ , \qquad \kappa_\alpha = \frac{2\,\kappa}{\sqrt{N_\alpha(N_\alpha^2-1)}} .$$

The classical (inflationary) dynamics around the "multi-giant vacua" decouple from each other and one may build an inflationary model around either of these.

If we start with a field which is initially in the sector specified by a given set of $\{N_\alpha\}$, then $N_\alpha$ remains a conserved quantity by the classical trajectory of the system. In general various fields in the same sector specified by a set of $N_\alpha$ can mix with each other.

⇓

That is, in general the inflationary trajectory in the space of $\phi_\alpha$ is curved.

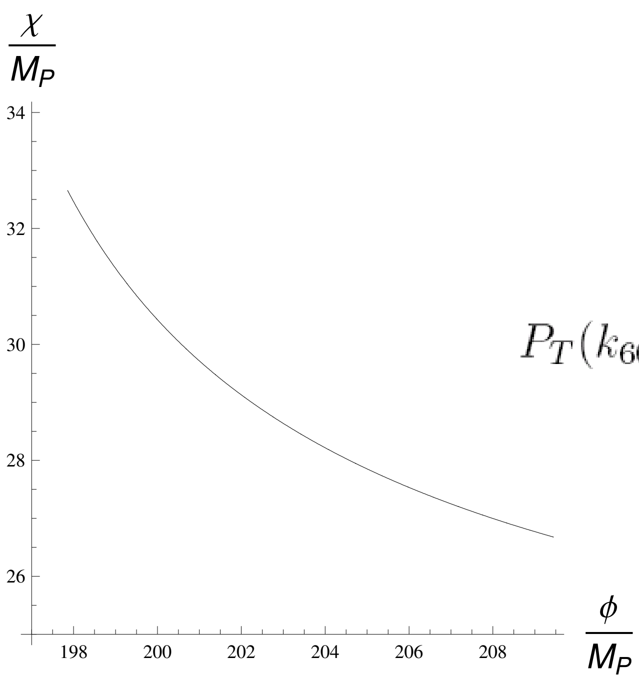
## Analysis of the potential for n-block vacua:

Various scenarios can occur in such a n-field inflationary model. Let us consider one two-block example that can arise in Matrix Inflation with

$$V = \frac{\lambda_\phi}{4}\phi^2(\phi-\mu_\phi)^2 + \frac{\lambda_\chi}{4}\chi^2(\phi-\mu_\chi)^2$$

$$\lambda_\phi = 2\times10^{-15}, \quad \mu_\phi = 196.168 M_P, \quad \mu_\chi = 36 M_P, \quad \lambda_\chi = \lambda_\phi\left(\frac{\mu_\phi}{\mu_\chi}\right)^2$$

$$\phi_i = 209.439\ M_P \quad , \quad \chi_i = 26.678\ M_P$$




$$P_S(k_{60}) \approx 4.4\times10^{-17} \quad n_S \approx 0.987 \quad C \approx 0.228$$

$$P_T(k_{60}) = 2.618\times10^{-10},\ i.e.\ r \simeq 0.107,\ \text{and}\ n_T \simeq -0.041$$

## Conclusions

• Matrix inflation is an interesting realization of inflation which is strongly supported from string theory. Matrix inflation can solve the fine-tunings associated with chaotic inflation and produce super-Planckian effective field excursions during inflation.

• Due to Matrix nature of the fields there would be many scalar fields in the model. This leads to the production of isocurvature productions at the CMB scales.

• Matrix inflation has a natural built-in mechanism of preheating to end inflation.

• In particular, if there is an isocurvature component (at a level still allowed by present data) but it is ignored in the CMB analysis, the sound horizon and cosmological parameters determination is biased, and, as a consequence, future surveys may incorrectly suggest deviations from a cosmological constant.

Take Isocurvature Perturbations Seriously!

A. Milligen, L. Verde, M. Beltran
arXiv:1006.3806 [astro-ph.CO]

*Thank you*