Inclusion of Thermal Conduction in Astrophysical Simulations – Applications to SNRs and HVCs

By

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Talk Outline:

A Bit on Thermal Conduction in Astrophysics

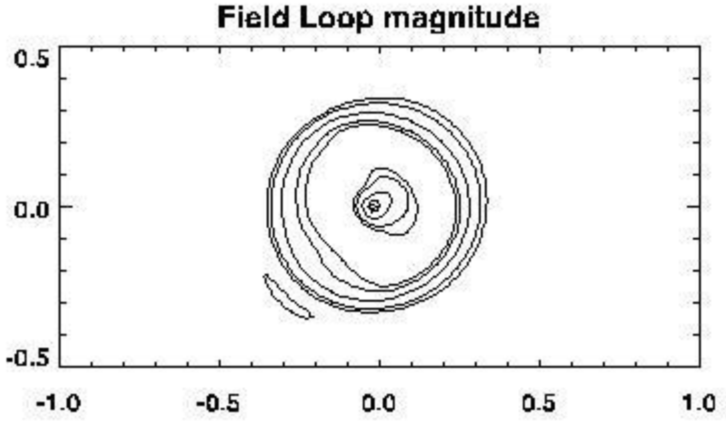
Emphasis on Fast & Stable SuperTimestepping Techniques

Application of these methods to Supernova Remnants

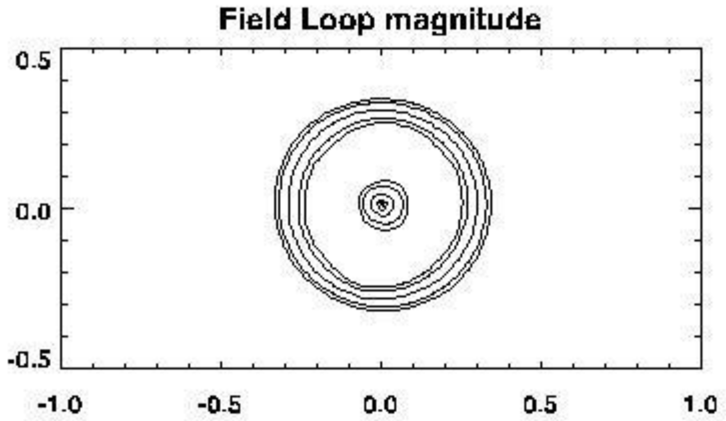
AMR-MHD Simulations of High Velocity Clouds

Development of Genuinely Multidimensional Riemann Solvers for CT

a)



b)



Gardiner & Stone (2005), D. Lee (2012)

Balsara (2010, 2012)

A Bit on Thermal Conduction in Astrophysics:

Present in various astrophysical systems (regulates energy transport):

Clusters of Galaxies
Shock-Cloud Interaction
ISM
Type I X-ray Bursts
Boundary Layers
Supernova Remnants
Solar Corona
Formation of Molecular Clouds
High Velocity Clouds

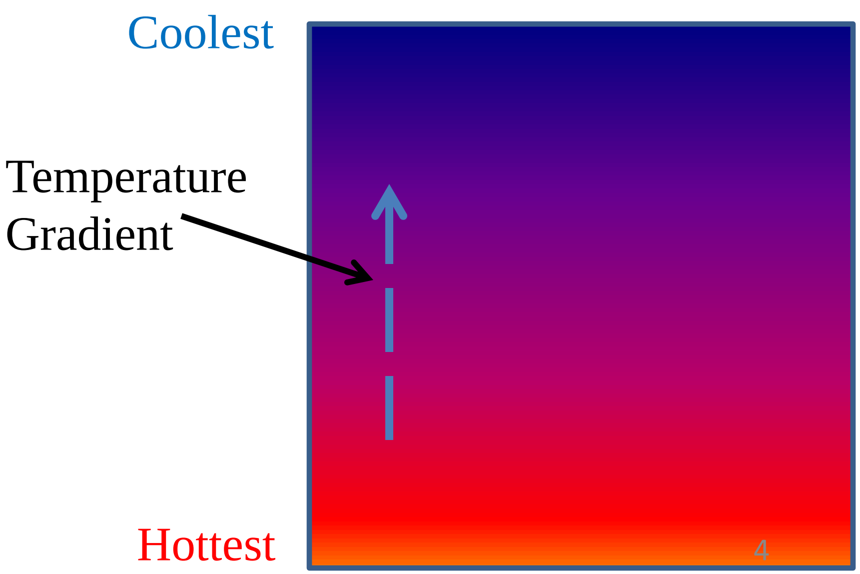
Challenges in its Numerical Implementation:

1) In the presence of magnetic fields, thermal conduction can become strongly anisotropic: (electrons carry heat; follow field lines; drift along *projected* temperature gradient)

$$\mathbf{F}_{\text{class}} = -\,\kappa\,\nabla T$$

2) Conduction coefficient is strongly non-linear; varies by several orders of magnitude in a few zones:

$$\kappa = a\,T^{5/2}$$



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(electrons carry heat; follow field lines; drift along *projected* temperature gradient)

$$\mathbf{F}_{\text{class}} = -\kappa \nabla T \quad \rightarrow \quad \mathbf{F}_{\text{class}} = -\kappa \, \mathbf{b} \left(\mathbf{b} \cdot \nabla T \right)$$
$$\mathbf{b} = \mathbf{B} / |\mathbf{B}|$$

2) Conduction coefficient is strongly non-linear; varies by several orders of magnitude in a few zones:

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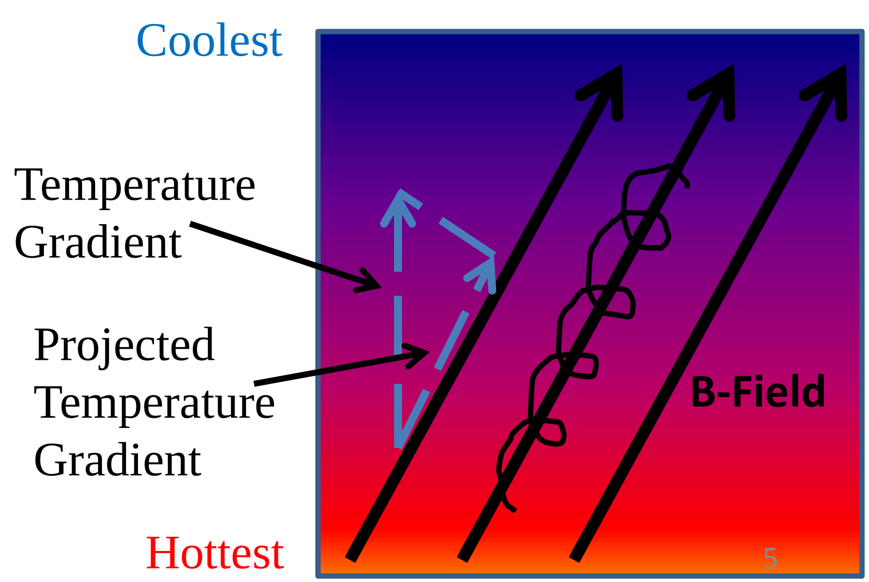


3) When the temperature gradient becomes large, plasma instabilities impede electron streaming.
Classical flux → Saturated flux

Problem: Saturated flux is hyperbolic while unsaturated flux is parabolic. Operator changes character.

$$\mathbf{F}_{sat} = -5\,\phi\,\rho\,c_s^3\,\mathrm{sgn}\left(\mathbf{b}\bullet\nabla T\right)\,\mathbf{b}$$

4) Can be solved by using Krylov methods.
Problem : The method is implicit & incredibly expensive!

$$\frac{\partial\,\varepsilon}{\partial\,t} + \nabla\bullet\left(Fluxes\right) = -\nabla\bullet\mathbf{F}_{net}$$

5) Can be solved by explicit timestep subcycling.
Problem : Insanely many timesteps needed in certain circumstances.

$$\Delta t^{parabolic} = \frac{\Delta x^2}{\kappa}$$

6) Can be solved by SuperTimestepping.
Problem: Prior versions of these methods were unstable.

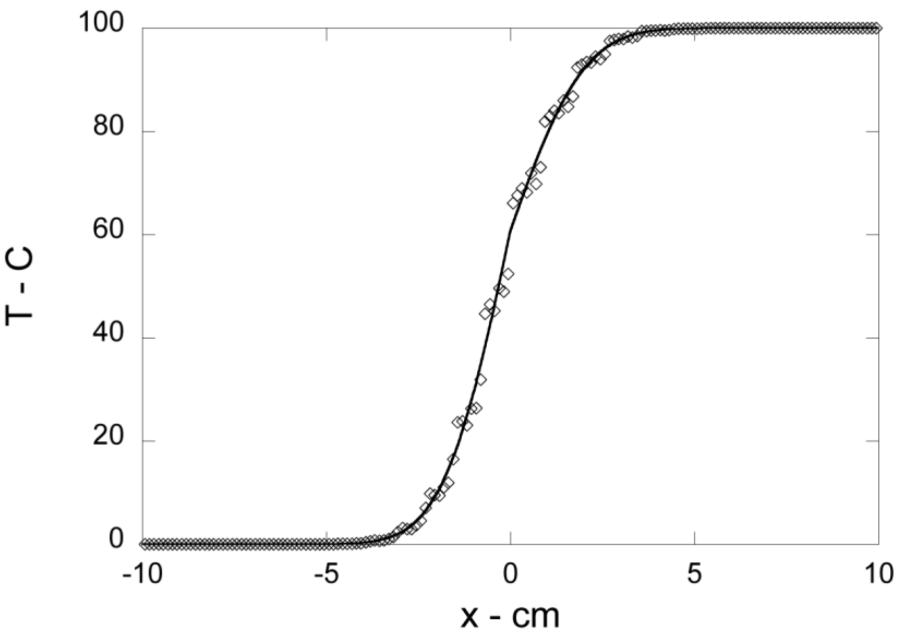
$$s \text{ explicit steps give } \Delta t^{explicit} \propto s^2\,\frac{\Delta x^2}{\kappa}$$

Wanted: A fast *explicit* method that has good *stability* properties in the *parabolic/hyperbolic* limits even as the thermal conduction coefficient varies over several *orders of magnitude*!

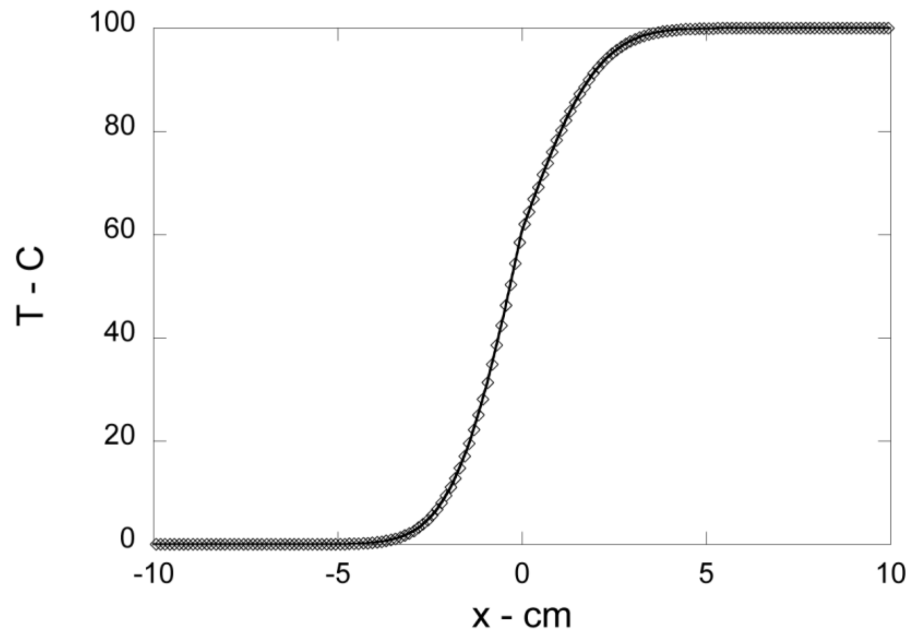
Fast & Stable SuperTimestepping Techniques:

——— Analytic Solution; ◇ Numerical Solution

Earlier Generation SuperTimestepping (RKC)



Recent Generation SuperTimestepping (RKL)



The problem is a loss of **Monotonicity** in the numerical solution.

The problem becomes more exaggerated as the thermal conduction coefficient varies in space.

The goal is to have a method that remains monotone even as the thermal conduction coefficient varies over a convex set of values – **Convex Monotonicity Preserving (CMP)** property.

$$\frac{\partial \varepsilon}{\partial t} = L^{class}(U) \quad \leftarrow \quad \text{Being parabolic, } L^{class}(U) \text{ has real, negative eigenvalues.}$$

The method can be written as an s-stage recursion sequence:

$$Y_0 = \varepsilon^n$$

$$Y_1 = Y_0 + \tilde{\mu}_1 \tau L^{class}(Y_0)$$

$$Y_j = \mu_j Y_{j-1} + \nu_j Y_{j-2} + (1 - \mu_j - \nu_j) Y_0 + \tilde{\mu}_j \tau L^{class}(Y_{j-1}) + \tilde{\gamma}_j \tau L^{class}(Y_0); \quad 2 \le j \le s$$

$$\varepsilon^{n+1} = Y_s$$

Question: How do we pick the terms in the recursion sequence?

The Trick: Pick them in keeping with known, stable, recursion sequences of analytic polynomial series!

Typically, we pick shifted Legendre polynomials. Legendre polynomials have good stability properties.

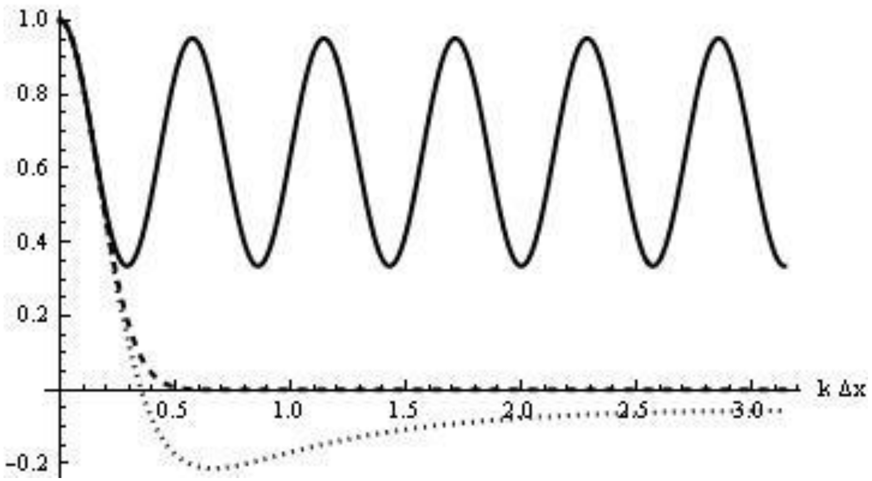
Bonus: An s-stage method will be stable for all $\Delta t^{explicit} \le \frac{s^2 + s - 2}{4} \frac{\Delta x^2}{\kappa}$ $\leftarrow$ SuperTimestepping advantage!

The methods are provably Convex Monotonicity Preserving.

Meyer, B. & Aslam (2012)

The methods have superior stability properties to the ones they replace for parabolic and mixed parabolic-hyperbolic operators!

Amplification factor v/s k Δx (RKC)



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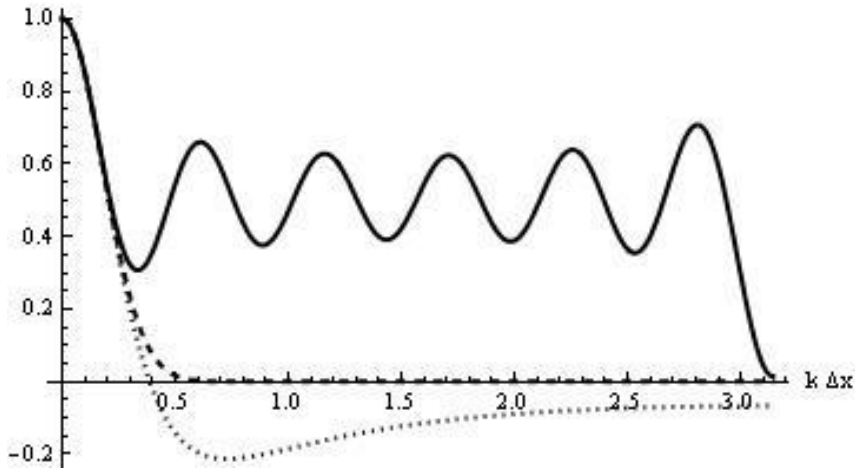
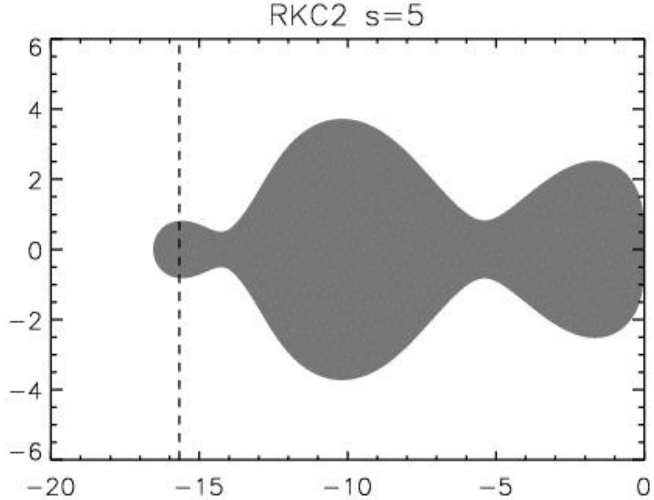


Stability in the complex plane (RKC)



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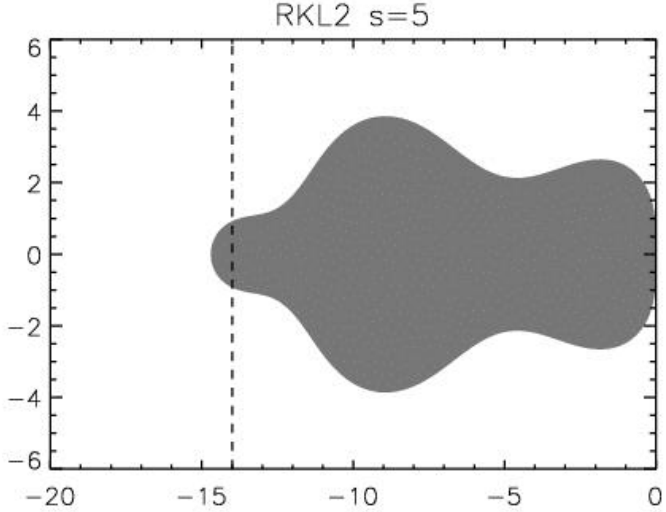


Application of these methods to Supernova Remnants

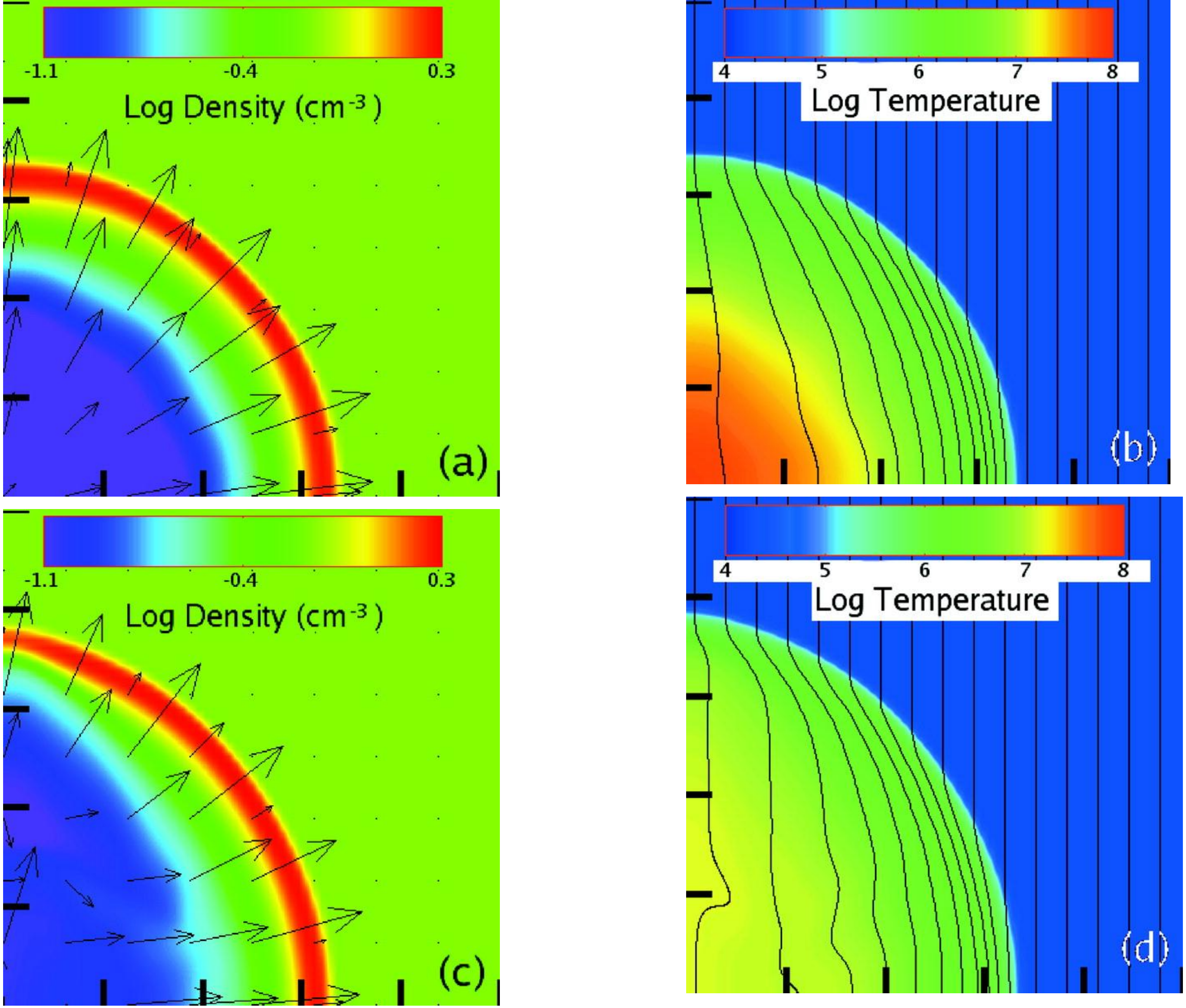
Simulations *without & with* thermal conduction: ρ = 0.7 amu/cm^3 ; T = 8000 K ; B = 3 μG

Movies not included

Rings mark 50 pc distances

Cooling;
No Thermal
Conduction

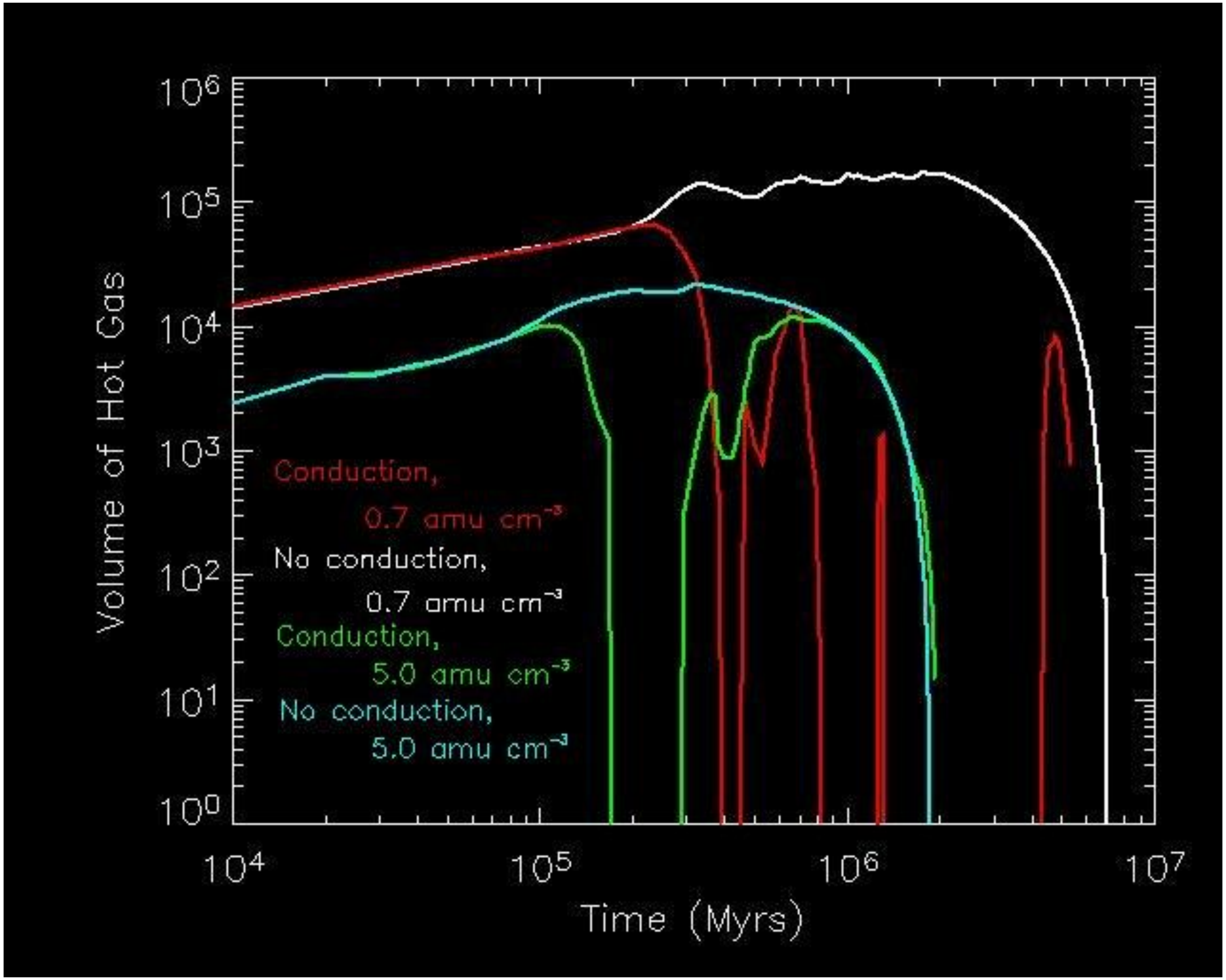
Cooling;
With
Thermal
Conduction



Focus on central hot-gas bubble (shown at 60Kyr) :
Temp down in 10^7 range; radiate in x-ray; density up by 10x ➔ more X-ray rad^n

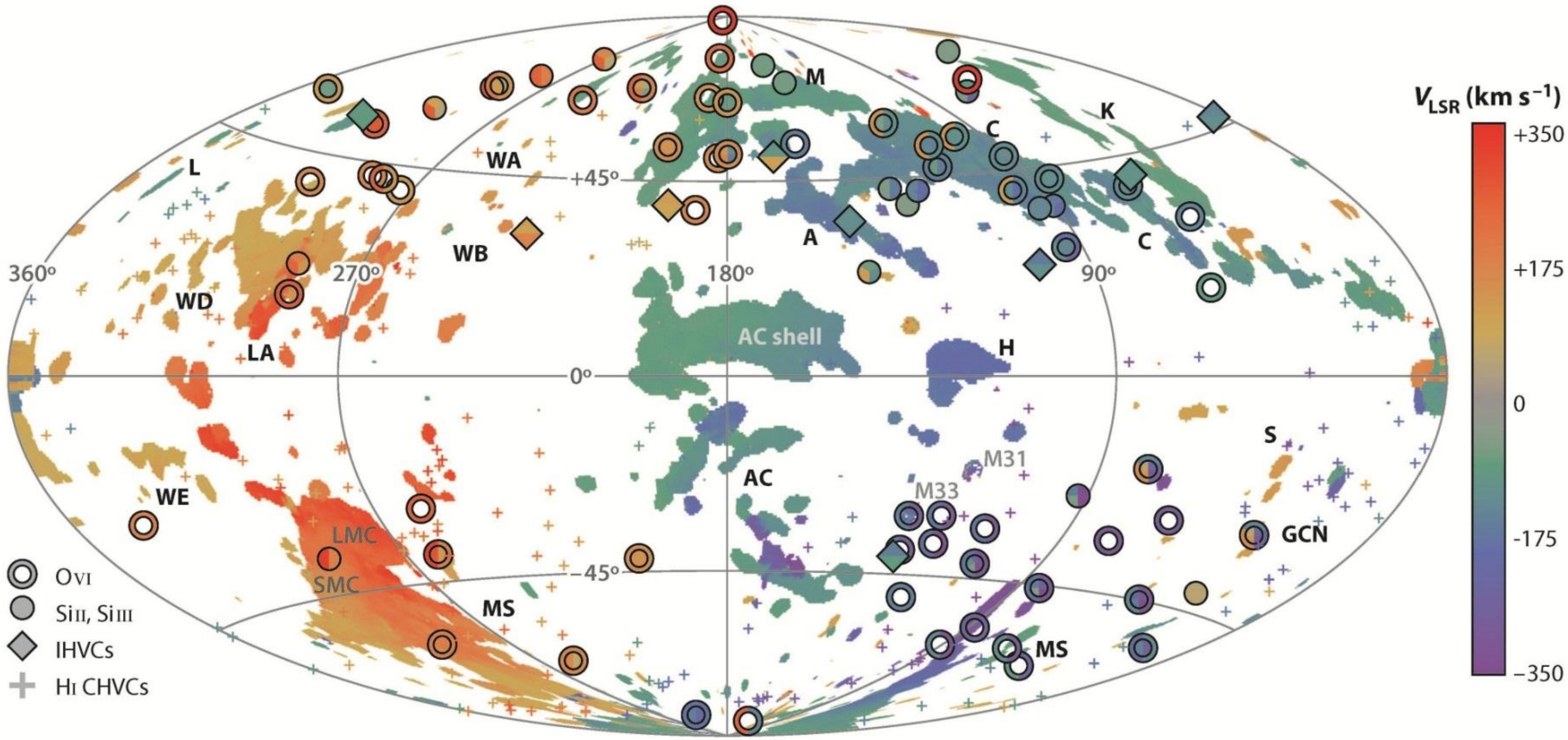
s=5-43 stages were needed, with $s_{avg} = 5$. On avg. SuperTimestepping added 39% to the cost of the computation.

Compare to Newton-Krylov methods that can be 20 times more expensive.



Importance for **hi-stage ions** (i.e. the observables): OVI (FUV) tracks $3x10^5$ K; OVII (x-ray) tracks $2.8x10^5$ to $1.8x10^6$ K gas; OVIII (x-ray) tracks $2.2x10^6$ K

AMR-MHD Simulations of High Velocity Clouds



Putman *et al*. (2012) + Putman *et al*. (2002), Sembach *et al*. (2003), Shull et al. (2009), Lehner & Howk (2011) Colors showing velocity w.r.t. LSR.

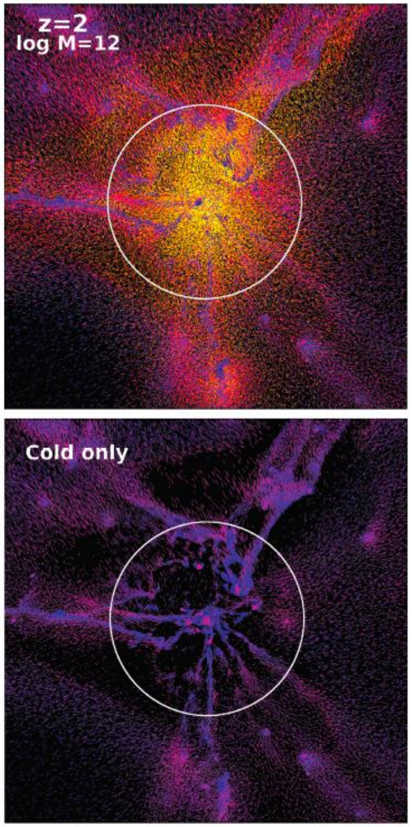
Provide between 0.1 to 0.4 M_{solar} /yr of fresh matter (Z~0.1-0.5) to the Galaxy. In steady state the Galaxy needs 1 M_{solar}/yr to offset starformation

GCE models require ~ 0.5 M_{solar} /yr of fresh matter .

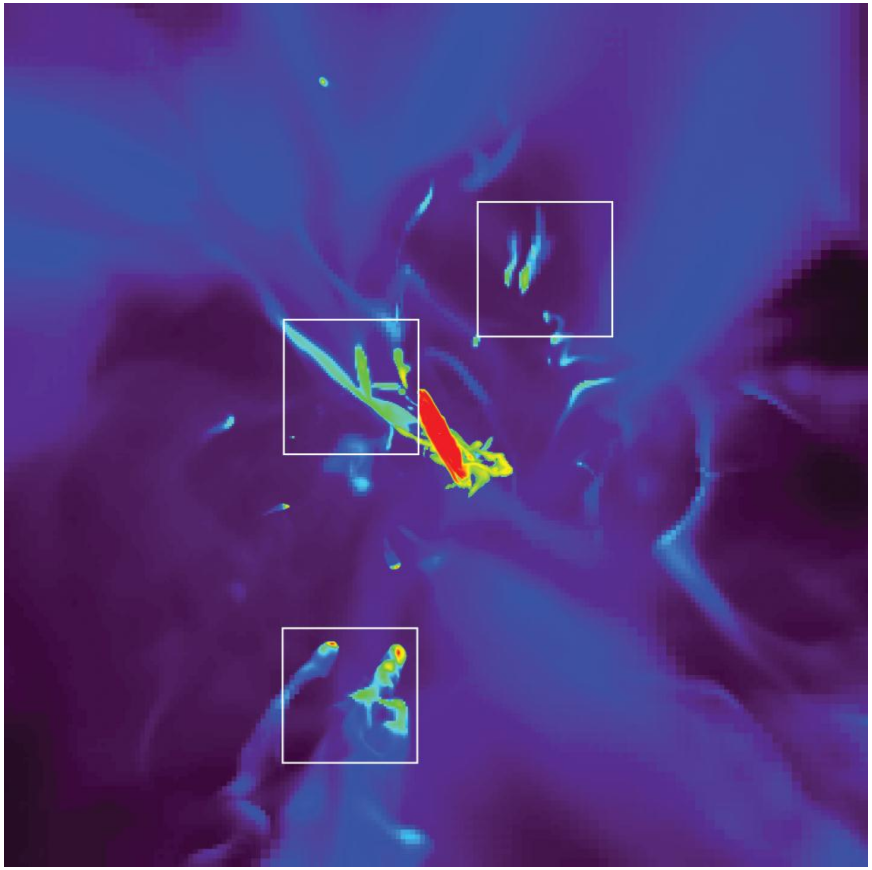
Hot v/s Cold Accretion: Lower mass galaxies seem to be acquiring their mass via cold ($T < T_{vir}$) filamentary accretion.

The survivability of these clouds, i.e. their ability to fuel the host galaxy, has been an issue in *global* and *local* simulations. ←Addressed in this talk.

Warm ➔ $T \sim 10^{5-6}$ K Hot ➔ $T > 10^{6}$ K



Keres *et al*. (2009)



Joung *et al*. (2012)

Temperatures of the accreting HVCs extremely well-studied in our Galaxy

Linewidths are very well-known, same for infall velocities.

Positions are becoming better known

Densities of the HVC material and halo reasonably well known

Clouds become denser and cooler as they infall.

Magnetic fields measured in a few instances.

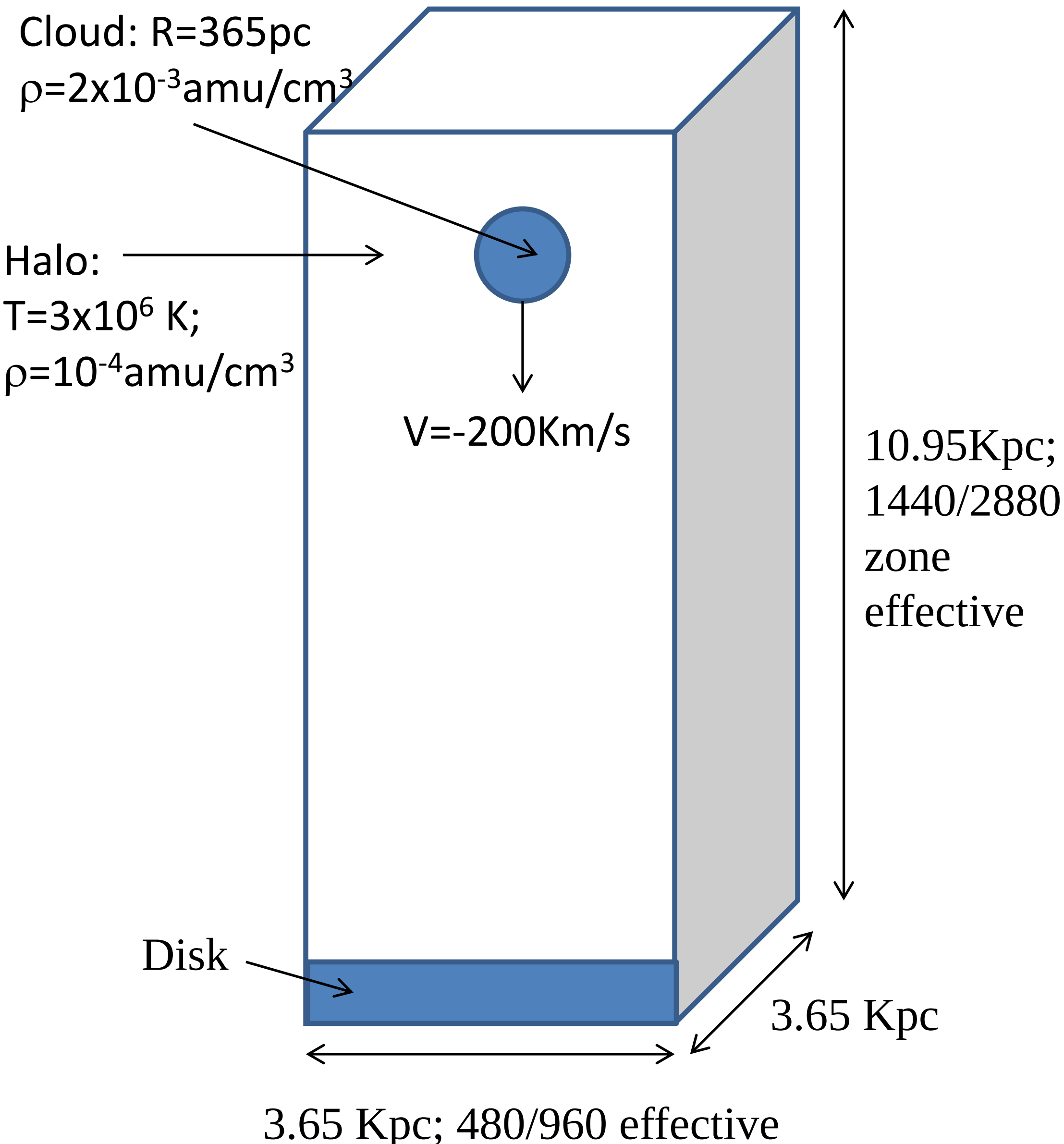
Would be good to match these to some reasonable extent.

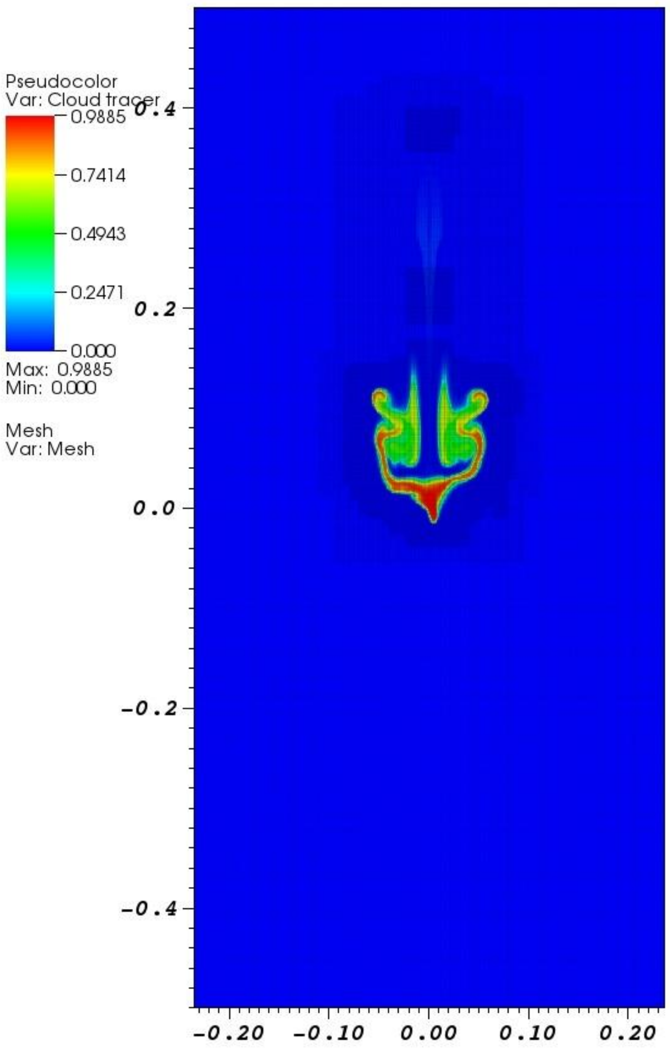
Prior simulations – clouds disrupt before they reach the disk – a fueling problem!

HVC, Schematic of Simulations

Self-consistent disk + stratified halo + cloud model; different cloud B-field and metallicities.

$M_{halo} = 1.5x10^4\ M_{solar}$; $H_{halo} = 5.5$ Kpc ; Plasma Beta = 400, when it is present.





Refinement covers entire cloud : $\Delta x \sim 1.7$ pc

Simulation Parameters:

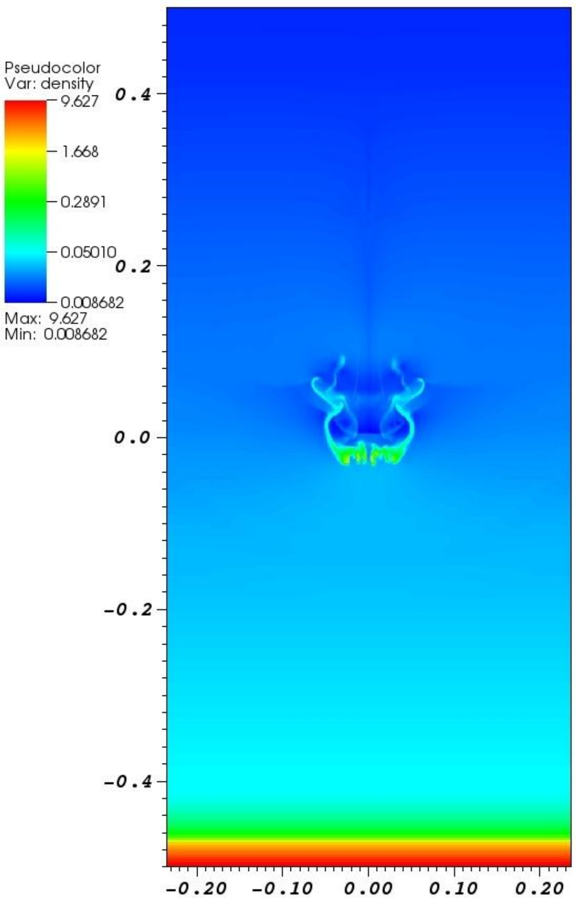
Non-magnetized	Magnetized; Uniform B-field	Magnetized; Magnetically Isolated Cloud
Run1 $Z_{cloud} = 0.5$; β = Infinity $Z_{halo} = 0.5$	Run2 $Z_{cloud} = 0.5$; $\beta = 400$; $Z_{halo} = 0.5$	Run3 $Z_{cloud} = 0.5$; $\beta_{HVC} = 400$; $Z_{halo} = 0.5$; β_{Halo} = Infinity
Run4 $Z_{cloud} = 0.1$; β = Infinity $Z_{halo} = 0.5$	Run5 $Z_{cloud} = 0.1$; $\beta = 400$; $Z_{halo} = 0.5$	Run6 $Z_{cloud} = 0.1$; $\beta_{HVC} = 400$ $Z_{halo} = 0.5$; β_{Halo} = Infinity

We focus on the first row of sims. Second row shows the same trends!

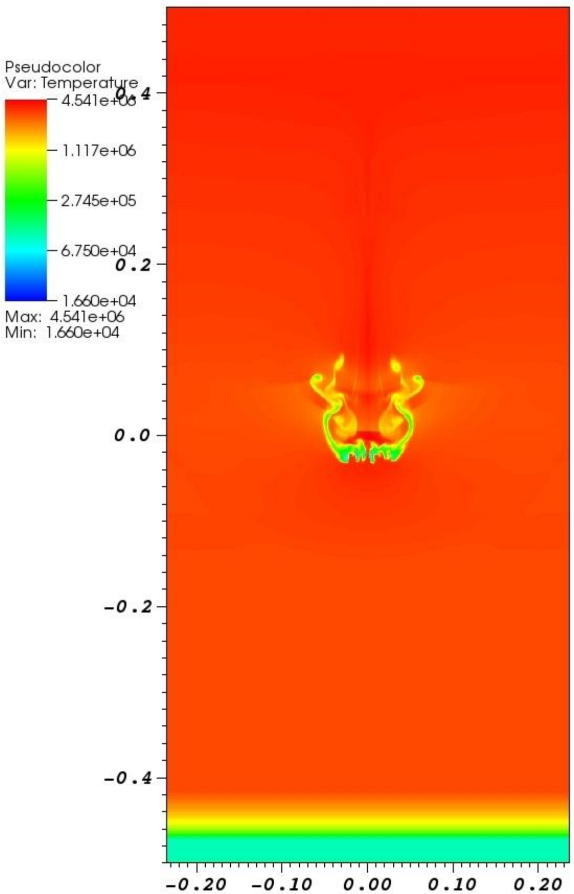
All runs included metallicity-dependent radiative cooling; with heating balancing cooling in the halo.

Run1: Unmagnetized HVC ; 22.1Myr

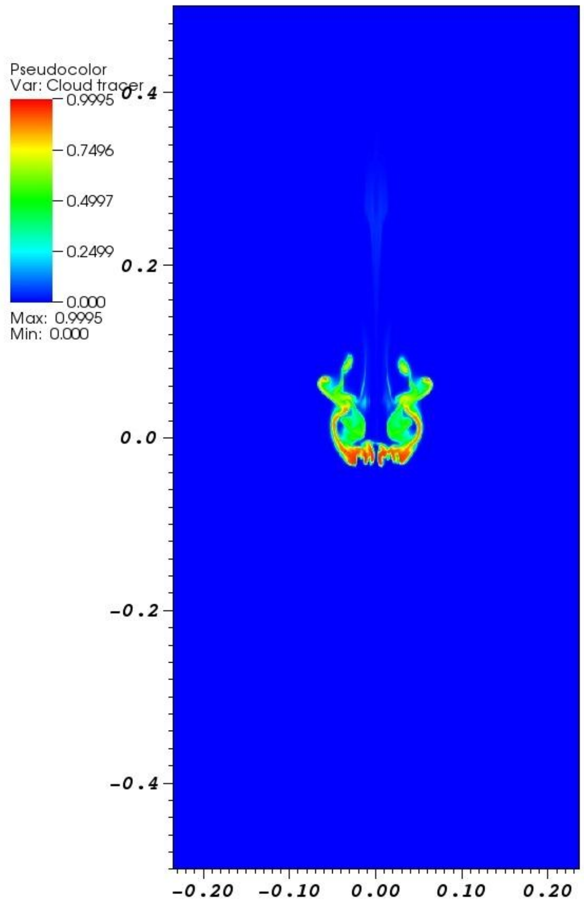
Density



Temperature

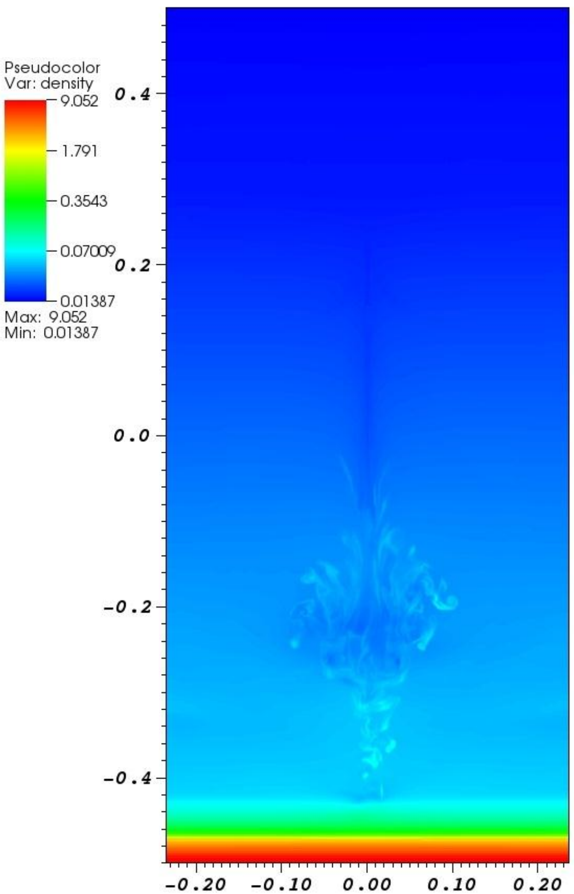


Species

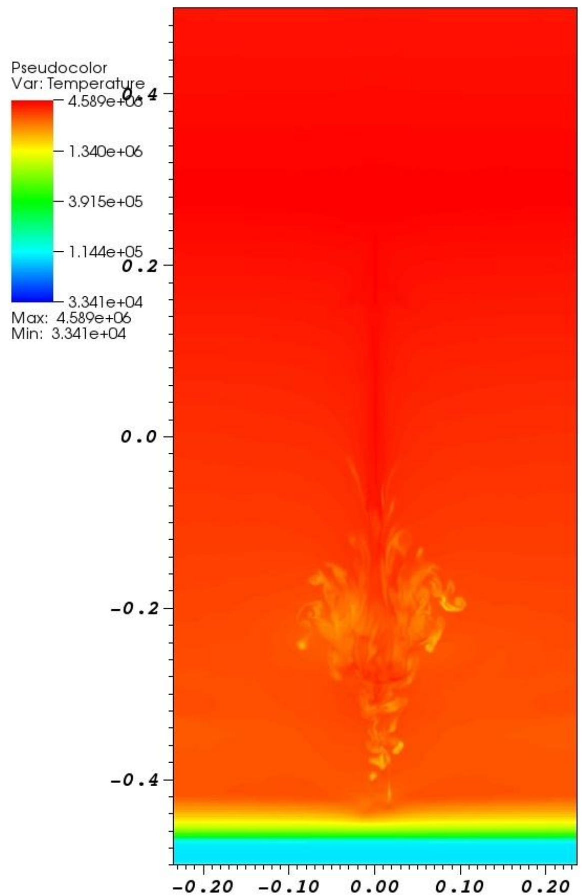


Run1: Unmagnetized HVC ; 34.8Myr

Density



Temperature



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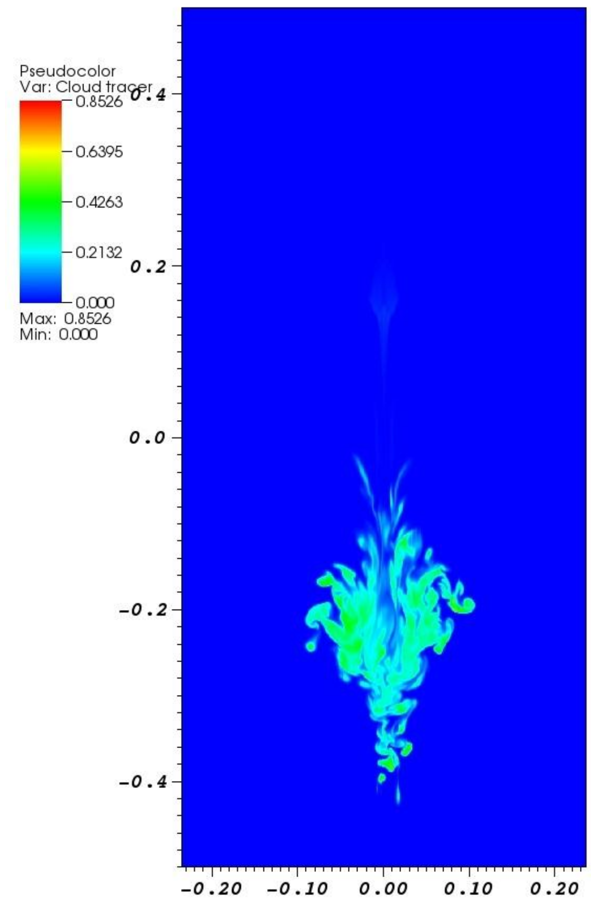
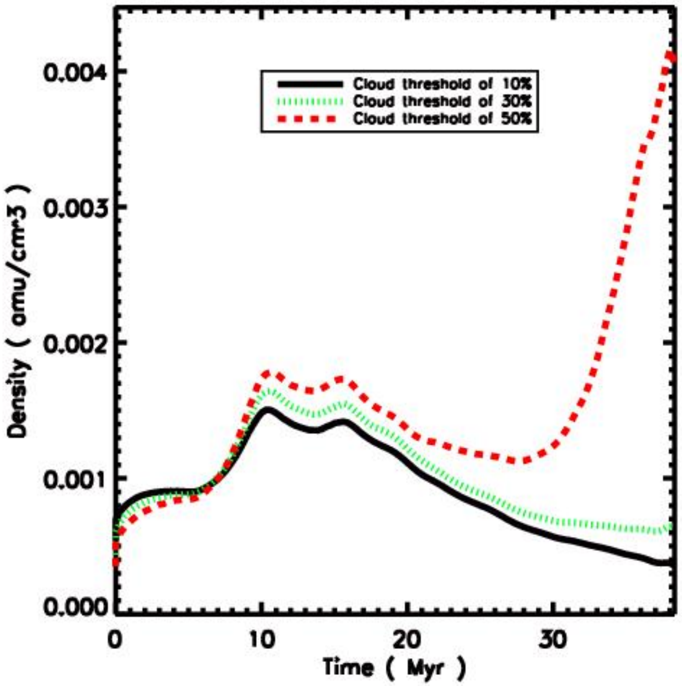
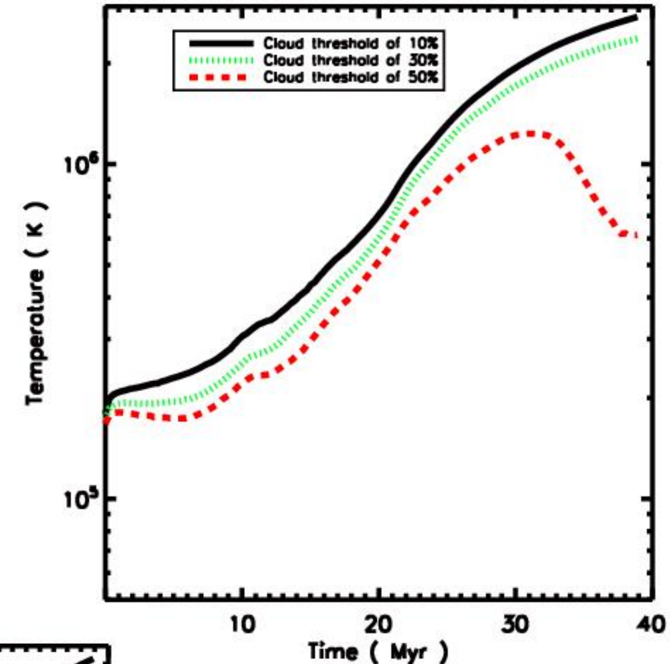


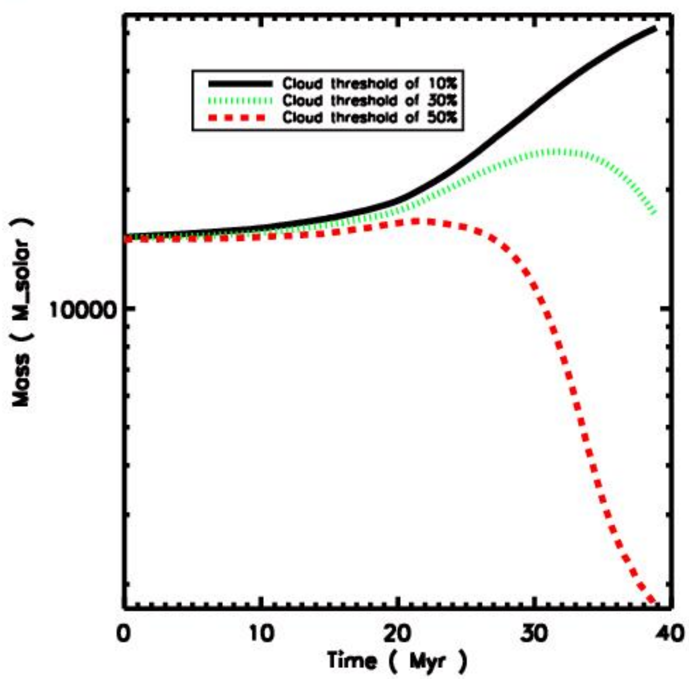
Density v/s time



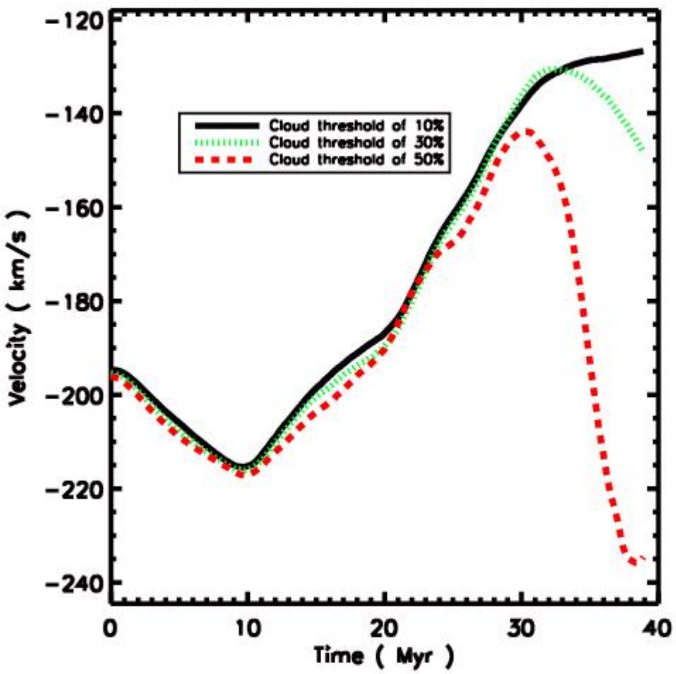
Temperature v/s time



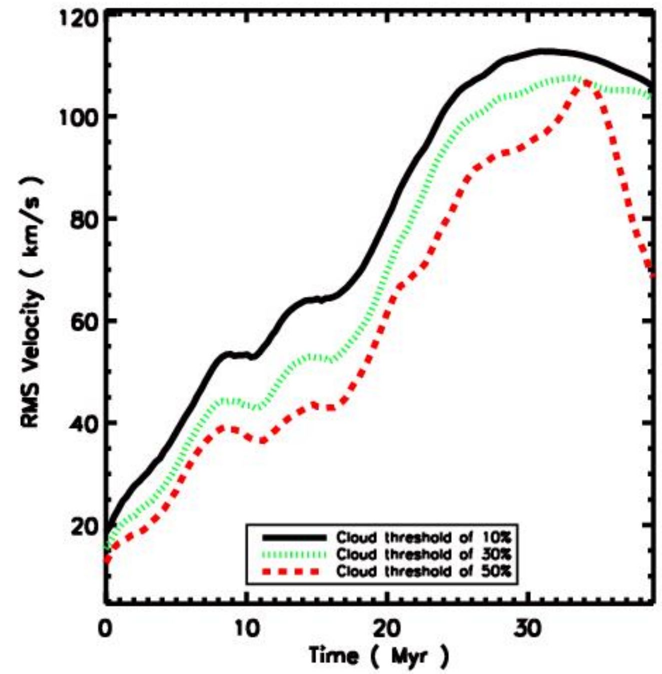
Mass v/s time



Z-Velocity v/s time

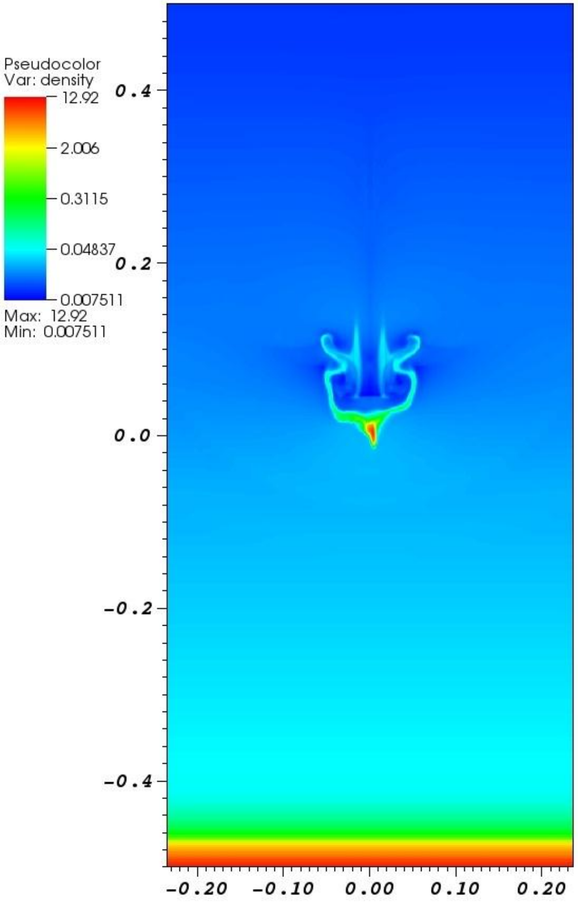


RMS Velocity v/s time

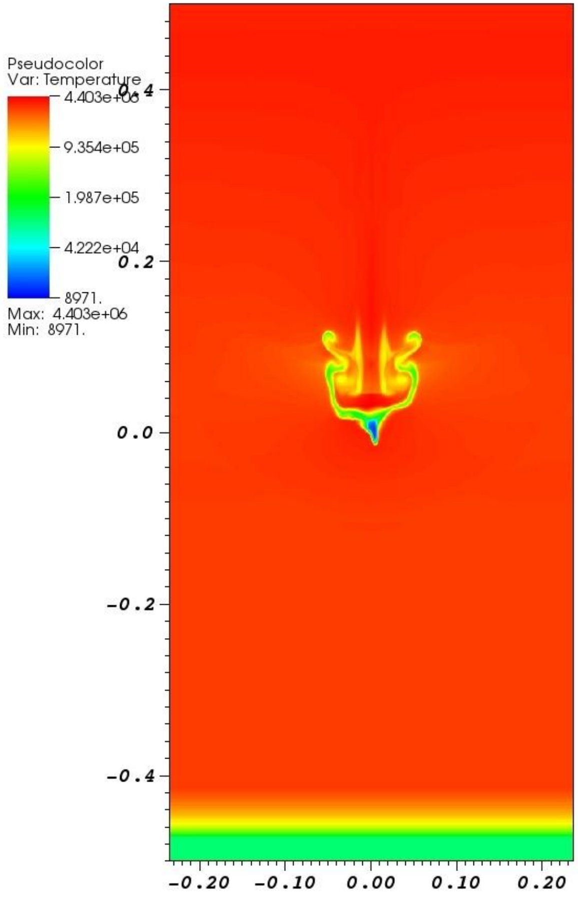


Run3: Magnetically Isolated HVC; Halo unmagnetized; 20 Myr

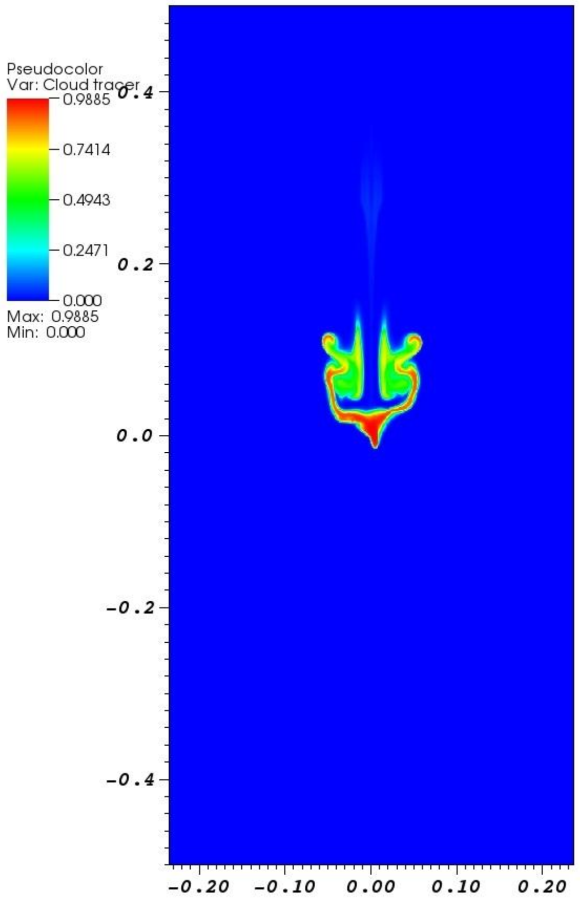
Density



Temperature

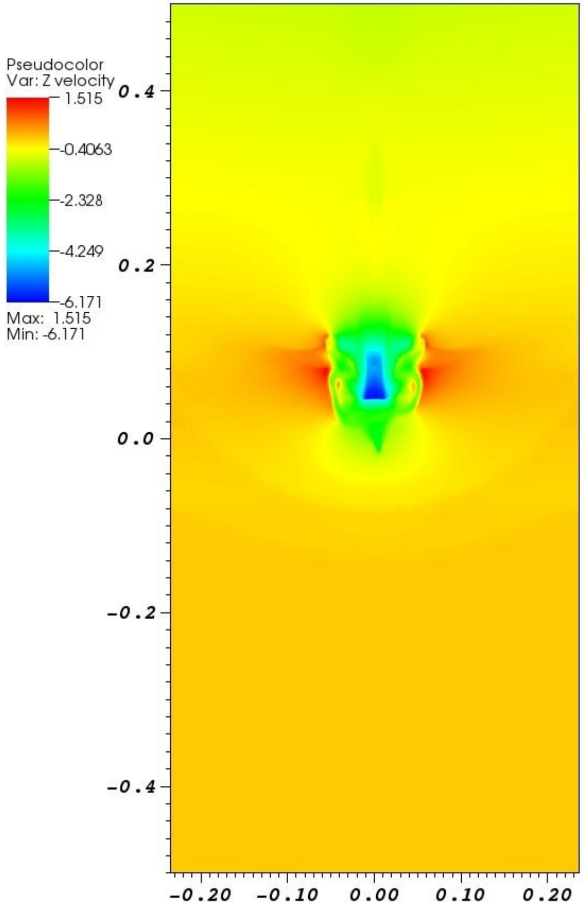


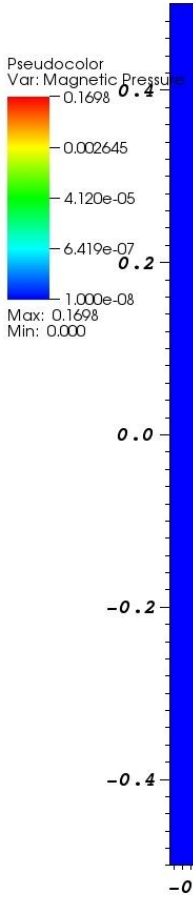
Species



Z-Velocity

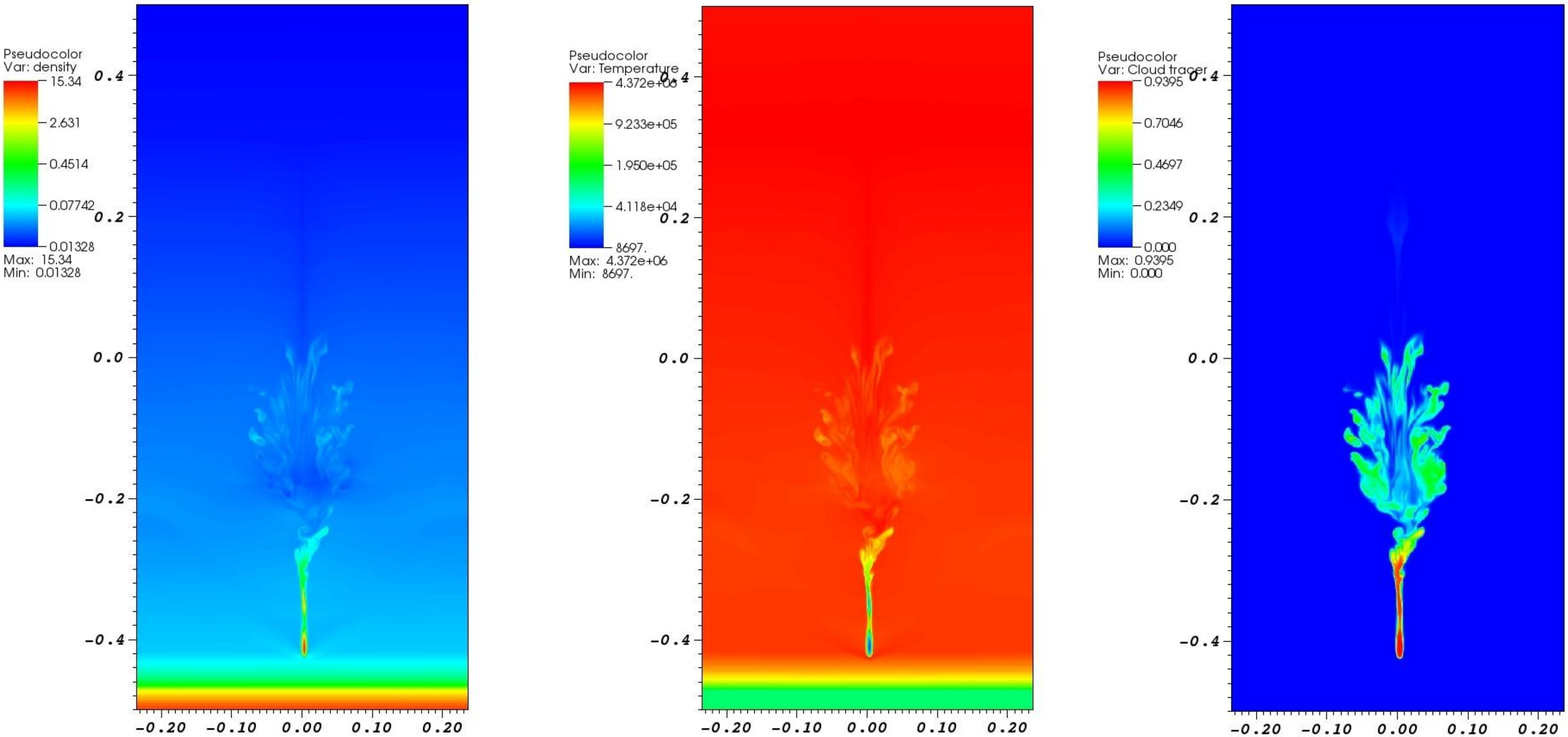
Magnetic Pressure





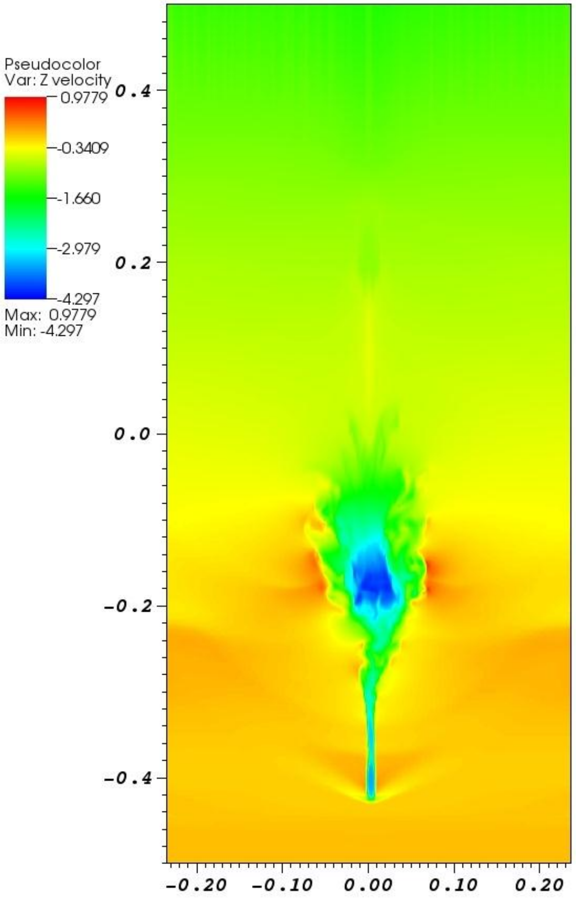
Run3: Magnetically Isolated HVC; Halo unmagnetized; 34.8 Myr

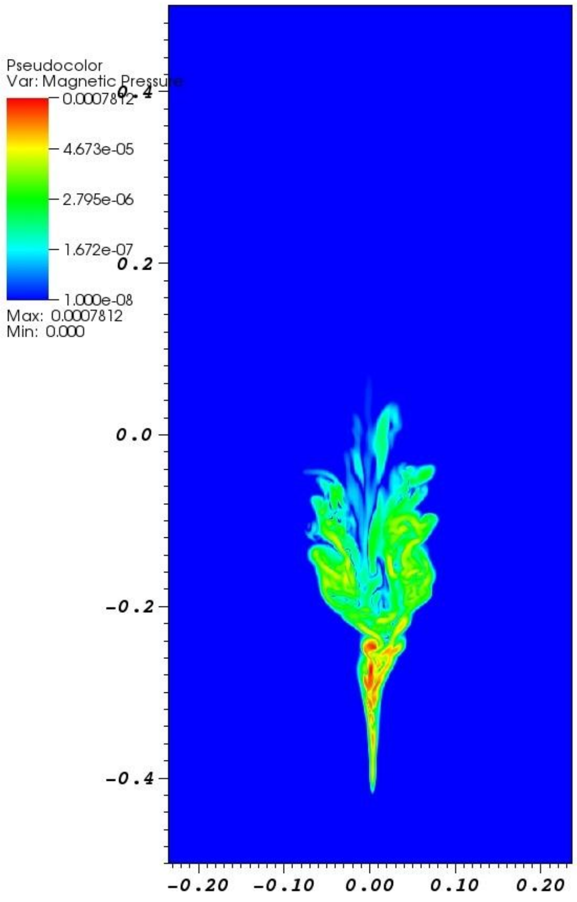
Density | Temperature | Species



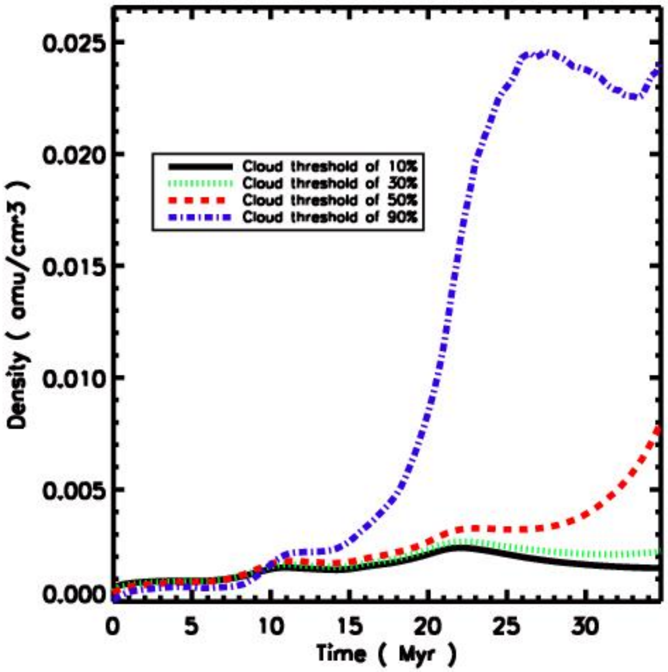
Z-Velocity

Magnetic Pressure

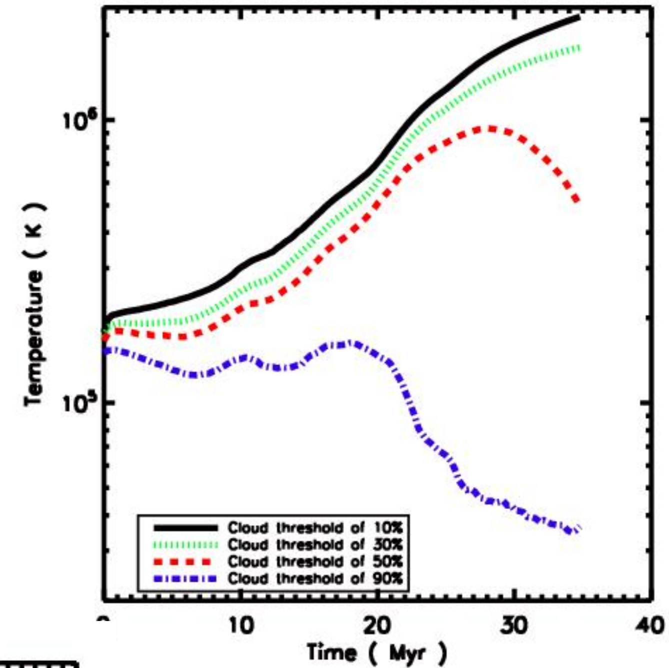




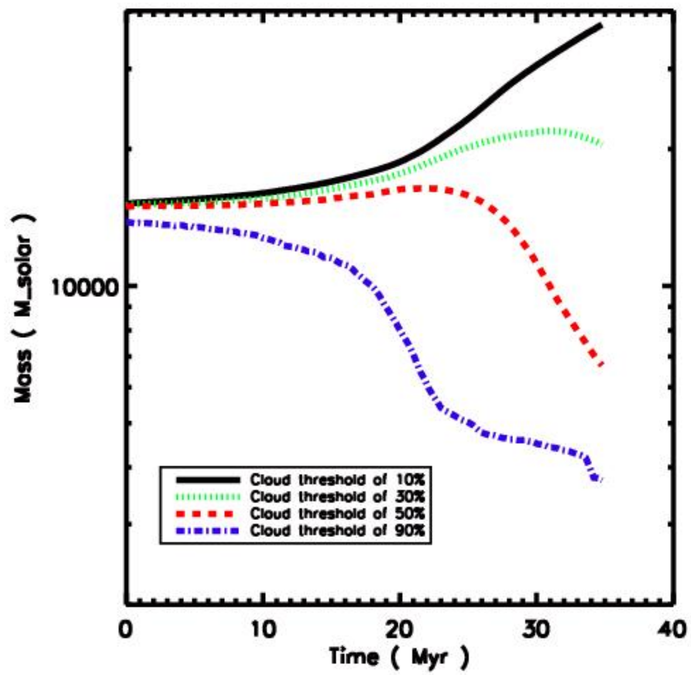
Density v/s time



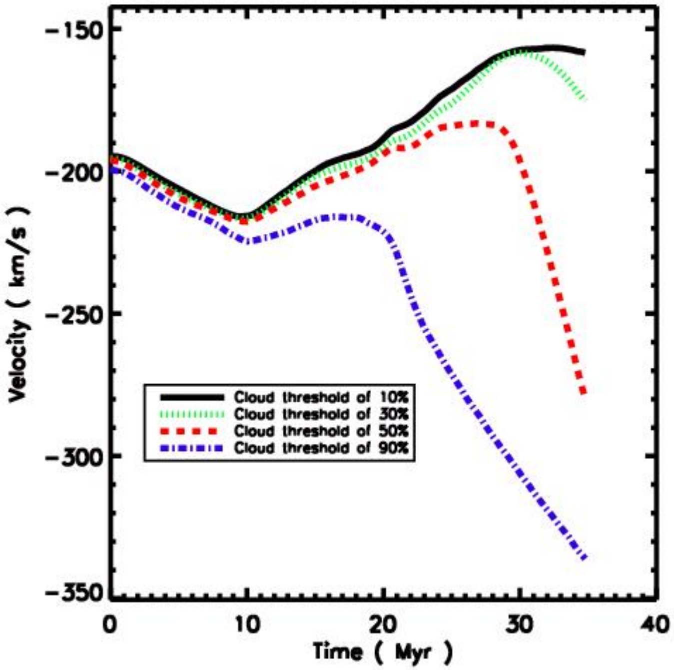
Temperature v/s time



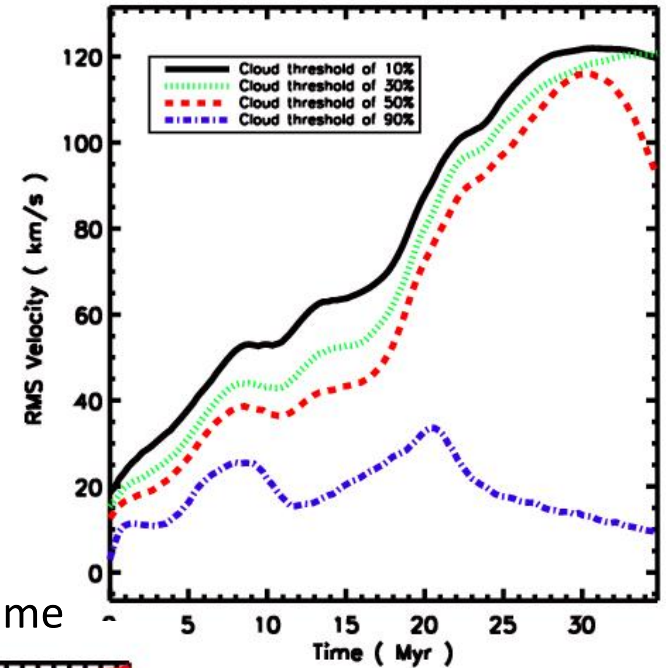
Mass v/s time



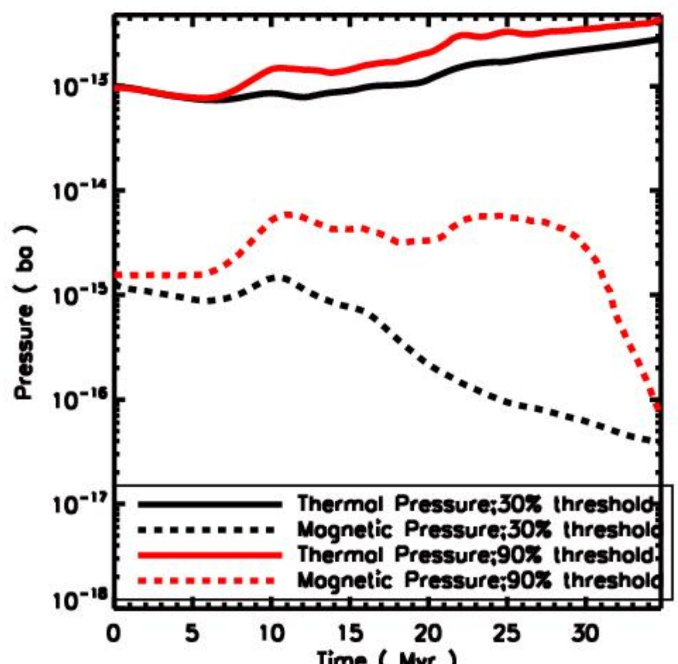
Z-Velocity v/s time



RMS Velocity v/s time



Gas & B pressure v/s time



Conclusions:

An efficient and stable SuperTimestepping strategy for incorporating anisotropic thermal conduction has been found.

Thermal conduction is very useful in SNR simulations.

- Dramatically reduces the filling fraction of very hot gas.
- Changes distribution of high stage ions.

The time evolution of HVCs is strongly influenced by the inclusion of magnetic fields.

- Field morphology strongly influences the survival of the cloud.
- Mesh resolution is also of paramount importance.
- Capturing the cloud-halo interface is very important.
- If all is done right, HVCs do convey most of their mass to the disk.