PBH, NON-THERMAL EARLY UNIVERSE, AND GRAVITATIONAL WAVES

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Modified cosmological thermal history: matter dominance (by nonrelativisitc PBH) in the early universe, all heavy relics are noticeably diluted.

Early period of structure formation at very small scales.

Universe heating by PBH evaporation, 2nd RD stage, "return to normality".

Possibly observable GW induced by PBH scattering and binaries.

Characteristic values:

$$\tau_{BH} < t_{BBN},$$
$$M_{BH} < 10^{10} g,$$
$$f \geq 10^{10}\, Hz.$$

Based on:
A.D. Dolgov, P.D. Naselsky,
I.D. Novikov, e-Print: astro-ph/0009407;
A.D. Dolgov, D. Ejlli, work in progress.

I. Cosmological story of PBH.

PBHs are formed if the density contrast at horizon scale is of the order of unity, $\delta\rho/\rho \sim 1$. Hence PBHs formed at cosmological time t_p, have masses:

$$M = t_p m_{Pl}^2, \quad t_p = r_g/2$$

where $r_g = 2M/m_{Pl}^2$ and

$$m_{Pl} = 1.22 \times 10^{19} GeV \approx 2.18 \times 10^{-5} g.$$

Relative cosmological energy density of BHs at production is

$$\Omega_{BH}(t_p) \equiv \Omega_p,$$

model dependent parameter.
Normally $\Omega_p \ll \Omega_{tot} \approx \Omega_R \approx 1$, thus the universe was at RD stage before and after production of BH with

$$\rho = 3m_{Pl}^2/\left(32\pi t^2\right),$$

till BH started to dominate, if they lived long enough.

Mass density of BH: $\rho_{BH} = n_{BH} M$.
Define the average distance between BHs as $d = n_{BH}^{-1/3}$.
Express the average distance at production d_p **through** t_p **and** Ω_p:

$$\Omega_p = \frac{\rho_p}{\rho_c} = \frac{32\pi t_p^2 M n_p}{3 m_{Pl}^2} = \frac{32\pi}{3} \left(\frac{t_p}{d_p}\right)^3$$

and $d_p = (4\pi/3)^{1/3} r_g \Omega_p^{-1/3}$.

Prior to BH decay $n_{BH}(t)a^3 = const$, if the coalescence can be neglected. Hence at RD stage:

$$n_{BH}(t) = n_p \left(\frac{a_p}{a(t)}\right)^3 = n_p \left(\frac{t_p}{t}\right)^{3/2},$$

while at MD stage

$$n_{BH}(t) = n_p \left(\frac{t_p}{t_{eq}}\right)^{3/2} \left(\frac{t_{eq}}{t}\right)^2,$$

where t_{eq} is the onset of BH dominance, if $t_{eq} > \tau_{BH}$.

Mass fraction of BH as a function of time behaves as

$$\Omega_{BH}(t) = \frac{n_{BH}(t)M}{\rho_c} = \frac{16\pi}{3} r_g t^2 n_{BH}(t)\,,$$

i.e. $\Omega_{BH} \sim t^{1/2}$ at RD stage. When $\Omega_{BH} \sim 1,$ MD stage started which lasted till BHs evaporated and $\Omega_{BH} \to 0.$

Condition of BH dominance (decay after onset of the MD stage).

$$\Omega_{BH} = \Omega_p \left(t/t_p\right)^{1/2}$$

Hence $\Omega_{BH} \sim 1$ at $t_{MD} = t_p/\Omega_p^2$.
Necessary: $t_{MD} < \tau_{BH} \sim M^3/m_{Pl}^4$.
Thus:

$$M > m_{Pl}/\Omega_p\,.$$

II. Initial GW emission.

BHs start to interact and thus to radiate GW when they enter inside horizon of each other. The corresponding time moment is t_h is determined by the condition $2t_h = d(t_h)$ so:

$$d_p(t_h/t_p)^{1/2} = t_h,$$

Correspondingly:

$$t_h = \frac{1}{2}\left(\frac{4\pi}{3}\right)^{2/3} r_g \Omega_p^{-2/3}.$$

For $t > t_h$, the curvature effects can be neglected and the BH motion is determined by the Newtonian gravity:

$$\ddot{r} = -\frac{M}{m_{Pl}^2 r^2}$$

with the initial condition $r_{in} = d(t_{in})$, where $d(t)$ is the average distance between BHs. E.g. at RD stage:
$d(t) = d_p(a_p/a(t)) = d_p(t_p/t)^{1/2}$,
and initial velocity equal to the Hubble one, $\dot{r}_{in} = H(t_{in})r_{in}$.

For $t_{in} = t_h$: $\dot{r}_{in} = c = 1$ and the non-relativistic approximation is invalid. To avoid that and we should choose $t_{in} > t_h$ such that $v_{in} \ll 1$.
The solution of the equation of motion demonstrates that the effects of mutual attraction at this stage and production of GW are weak.

The acceleration of BH towards each other would be noticeable when their Hubble velocity becomes smaller than the capture velocity. The corresponding time moment, t_c, when it started is determined from the condition:

$$\frac{1}{2}v_H^2 \equiv \frac{1}{2}(H_c d_c)^2 = \frac{M}{m_{Pl}^2 d_c},$$

where sub-index "c" means that the corresponding quantity is taken at t_c.

At RD-stage we obtain:

$$\frac{d_p^3}{4t_c^2}\left(\frac{t_c}{t_p}\right)^{3/2} = r_g$$

and thus:

$$t_c = \frac{8\pi^2}{9}\frac{r_g}{\Omega_p^2}.$$

However, the mass fraction of BHs at $t = t_c$ is:

$$\Omega_c = \Omega_p \left(\frac{t_c}{t_p} \right)^{1/2} = \frac{4\pi}{3} > 1 \, .$$

Thus at $t = t_c$ the universe is already matter dominated and we need to use another law for the scale factor, namely $a \sim t^{2/3}$.

Let us find time t_{eq} when the energy density of relativistic matter and non-relativistic BHs became equal. This can be found from the condition $\Omega_{BH}(t_{eq}) = \Omega_p \left(t_{eq} t_p\right)^{1/2} = 1$ and thus:

$$t_{eq} = \frac{t_p}{\Omega_p^2} = \frac{r_g}{2\Omega_p^2}.$$

We take $\Omega_{BH}(t_{eq}) = 1/2$, or even better, solve analytically the Friedman equations...

Recalculate now t_c with an account of the change of expansion regime from RD to MD. The average distance between BHs when $t > t_{eq}$, behaves as:

$$d(t) = d_p \left(\frac{t_{eq}}{t_p}\right)^{1/2} \left(\frac{t}{t_{eq}}\right)^{2/3} .$$

Now we find that the condition that the Hubble velocity is smaller than the virial one, for average values, reads:

$$\frac{4d_p^3}{9r_g t_p^{3/2} t_{eq}^{1/2}} < 1\,.$$

Here we took $H(t_c) = 2/(3t_c)$. But this condition is never fulfilled.

However, this negative result does not mean that the acceleration of BHs and GW emission are suppressed.

III. Rising perturbations and GW.
At MD stage primordial density perturbations rise as $\Delta \equiv \delta\rho/\rho \sim a(t)$. For sufficiently long MD stage, Δ would reach unity and after that quickly rises to $\Delta \gg 1$. Density of PBH grows and GW emission is strongly amplified.

The regions with high n_{BH} would emit GW much more efficiently than in the homogeneous case. The emission of GW is proportional to vn_{BH}^2 and, both the BH velocity in dense regions and n_{BH} would be by several orders of magnitude larger than those in the homogeneous universe.

The BH life-time, τ_{BH}, must be large enough, so that the density fluctuations in BH matter would rise up to the values of the order unity:

$$\left(\frac{\delta\rho}{\rho}\right)_{in} \left(\frac{\tau_{BH}}{t_{eq}}\right)^{2/3} \sim 1\,,$$

where $(\delta\rho/\rho)_{in} \sim 10^{-5} - 10^{-4}$ is the magnitude of primordial density perturbations, not necessarily true at small scales.

The life-time of BH w.r.t. evaporation is

$$\tau_{BH} \approx \frac{3 \times 10^3}{N_{eff}} \frac{M^3}{m_{Pl}^4},$$

where $N_{eff} \sim 100$ is the number of species with $m < T_{BH} = m_{Pl}^2/(8\pi M)$.

Perturbations would become large if

$$\frac{M}{m_{Pl}} > 2 \cdot 10^7 \left(\frac{10^{-6}}{\Omega_p}\right) \sqrt{\frac{10^{-4}}{\Delta_{in}} \frac{N_{eff}}{100}}\,.$$

After Δ reached unity, rapid structure formation would take place; violent relaxation with non-dissipating dark matter.

As a guiding example take the size of the high density bunch of BHs equal to horizon at $t = t_{eq}$ and the density contrast equal $\Delta = 10^6$ as in contemporary galaxies.

The size of the bunch at $t = \tau_{BH}$:

$$R_b = 2t_{eq} \left(\frac{\tau_{BH}}{t_{eq}} \right)^{2/3} \Delta_b^{-1/3},$$

where $\Delta_b = \rho_b/\rho_c$ and ρ_c and ρ_b are the average cosmological energy density and the density of BHs in the bunch. Thus:

$$R_b = \frac{0.2\Omega_p^{-\frac{2}{3}}}{m_{Pl}} \left(\frac{M}{m_{Pl}} \right)^{\frac{7}{3}} \left(\frac{100}{N_{eff}} \right)^{\frac{2}{3}} \left(\frac{10^6}{\Delta_b} \right)^{\frac{1}{3}}$$

Such high density bunch of PBH would have mass:

$$M_b = \frac{16}{9} m_{Pl}^2 t_{eq} = \frac{M}{\Omega_p^2},$$

i.e. the mass inside horizon at $t = t_{eq}$.
The virial velocity inside this bunch is

$$v = \sqrt{\frac{2M_b}{m_{Pl}^2 R_b}} = \frac{4}{3}\Delta^{\frac{1}{6}} \left(\frac{m_{Pl}}{\Omega_p M}\right)^{\frac{2}{3}} \left(\frac{N_{eff}}{3000}\right)^{\frac{1}{3}}$$

Maximum velocity in the bunch is limited by the condition of sufficiently large M i.e. that $\Delta \equiv \delta\rho/\rho \geq 1$ (p. 24) and reads:

$$v_{max} \approx 0.06 \Delta^{1/6} \left(\frac{\Delta_{in}}{10^{-4}} \right)^{-1/3}$$

and with Δ as large as 10^6 BHs can be moderately relativistic.

IV. Bremsstrahlung of gravitons.

Bakker, Gupta, Kaskas, Phys. Rev. 182 (1969), 1391:

$$d\sigma = \frac{64M_1^2 M_2^2}{15m_{Pl}^6}\left(5 + \frac{3}{2}ln\frac{4E_{kin}}{\omega}\right)\frac{d\omega}{\omega},$$

where $E_{kin} = Mv^2/2$, BHs are non-relativistic, and $M_1 \gg M_2$.

Weizsäcker-Williams approximation does not work.

However we use this result for an order of magnitude estimate if $M_1 \sim M_2$ and approximate

$$\langle \sigma \omega \rangle = \int_0^{\omega_{max}} d\sigma \omega \approx Q \frac{M^4 \omega_{max}}{m_{Pl}^6},$$

where $Q \sim 10^2 - 10^3$ is a numerical coefficient.

NB: The Sommerfeld enhancement is not taken into account.

The rate of GW emission due to bremsstrahlung is:

$$\dot{\rho}_{GW} = \langle \sigma \omega \rangle v n_{BH}^2$$

The effective time of GW emission is of the order of cosmological time, so we take $\rho_{GW} = t_e \dot{\rho}_{GW}$, where t_e is the time of emission, which will be taken equal to τ_{BH}.

Thus the cosmological energy density of GW at the time of emission is:

$$\Omega_{GW}(t_e) = \langle \sigma\omega \rangle v t_e \Delta \frac{\Omega_{BH}^2(t_e)\rho_c(t_e)}{M^2},$$

where $\rho_c(t_e) = m_{Pl}^2/(6\pi t_e^2)$ **and** $\Omega_{BH}(t_e) \approx 1$.

Taking $t_e = \tau_{BH}$, we find:

$$\Omega_{GW}(\tau_{BH}) = \frac{QN_{eff}}{18\pi \cdot 10^3}\frac{\omega_{max}}{M} v\Delta\,.$$

The first numerical factor is of order unity, ω_{max} can be about $r_g^{-1} \sim m_{Pl}^2/M$, $v \lesssim 0.1$, and $\Delta \sim 10^6$. So

$$\Omega_{max}(\tau_{BH}) \sim 10^5 \left(\frac{m_{Pl}}{M}\right)^2$$

NB: recall the bounds $m_{Pl}/M < \Omega_p/20$ and $m_{Pl}/M < 10^{-7}(\Omega_p/10^{-6})$.

Maximum value of ω is determined by the sensitivity of GW detectors. The red-shifted ω today:

$$\omega_0 = \omega \frac{T_0}{T(\tau_{BH})}.$$

$T(\tau_{BH})$ is found approximately from:

$$\rho = \frac{m_{Pl}^2}{6\pi\tau_{BH}^2} = \frac{\pi^2 g_* T^4}{30},$$

where $g_* = g_*(T) \approx 10^2$ is the number of particle species at this T.

$$T(\tau_{BH}) = \left(\frac{30}{6\pi g_*}\right)^{\frac{1}{4}} \left(\frac{N_{eff}}{3\cdot 10^3}\right)^{\frac{1}{2}} \frac{m_{Pl}^{\frac{5}{2}}}{M^{\frac{3}{2}}}.$$

Substituting numbers:

$$T(\tau_{BH}) \approx 0.06 m_{pl} \left(\frac{m_{Pl}}{M}\right)^{3/2}.$$

For comparison at production moment:

$$T_p \approx 0.2 m_{Pl} \left(\frac{m_{Pl}}{M}\right)^{1/2}.$$

The frequency of such GW today:

$$\omega_0 = 2.7^o \left(\frac{M}{m_{Pl}}\right)^{3/2} \frac{\omega}{0.06 m_{Pl}}.$$

If we take maximum $\omega \sim m_{Pl}^2/M$, the GW frequency today would be:
$\omega_0 \sim (6 \cdot 10^{12}/s)\,(M/m_{Pl})^{1/2}$,
i.e. for $M = 10^5 g$, $\omega_0 \sim 1$ keV.
Usually results are expressed in terms of $f = \omega/(2\pi)$.

Now we can calculate Ω_{GW} today as a function of measured $f = \omega_0/(2\pi)$:

$$\Omega_{GW}^{(0)} = 10^{-5} v \Delta \left(\frac{f}{10^{11} Hz} \right) \left(\frac{m_{Pl}}{M} \right)^{5/2} .$$

In this expression the factor 10^{-4} is included which comes from dilution of relativistic gravitons at the late MD stage and it is assumed that $QN_{eff}/(18 \cdot 10^3 \pi) \approx 1$.

As we saw above, $10^{-5} v\Delta \sim 1$.
However, this product may be larger due to possible Sommerfeld enhancement and central cusps in BH bunches.
The existing and near-future detectors are not sensitive to such GW but Ultimate DECIGO (2035), which will be sensitive to $\Omega = 10^{-20}$ at $f = 1$ Hz may put the limit:

$$M > 10^{3.6} m_{Pl}$$

or discover them.

High frequency detectors, not of existing projects, but of next generations may have better chance.

V. Classical (in contrast to quantum) emission of gravitational waves:

$$\frac{dE}{dt} = \frac{1}{45 m_{Pl}^2} \frac{d^3 D}{dt^3},$$

where $D \sim M r^2$ is the quadrupole moment of two colliding BHs.

We calculate D using equation of motion:

$$\ddot{r} = -\frac{M}{m_{Pl}^2 r^2}.$$

Convenient variables:

$$z = r/R, \;\; t = m_{Pl} R \sqrt{r/M}\, t$$

where R is the initial value of the distance between black holes taken to be equal to the average distance.
In this variables the equation of motion takes a very simple form:

$$z'' = -1/z^2 \, ,$$

where prime means differentiation w.r.t. dimensionless time η.

The estimated energy loss in a collision of two black holes:

$$\delta E = \frac{M}{315}\left(\frac{r_g}{r_{min}}\right)^{7/2},$$

where r_{min} is the minimal distance between the black holes during the collision.

A reasonable estimate $r_{min} = Sr_g$,, where S is a numerical factor larger than 1 but probably smaller than 10. For $r \sim r_g$ our flat-space estimates are invalid.

The number of collisions per unit time and volume is:

$$\dot{n}_{coll} = \sigma n_{BH}^2 v\,,$$

where σ is the cross-section of the collision, which we take as $\sigma = S^2 r_g^2$; probably it is an underestimate.

Now we can evaluate the cosmological fraction of energy of the produced gravitational waves:

$$\Omega_{GW} = \frac{4S^{-3/2}}{1890\pi K} \left(\frac{m_{Pl}}{M} \right)^2 \Omega^2_{BH} v\Delta$$

Taking into account that $\Omega_{BH} = 1$, $v\Delta \sim 10^{-5}$, $K = N_{eff}/3000 \approx 0.03$, and that relativistic matter is diluted by 10^4 at the contemporary matter dominated stage, we obtain:

$$\Omega^{(0)}_{GW} \approx 0.3 S^{-3/2} (m_{Pl}/M)^2 .$$

The expected frequency of such GW today can be estimated as follows. The characteristic frequency at production is $\omega_{prod} \sim 1/Sr_g$.

It is redshifted by the ratio of the temperature at production, T_{prod} to the present day $T_0 = 2.7^o$. Thus the detected frequency today should be:

$$\omega_{obs} \sim 7 \cdot 10^9 S^{-1} (M/m_{Pl})^{1/2} \, Hz$$

VI. GW from PBH binaries.

Gravitationally bound systems of PBH pairs captured by dynamical friction.

Luminosity of GW radiation from a single binary:

$$L = \frac{32 M_1^2 M_2^2 (M_1 + M_2)}{5 r^5} \approx \frac{64}{5} \frac{M^5}{r^5 m_{Pl}^8}.$$

If we take $r = \kappa r_g$, $\kappa \gg 1$, then

$$L = \frac{2 m_{Pl}^2}{5 \kappa^5}.$$

Average distance between BHs in the high density bunch:

$$d_b = 0.1 r_g \Omega_p^{\frac{2}{3}} \left(\frac{M}{m_{Pl}}\right)^{\frac{4}{3}} \left(\frac{100}{N}\right)^{\frac{2}{3}} \left(\frac{10^6}{\Delta}\right)^{\frac{1}{3}},$$

so $\kappa < 0.1 \Omega_p^{2/3} (M/m_{Pl})^{4/3}$.

Total rate of GW radiation from the binaries, if the binaries make fraction ϵ in the high density bunch:

$$\dot\rho_{GW} = \epsilon L n^b_{BH} = 10^6 \epsilon \rho_c(\tau_{BH}) \frac{\Delta}{10^6} \frac{L}{M}.$$

Total emitted energy $\rho_{GW} = \dot\rho_{GW} t_{eff}$, $t_{eff} \sim \kappa r_g$ is the coalescence time. Total cosmological fraction of GW energy now:

$$\Omega^{(0)}_{GW} = 10^{-4+6} \epsilon t_{eff} \frac{\Delta}{10^6} \frac{L}{M}.$$

Substituting proper numbers and expressions:

$$\Omega_{GW}^{(0)} = 10^2 \epsilon \kappa^{-4} > \frac{10^6 \epsilon}{\Omega_p^{8/3}} \left(\frac{m_{Pl}}{M} \right)^{16/3} .$$

This is a lower bound because we took maximum κ.

Since $M/m_{Pl} > 20/\Omega_p$:

$$\Omega_{GW}^{(0)} \sim 10^6 \epsilon \left(\frac{\Omega_p}{400} \right)^{8/3}$$

VII. Gravitons from BH evaporation.

Average graviton energy:

$$\omega_{av} = 3T_{BH} = \frac{3m_{Pl}^2}{8\pi M}.$$

Gravitons carry about 1% of the total evaporated energy and thus their contribution into cosmological energy density would be about 10^{-6}.

For $\omega < \omega_{av}$ the graviton density fraction drops down to $10^{-6}(\omega/\omega_{av})^4$.

VIII. BH formation mechanism.

With flat spectrum of perturbations the probability of BH formation is low, $\Omega_p \ll 1$. Large density perturbations at small scales after inflation could be created by a massive scalar field with general renormalizable coupling to the inflaton (AD, J. Silk):

$$\lambda(\Phi - \Phi_1)^2 |\chi|^2 .$$

Log-normal mass spectrum of BH.

Assume that χ lives in CW potential.
If $m_\chi < H_{infl}$ but $m_\chi > H_{heat}$,
χ would acquire large value during inflation but with low probability.
MD bubbles of χ in small fraction of volume could be formed with large density contrast.
The bubbles are matter dominated and cold, hence the bounds presented above may be not applicable.

IX. Cosmological evolution with of BH dominance at an early stage.

To create BH with mass M the temperature before the production moment should be:

$$T_{heat} > 0.2 m_{Pl}^{3/2}/M^{1/2}.$$

Demanding $T_{heat} < T_{GUT} = 10^{15}$ GeV gives: $M > 10^7 M_{Pl}$.

If $M > 10^7 m_{Pl} \approx 10^2$ g, the BH temperature

$$T_{BH} = \frac{m_{Pl}^2}{8\pi M} < 5 \times 10^{10}\ GeV\,.$$

Reheating temperature after BH evaporation:

$$T_{reh} \approx 0.1 m_{Pl} \left(\frac{m_{Pl}}{M}\right)^{\frac{3}{2}} < 2 \times 10^7\ GeV.$$

However, lighter BHs are also possible due to matter accretion.

The bounds presented in the previous page may not be valid in the case of the mechanism based on inhomogeneous Affleck-Dine baryogenesis.

Baryogenesis might proceed according to AD or through asymmetric BH evaporation.

Heating after inflation is almost forgotten.

DM is produced by BH evaporation or by secondary thermalization.

The universe would be clumpy at very small scales, $M_b \sim M/\Omega_p^2$.

Formation of larger BH from bunches of small BH by dynamical friction?

X. Conclusion

Cosmological scenario with dominance of PBH is plausible.

This early MD-stage may be observable through high frequency GW.

Heavy relics from after-inflationary heating would be forgotten.

Baryogenesis might successfully proceed in the course of BH evaporation.

If DM and baryon asymmetry are produced in BH evaporation, it is natural to expect that $\Omega_{DM} \sim \Omega_b$.

Cosmological energy density of dark matter particles is not related to their annihilation cross-section.

BBN is safe, though some distortion is possible.

Impact on CMB is weak or high frequency GW could distort it (?).