A New CT-MHD Algorithm in FLASH & FLASH's Applications to HEDP

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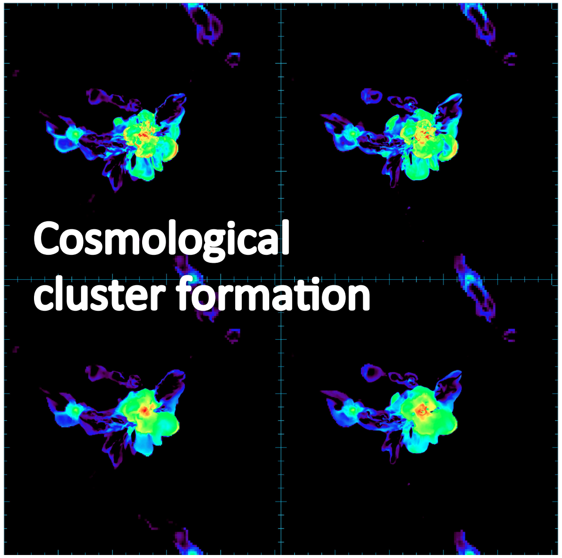
Outline

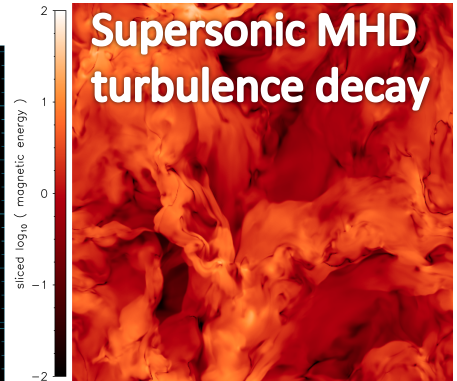
- ***Part 1: Applications***
  - Applications of new physics in FLASH
  - High-Energy-Density Laboratory experiments: LULI/RAL
  - FLASH code validation and predictive science

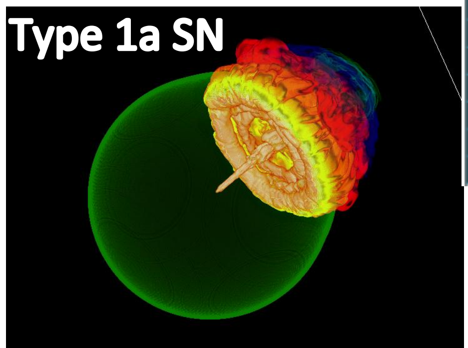
- ***Part 2: Algorithms***
  - 3D Unsplit Staggered Mesh (USM) MHD Scheme
  - A new upwind biased constrained-transport (CT) scheme (Lee, JCP, 2012, under review)
  - Small angle weakly magnetized field loop advection tests

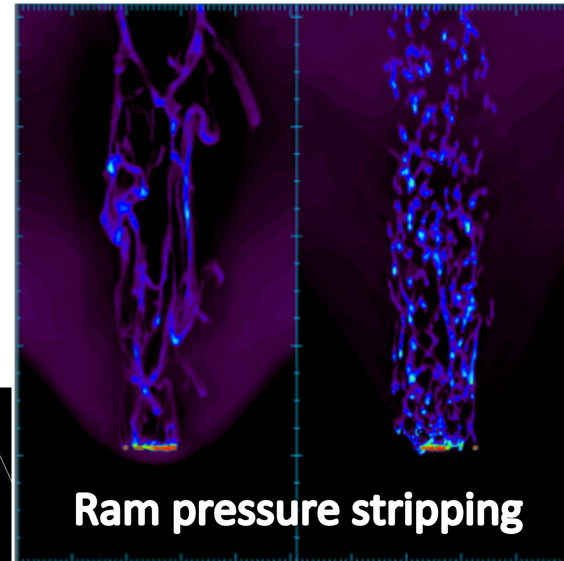
Applications:
FLASH Simulations of Astrophysics & Validations of HEDP

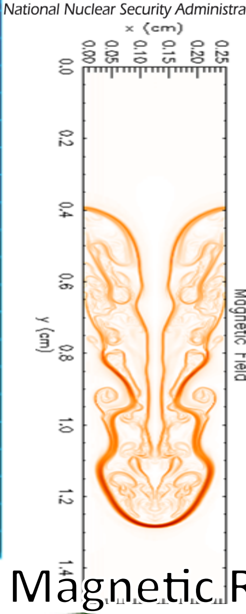
Astrophysical Applications

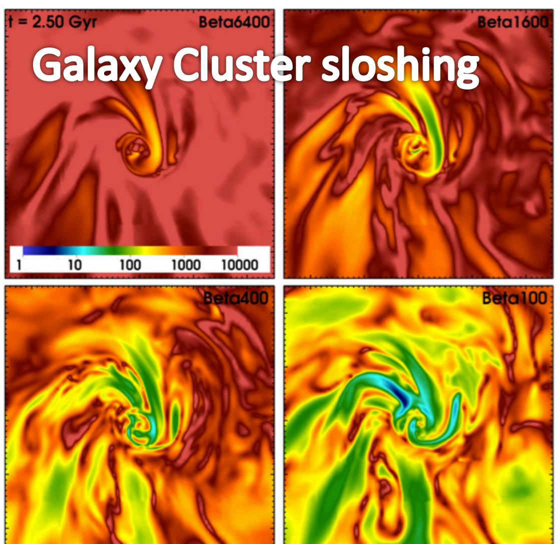


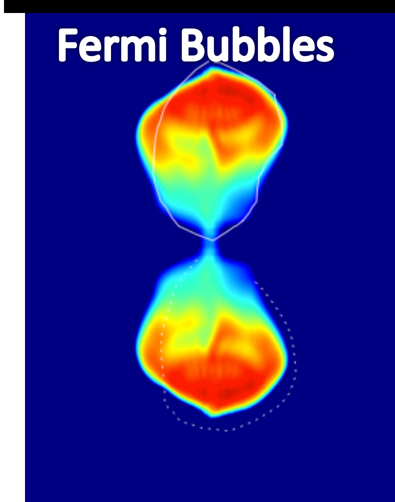


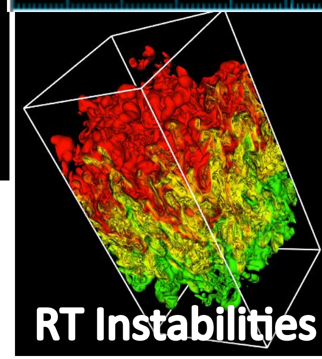


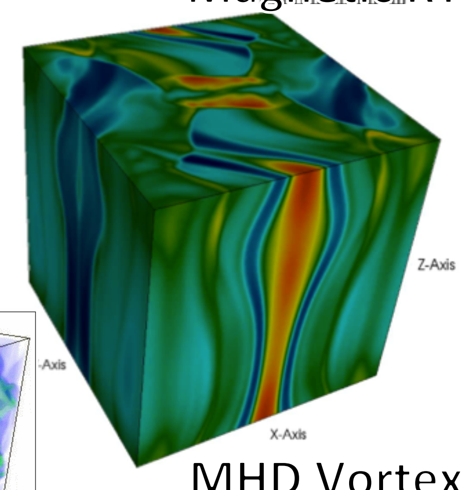


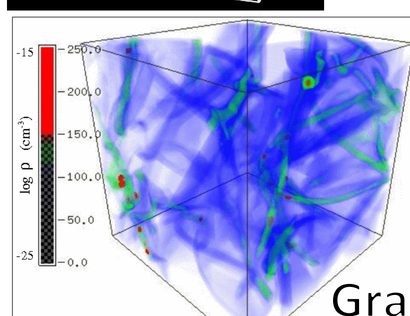












FLASH Basics

- An application code, composed of units/modules
- Particular modules are set up together to run different physics problems
  - Astrophysics
  - High-energy-density physics
  - Incompressible Navier-Stokes (GWU)
- Fortran, C, Python, …
  - More than 1.2 million lines of code, 75% code, 25% comments
- Very portable, scales to tens of thousand processors

FLASH Capabilities

Astrophysics

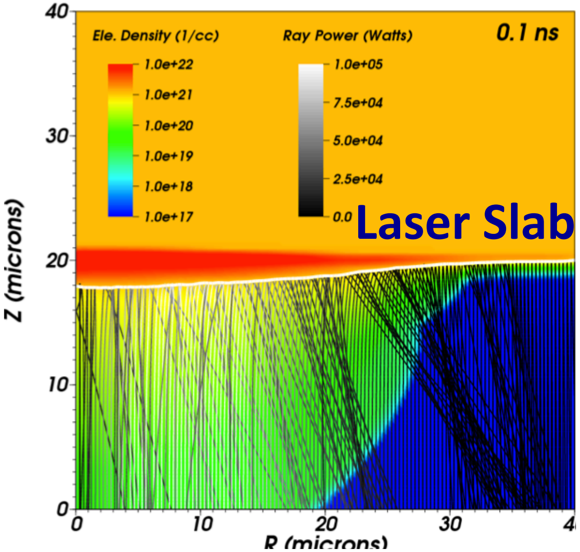
- Hydrodynamics, MHD, RHD, hybrid PIC
- Equation of States (Gamma laws, multigamma, Helmholtz)
- Nuclear Physics and other local source terms
- External gravity, Self-gravity
- Particles: active and passive
- Material Properties
- Cosmology

HEDP

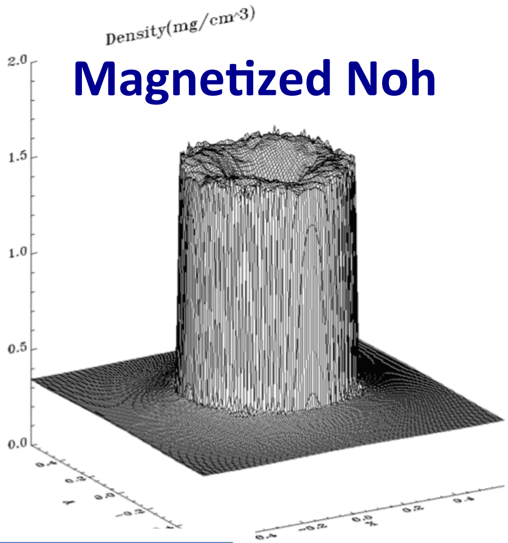
- Advancing 1T (single temperature), 2T (distinct ion + electron), and 3T (ion+ electron + radiation temperatures) in HD and MHD
- Implicit thermal conduction and flux limited multi-group radiation diffusion with HYPRE
- Multi-material support: EOS and opacities can combine tabular and analytic models for different materials
- Laser Ray Tracing for energy deposition
- Preliminary hybrid particle-in-cell capability (only in uniform grid)
- Ability to represent rigid bodies inside the flow

HEDP Applications

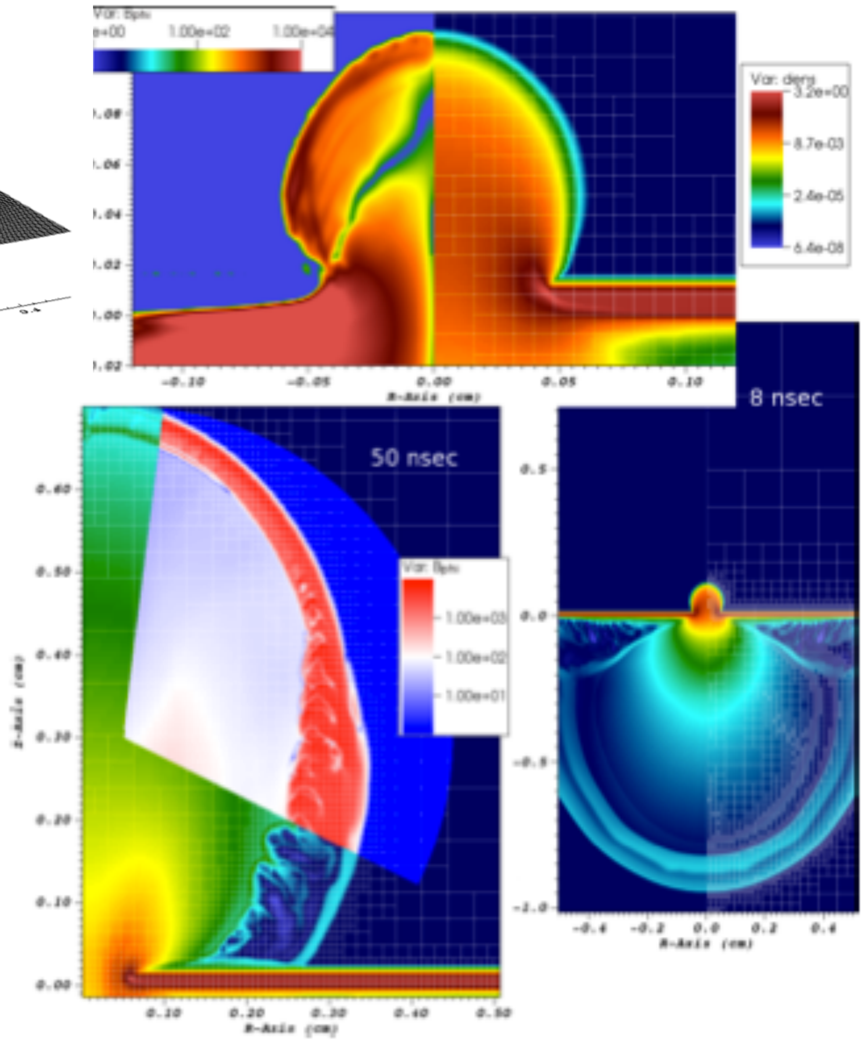
Laser Slab

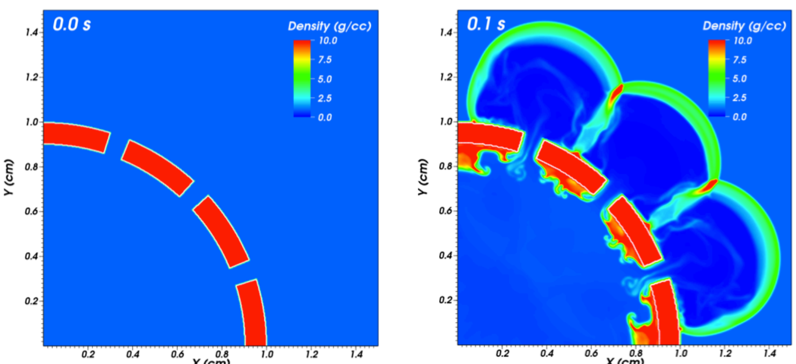


Magnetized Noh

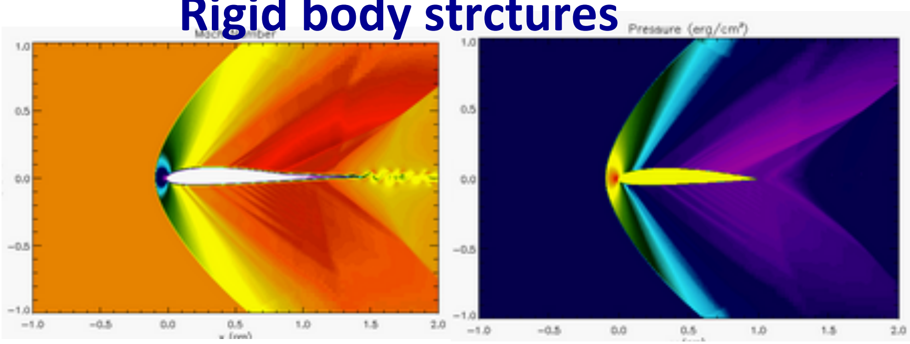


LULI/Vulcan experiments: B field generation/ amplification

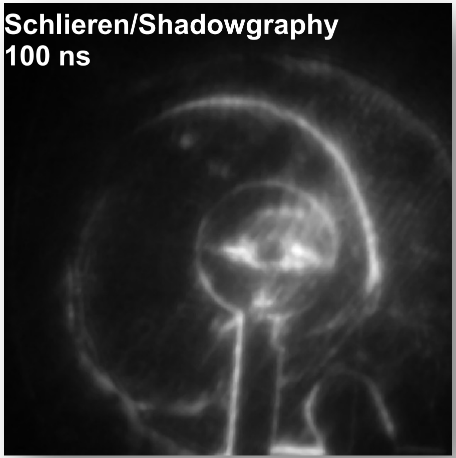


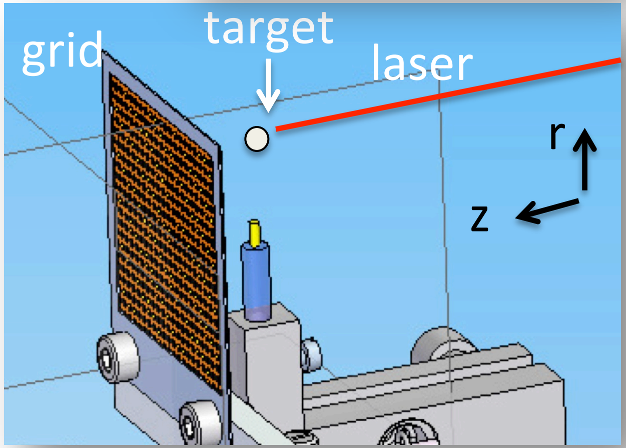


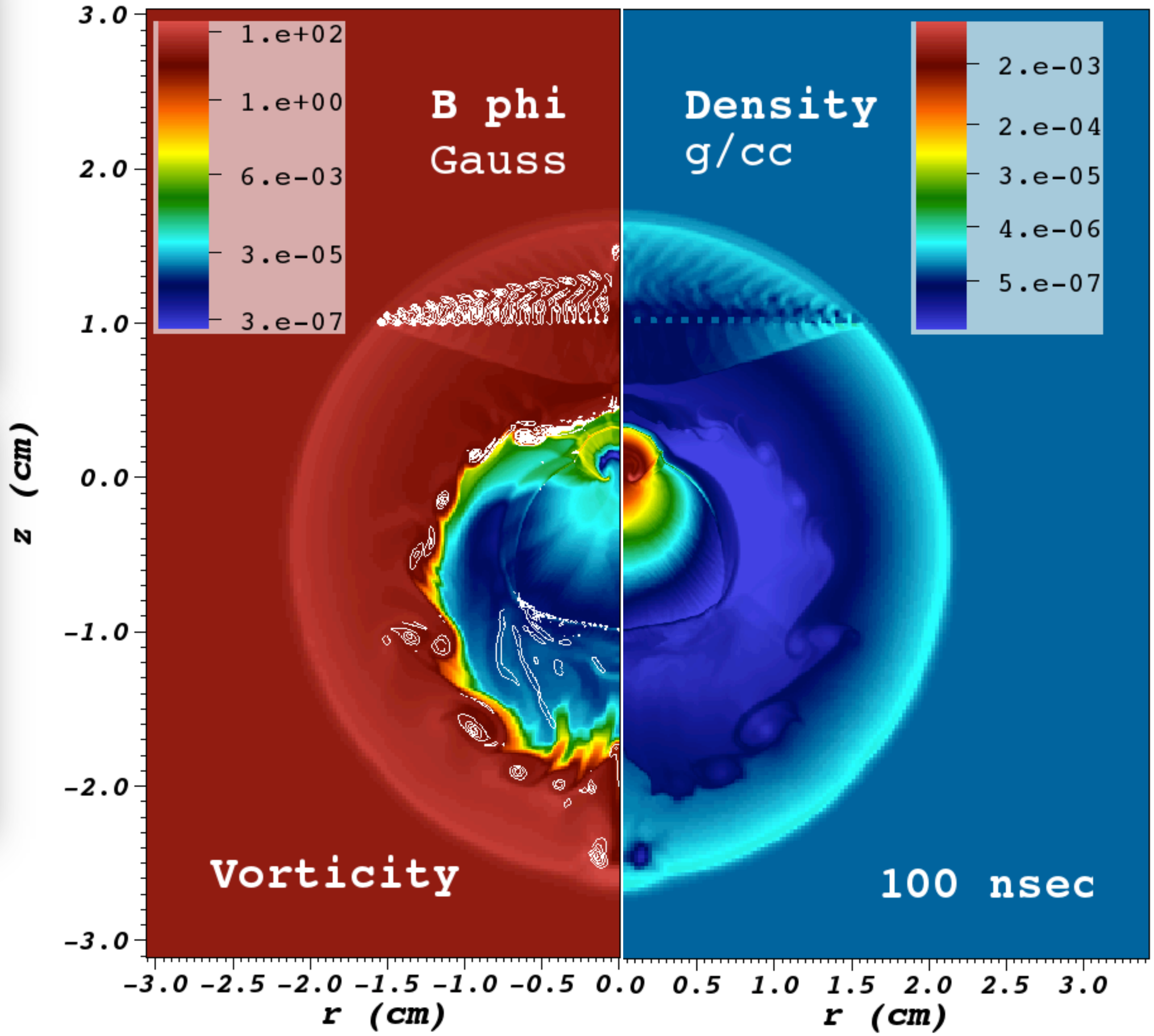
Rigid body strctures



B Field Generation Experiments at LULI



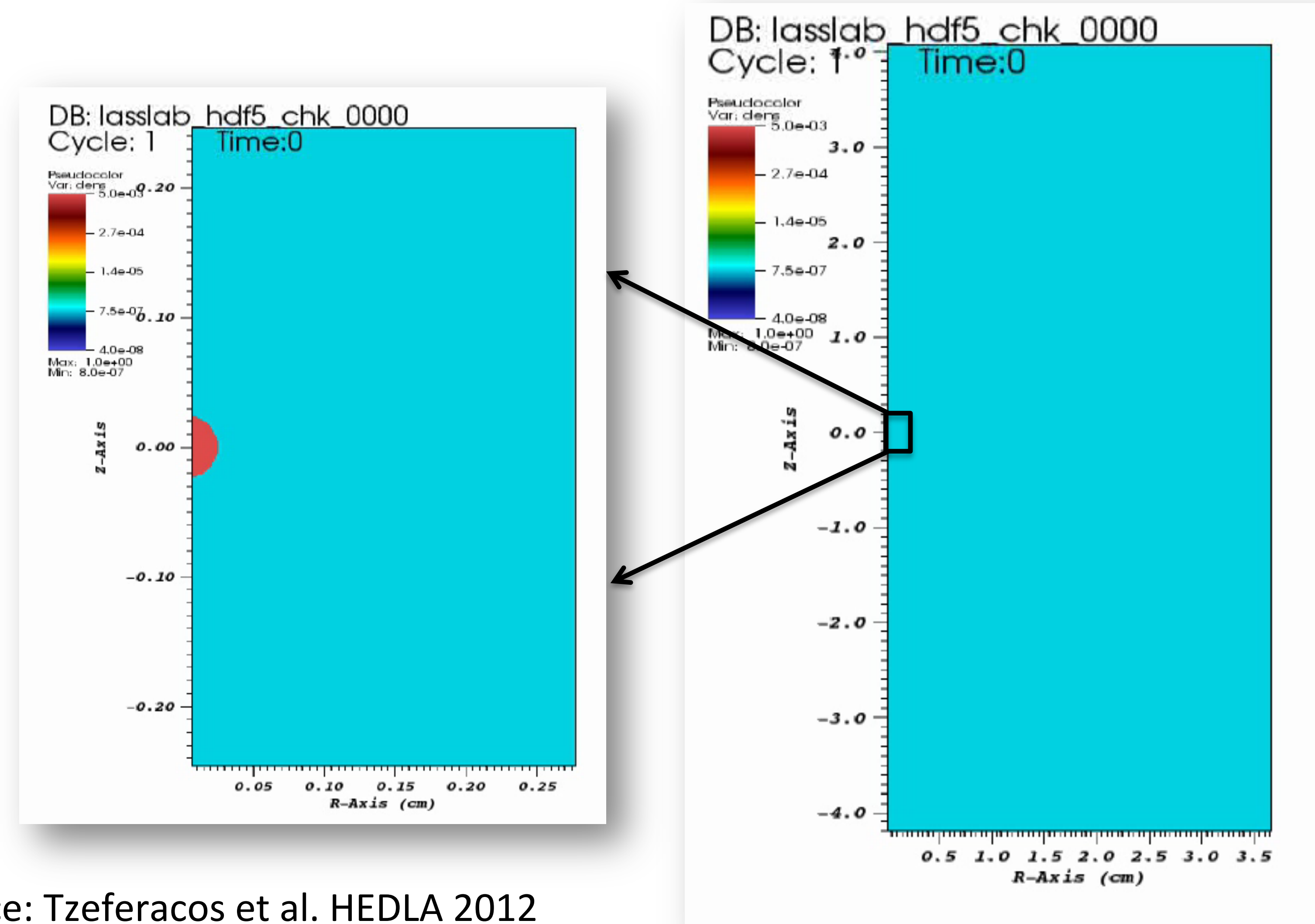




Sources:

- Gregori et al. Nature 2012
- Fatenejad et al. HEDLA 2012,
- Tzeferacos et al. HEDLA 2012

B Field Generation Experiments at LULI



Source: Tzeferacos et al. HEDLA 2012

Divergence-Free Fields: A New Upwind Constrained Transport (CT)-MHD Algorithm

Two USM Method Papers

An unsplit staggered mesh scheme for multidimensional magnetohydrodynamics

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A R T I C L E I N F O

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Magnetohydrodynamics
Constrained transport
Corner transport upwind
Unsplit scheme
Staggered mesh
High-order Godunov method

A B S T R A C T

We introduce an unsplit staggered mesh scheme (US
drodynamics (MHD) that uses a constrained transpor
nov fluxes and incorporates a new data reconstruct
order MHD interface states. In this new algorithm,
"multidimensional MHD terms", proportional to $\nabla \cdot$
a single update. This data reconstruction–evolution s
port upwind (CTU) approach of Colella, maintains in
by the advection of a very weak magnetic field loop i
tion algorithm is also of advantage in its consistency
The scheme maintains the $\nabla \cdot \mathbf{B} = 0$ constraint by sol
tions using the standard CT approach, where the ac
directly influences the quality of the magnetic field s
dissipative behavior in the simple electric field avera
ified electric field construction (MEC) that includes r
tion and enhances solution accuracy. A series of c
excellent performance of the full USM–MEC scheme
MHD test problems chosen from the literature. The s
freely available in the University of Chicago ASC FLA
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2D paper, JCP, 2009

A Solution Accurate, Efficient and Stable Unsplit Staggered Mesh Scheme for Three Dimensional Magnetohydrodynamics

Dongwook Lee
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3D paper, submitted to JCP, 2012

Abstract

In this paper, we extend the unsplit staggered mesh scheme (USM) for 2D magnetohydrodynamics (MHD) [D. Lee, A. Deane, An Unsplit Staggered Mesh Scheme for Multidimensional Magnetohydrodynamics, J. Comput. Phys. 228 (2009) 952–975] to a full 3D MHD scheme. The 3D scheme uses the same set of fundamental algorithmic ideas that have been developed in the 2D USM scheme. The scheme is a finite-volume Godunov method consisting of (1) a constrained transport (CT) method for preserving the solenoidal magnetic field evolution on a staggered grid, and (2) an efficient and accurate single-step, directionally unsplit multidimensional data reconstruction-evolution algorithm, which extends Colella's original 2D corner transport upwind (CTU) method [P. Colella, Multidimensional Upwind Methods for Hyperbolic Conservation Laws, J. Comput. Phys. 87 (1990) 446–466]. We present two types of data reconstruction-evolution algorithms for 3D: a reduced CTU scheme and a full CTU scheme. The reduced 3D CTU scheme is a variant of a direct 3D extension of Collela's 2D CTU method, whereas our full 3D CTU approach is a variant of the 3D unsplit CTU method by Saltzman [J. Saltzman, An unsplit 3D upwind method for hyperbolic conservation laws, J. Comput. Phys. 115 (1994) 153–168] for hyperbolic conservation laws. The key novelty in our algorithms is a new approach to account for transverse fluxes that stabilize an unsplit hyperbolic system *without* solving intermediate Riemann problems. The two schemes use multidimensional characteristic tracing to account for the stabilizing effects provided by incorporating the transverse fluxes in CTU. The proposed algorithms are simple and efficient especially when including multidimensional MHD source terms that maintain in-plane magnetic field dynamics. We also introduce a new CT scheme that makes use of proper upwind information in taking an average of electric fields. This method enhances numerical accuracy in solving the induction equations using the third-order spatially accurate modified electric field construction (MEC) scheme developed in 2D USM. We show that the new 3D USM-MHD algorithm provides a full CFL stability limit (CFL number ≤ 1) in most of the MHD test problems available in the literature, only requiring three Riemann problems (except for the extra three Riemann solves for the intermediate magnetic field update) per zone per time step in 3D. Our 3D USM schemes can be easily combined with various reconstruction methods (e.g., first-order Godunov, second-order MUSCL-Hancock, third-order PPM and fifth-order WENO), and a wide choice of Riemann solvers (e.g., local Lax-Friedrichs, HLLE, HLLC, HLLD, and Roe). The 3D USM-MHD solver is available in the University of Chicago Flash Center's official FLASH release.

Key words: MHD; Magnetohydrodynamics; Constrained Transport; Corner Transport Upwind; Unsplit Scheme; Staggered Mesh; High-Order Godunov Method; Large CFL Number.

Four (Three + One) CT Methods

- CT scheme by Balsara and Spicer, JCP, 1999:

$$E^{n+1/2}_{z,i+1/2,j+1/2,k} = \frac{1}{4}(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k})$$

- ε^c_z CT scheme by Gardiner and Stone, JCP, 2005

- MEC (modified electric field construction) scheme by Lee and Deane, JCP, 2009

➔ ***These three CT approaches ALL fail for a special case of field loop advection!!!***

- … and a new ***upwind biased MEC*** scheme, Lee, JCP, 2012, submitted

1. CT by Balsara & Spicer (BSCT)

- CT scheme by Balsara and Spicer, JCP, 1999:

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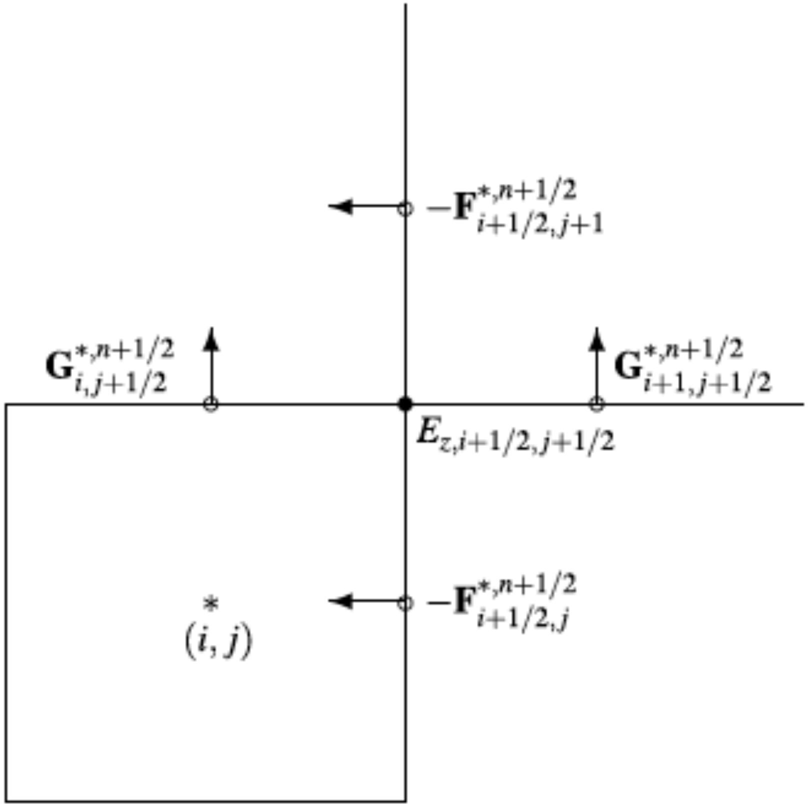

Lack of Upwind Consideration in BSCT

- CT scheme by Balsara and Spicer, 1998:

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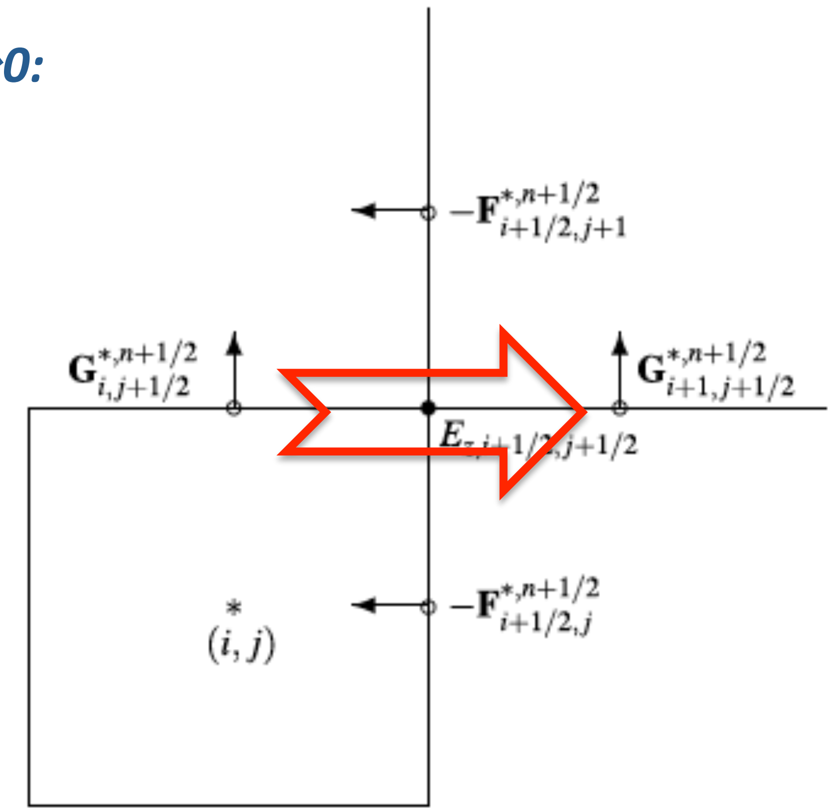
- Consider ***u>0, v→0:***



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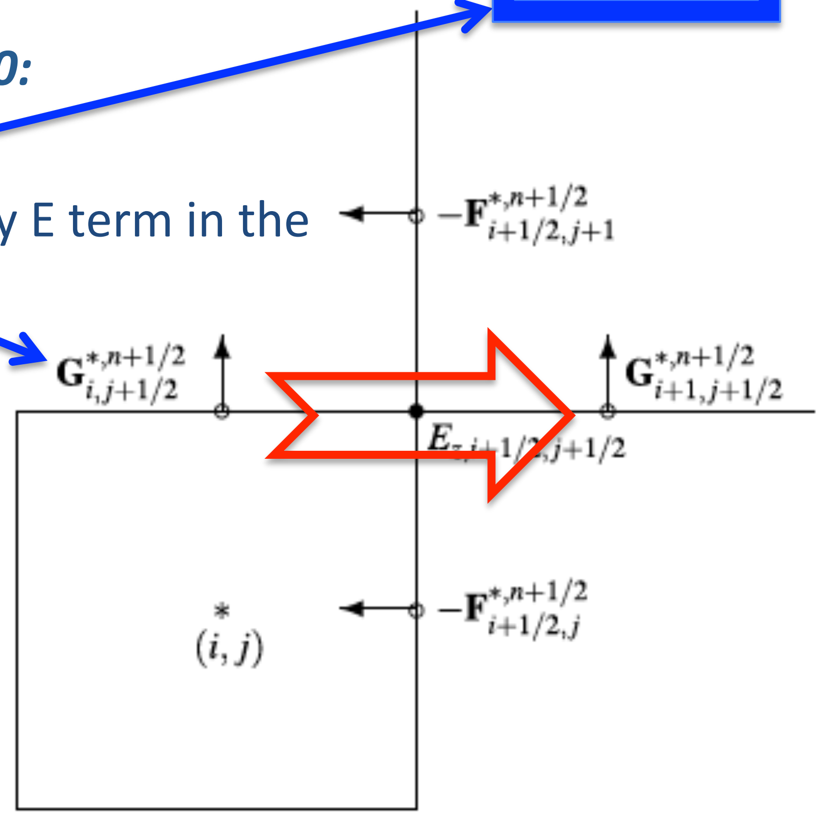
- Consider ***u>0, v→0:***

- $E^{*,n+1/2}_{z,i,j+1/2,k}$ is the only E term in the upwind direction!



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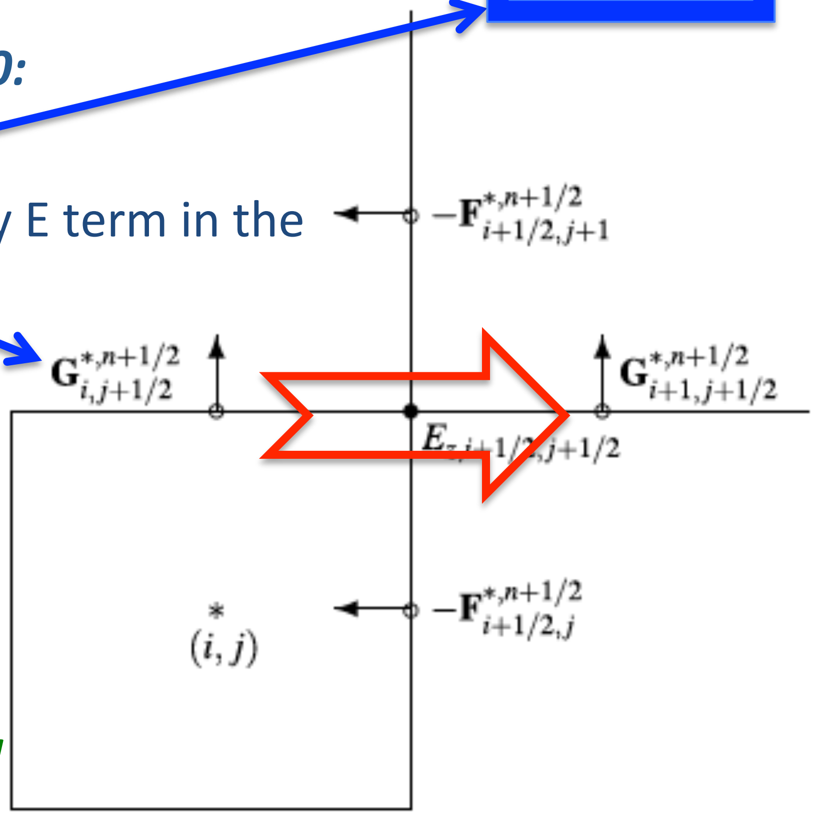
- Consider ***u>0, v→0:***
- $E^{*,n+1/2}_{z,i,j+1/2,k}$ is the only E term in the upwind direction!
- ***Numerical Oscillations!***
- ***This is true in most CT schemes!***



2. Gardiner & Stone's CT (GSCT)

- The method is consistent for plane-parallel, grid-aligned flows:

$$E_{z,i+1/2,j+1/2,k}^{n+1/2} = \frac{1}{4}\left(E_{z,i+1/2,j,k}^{*,n+1/2} + E_{z,i+1/2,j+1,k}^{*,n+1/2} + E_{z,i,j+1/2,k}^{*,n+1/2} + E_{z,i+1,j+1/2,k}^{*,n+1/2}\right)$$
$$+ \frac{\Delta y}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+1/4,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+3/4,k}\right)$$
$$+ \frac{\Delta x}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+1/4,j+1/2,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+3/4,j+1/2,k}\right)$$

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$$
\begin{aligned}
E^{n+1/2}_{z,i+1/2,j+1/2,k} &= \frac{1}{4}\left(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k}\right) \\
&+ \frac{\Delta y}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+1/4,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+3/4,k}\right) \\
&+ \frac{\Delta x}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+1/4,j+1/2,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+3/4,j+1/2,k}\right) \\
&= \frac{1}{4}(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k}) \\
&+ \frac{1}{4}(E^{*,n+1/2}_{z,i,j+1/2,k} - E^{*,n+1/2}_{z,i,j,k} - E^{*,n+1/2}_{z,i,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k}) \\
&+ \frac{1}{8}(E^{*,n+1/2}_{z,i+1/2,j,k} - E^{*,n+1/2}_{z,i,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} - E^{*,n+1/2}_{z,i,j+1,k}) \\
&- \frac{1}{8}(E^{*,n+1/2}_{z,i+1,j,k} - E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1,j+1,k} - E^{*,n+1/2}_{z,i+1/2,j+1,k}).
\end{aligned}
$$

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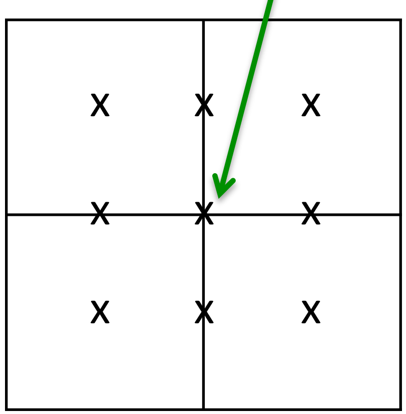
$$
\begin{aligned}
E^{n+1/2}_{z,i+1/2,j+1/2,k} = & \frac{1}{4}\left(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k}\right) \\
& + \frac{\Delta y}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+1/4,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+3/4,k}\right) \\
& + \frac{\Delta x}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+1/4,j+1/2,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+3/4,j+1/2,k}\right) \\
= & \frac{1}{4}(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k}) \\
& + \frac{1}{4}(E^{*,n+1/2}_{z,i,j+1/2,k} - E^{*,n+1/2}_{z,i,j,k} - E^{*,n+1/2}_{z,i,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k}) \\
& + \frac{1}{8}(E^{*,n+1/2}_{z,i+1/2,j,k} - E^{*,n+1/2}_{z,i,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} - E^{*,n+1/2}_{z,i,j+1,k}) \\
& - \frac{1}{8}(E^{*,n+1/2}_{z,i+1,j,k} - E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1,j+1,k} - E^{*,n+1/2}_{z,i+1/2,j+1,k}).
\end{aligned}
$$



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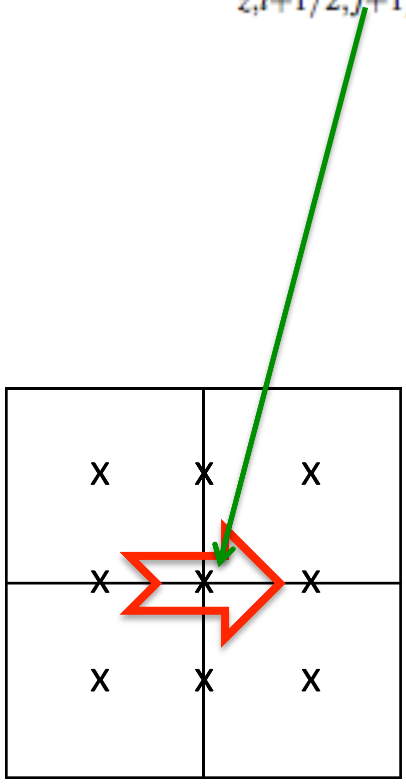
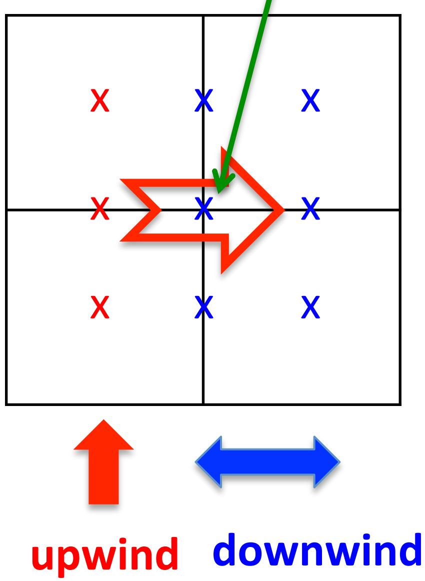
$$+ \frac{\Delta y}{8}\left(\left.\frac{\partial E^{*,n+1/2}_z}{\partial y}\right|_{i+1/2,j+1/4,k} - \left.\frac{\partial E^{*,n+1/2}_z}{\partial y}\right|_{i+1/2,j+3/4,k}\right)$$
$$+ \frac{\Delta x}{8}\left(\left.\frac{\partial E^{*,n+1/2}_z}{\partial x}\right|_{i+1/4,j+1/2,k} - \left.\frac{\partial E^{*,n+1/2}_z}{\partial x}\right|_{i+3/4,j+1/2,k}\right)$$
$$= \frac{1}{4}(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k})$$
$$+ \frac{1}{4}(E^{*,n+1/2}_{z,i,j+1/2,k} - E^{*,n+1/2}_{z,i,j,k} - E^{*,n+1/2}_{z,i,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k})$$
$$+ \frac{1}{8}(E^{*,n+1/2}_{z,i+1/2,j,k} - E^{*,n+1/2}_{z,i,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} - E^{*,n+1/2}_{z,i,j+1,k})$$
$$- \frac{1}{8}(E^{*,n+1/2}_{z,i+1,j,k} - E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1,j+1,k} - E^{*,n+1/2}_{z,i+1/2,j+1,k}).$$



2. Gardiner & Stone's CT (GSCT)

- The method is consistent for plane-parallel, grid-aligned flows:

$$E^{n+1/2}_{z,i+1/2,j+1/2,k} = \frac{1}{4}\left(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k}\right)$$
$$+ \frac{\Delta y}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+1/4,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial y}\right|_{i+1/2,j+3/4,k}\right)$$
$$+ \frac{\Delta x}{8}\left(\left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+1/4,j+1/2,k} - \left.\frac{\partial E_z^{*,n+1/2}}{\partial x}\right|_{i+3/4,j+1/2,k}\right)$$
$$= \frac{1}{4}(E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k} + E^{*,n+1/2}_{z,i+1,j+1/2,k})$$
$$+ \frac{1}{4}(E^{*,n+1/2}_{z,i,j+1/2,k} - E^{*,n+1/2}_{z,i,j,k} - E^{*,n+1/2}_{z,i,j+1,k} + E^{*,n+1/2}_{z,i,j+1/2,k})$$
$$+ \frac{1}{8}(E^{*,n+1/2}_{z,i+1/2,j,k} - E^{*,n+1/2}_{z,i,j,k} + E^{*,n+1/2}_{z,i+1/2,j+1,k} - E^{*,n+1/2}_{z,i,j+1,k})$$
$$- \frac{1}{8}(E^{*,n+1/2}_{z,i+1,j,k} - E^{*,n+1/2}_{z,i+1/2,j,k} + E^{*,n+1/2}_{z,i+1,j+1,k} - E^{*,n+1/2}_{z,i+1/2,j+1,k}).$$



3. Standard-MEC (LD-MEC)

- 3rd order modified electric field construction (standard-MEC), Lee & Deane, JCP, 2009:

$$
\begin{aligned}
E^{n+1/2}_{z,i+1/2,j+1/2,k} = \Bigg[& \left(E^{*,n+1/2}_{z,i+1/2,j,k} + \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y^2} \right) + \\
& \left(E^{*,n+1/2}_{z,i+1/2,j+1,k} - \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y^2} \right) + \\
& \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right) + \\
& \left(E^{*,n+1/2}_{z,i+1,j+1/2,k} - \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x^2} \right) \Bigg].
\end{aligned}
$$

3. Standard-MEC (LD-MEC)

- 3rd order modified electric field construction (standard-MEC), Lee & Deane, JCP, 2009:

$$
\begin{aligned}
E^{n+1/2}_{z,i+1/2,j+1/2,k} = \Bigg[& \left(E^{*,n+1/2}_{z,i+1/2,j,k} + \frac{\Delta y}{2}\frac{\partial E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y} + \frac{\Delta y^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y^2} \right) + \\
& \left(E^{*,n+1/2}_{z,i+1/2,j+1,k} - \frac{\Delta y}{2}\frac{\partial E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y} + \frac{\Delta y^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y^2} \right) + \\
& \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2}\frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right) + \\
& \left(E^{*,n+1/2}_{z,i+1,j+1/2,k} - \frac{\Delta x}{2}\frac{\partial E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x^2} \right) \Bigg].
\end{aligned}
$$

Only upwind term!

4. New Upwind-MEC (U-MEC)

- New upwind biased modified electric field construction(upwind-MEC), Lee, JCP, submitted, 2012

$$E^{n+1/2}_{z,i+1/2,j+1/2,k} = \alpha\Bigg[v_P\left(E^{*,n+1/2}_{z,i+1/2,j,k} + \frac{\Delta y}{2}\frac{\partial E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y} + \frac{\Delta y^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y^2}\right) +$$
$$v_N\left(E^{*,n+1/2}_{z,i+1/2,j+1,k} - \frac{\Delta y}{2}\frac{\partial E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y} + \frac{\Delta y^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y^2}\right) +$$
$$u_P\left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2}\frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2}\right) +$$
$$u_N\left(E^{*,n+1/2}_{z,i+1,j+1/2,k} - \frac{\Delta x}{2}\frac{\partial E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x^2}\right)\Bigg].$$

$$u_P = \frac{1}{2}(1+\mathrm{sign}(u_{i+1/2,j+1/2}))|\mathrm{sign}(u_{i+1/2,j+1/2})|,$$
$$u_N = \frac{1}{2}(1-\mathrm{sign}(u_{i+1/2,j+1/2}))|\mathrm{sign}(u_{i+1/2,j+1/2})|$$
$$v_P = \frac{1}{2}(1+\mathrm{sign}(v_{i+1/2,j+1/2}))|\mathrm{sign}(v_{i+1/2,j+1/2})|$$
$$v_N = \frac{1}{2}(1-\mathrm{sign}(v_{i+1/2,j+1/2}))|\mathrm{sign}(v_{i+1/2,j+1/2})|,$$

$$\mathrm{sign}(x) = \begin{cases} 1 & \text{if } x>0, \\ 0 & \text{if } x=0, \\ -1 & \text{if } x<0. \end{cases}$$

4. New Upwind-MEC (U-MEC)

- New upwind biased modified electric field construction(upwind-MEC), Lee, JCP, submitted, 2012

$$
\begin{aligned}
E^{n+1/2}_{z,i+1/2,j+1/2,k} = \alpha \Bigg[& v_P \left(E^{*,n+1/2}_{z,i+1/2,j,k} + \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y^2} \right) + \\
& v_N \left(E^{*,n+1/2}_{z,i+1/2,j+1,k} - \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y^2} \right) + \\
& u_P \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right) + \\
& u_N \left(E^{*,n+1/2}_{z,i+1,j+1/2,k} - \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x^2} \right) \Bigg].
\end{aligned}
$$

Upwinding Weights

$$u_P = \frac{1}{2}\left(1+\mathrm{sign}(u_{i+1/2,j+1/2})\right)\left|\mathrm{sign}(u_{i+1/2,j+1/2})\right|,$$

$$u_N = \frac{1}{2}\left(1-\mathrm{sign}(u_{i+1/2,j+1/2})\right)\left|\mathrm{sign}(u_{i+1/2,j+1/2})\right|$$

$$v_P = \frac{1}{2}\left(1+\mathrm{sign}(v_{i+1/2,j+1/2})\right)\left|\mathrm{sign}(v_{i+1/2,j+1/2})\right|$$

$$v_N = \frac{1}{2}\left(1-\mathrm{sign}(v_{i+1/2,j+1/2})\right)\left|\mathrm{sign}(v_{i+1/2,j+1/2})\right|,$$

$$\mathrm{sign}(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x = 0, \\ -1 & \text{if } x < 0. \end{cases}$$

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- New upwind biased modified electric field construction(upwind-MEC), Lee, JCP, submitted, 2012. **Considering *u>0, v→0*:**

$$E^{n+1/2}_{z,i+1/2,j+1/2,k} = \alpha \Bigg[v_P \left(E^{*,n+1/2}_{z,i+1/2,j,k} + \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j,k}}{\partial y^2} \right) +$$
$$v_N \left(E^{*,n+1/2}_{z,i+1/2,j+1,k} - \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y^2} \right) +$$
$$u_P \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right) +$$
$$u_N \left(E^{*,n+1/2}_{z,i+1,j+1/2,k} - \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x^2} \right) \Bigg].$$

Upwinding Weights

$$u_P = \frac{1}{2}(1+\mathrm{sign}(u_{i+1/2,j+1/2}))|\mathrm{sign}(u_{i+1/2,j+1/2})|,$$
$$u_N = \frac{1}{2}(1-\mathrm{sign}(u_{i+1/2,j+1/2}))|\mathrm{sign}(u_{i+1/2,j+1/2})|$$
$$v_P = \frac{1}{2}(1+\mathrm{sign}(v_{i+1/2,j+1/2}))|\mathrm{sign}(v_{i+1/2,j+1/2})|$$
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$$v_N \left(E^{*,n+1/2}_{z,i+1/2,j+1,k} - \frac{\Delta y}{2} \frac{\partial E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y} + \frac{\Delta y^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1/2,j+1,k}}{\partial y^2} \right) +$$

$$u_P \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right) +$$

$$u_N \left(E^{*,n+1/2}_{z,i+1,j+1/2,k} - \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i+1,j+1/2,k}}{\partial x^2} \right) \Bigg].$$

$$u_P = \frac{1}{2}(1 + \text{sign}(u_{i+1/2,j+1/2}))|\text{sign}(u_{i+1/2,j+1/2})|,$$

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$$v_P = \frac{1}{2}(1 + \text{sign}(v_{i+1/2,j+1/2}))|\text{sign}(v_{i+1/2,j+1/2})|$$

$$v_N = \frac{1}{2}(1 - \text{sign}(v_{i+1/2,j+1/2}))|\text{sign}(v_{i+1/2,j+1/2})|,$$

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- New upwind biased modified electric field construction(upwind-MEC), Lee, JCP, submitted, 2012. **Considering *u>0, v→0*:**

$$E^{n+1/2}_{z,i+1/2,j+1/2,k} =$$

$$\left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2}\frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2}\right)$$

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$$E^{n+1/2}_{z,i+1/2,j+1/2,k} = \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2} \frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8} \frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right)$$

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- New upwind biased modified electric field construction(upwind-MEC), Lee 2012:

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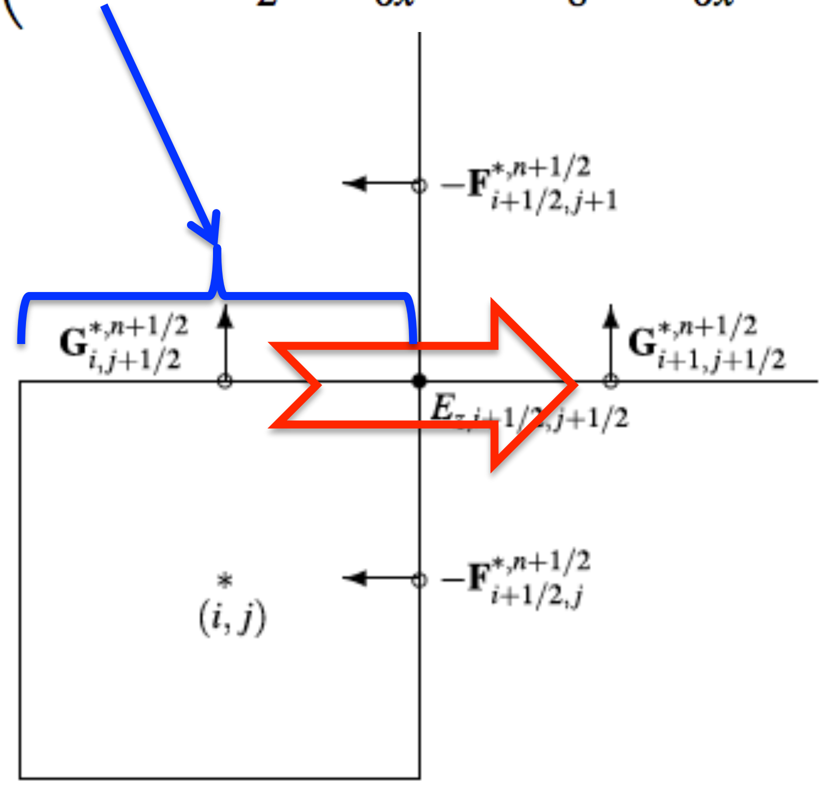
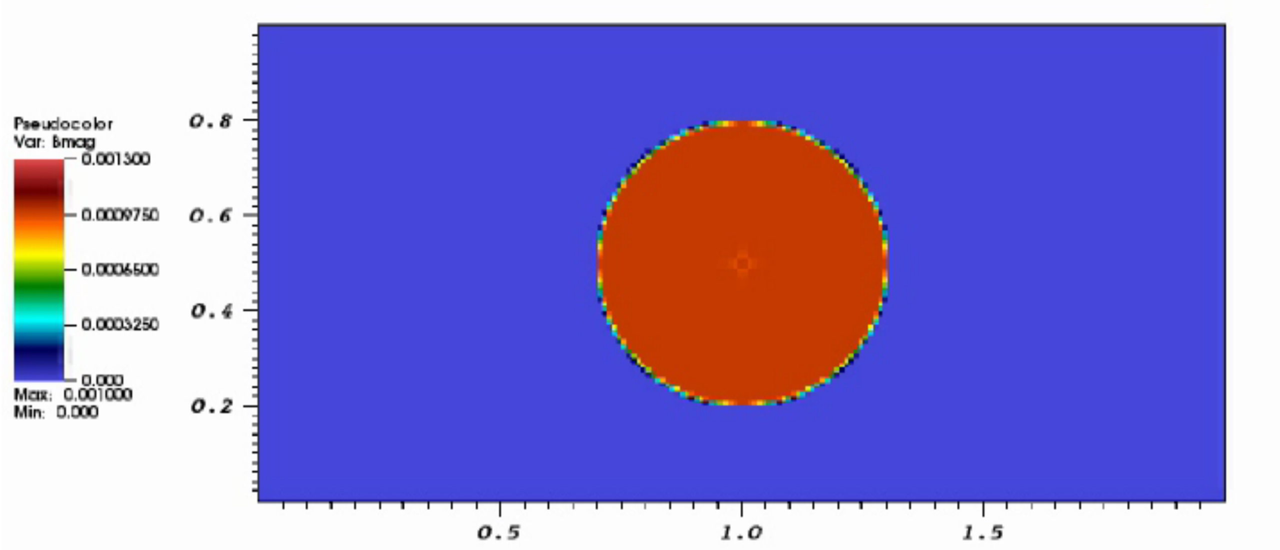
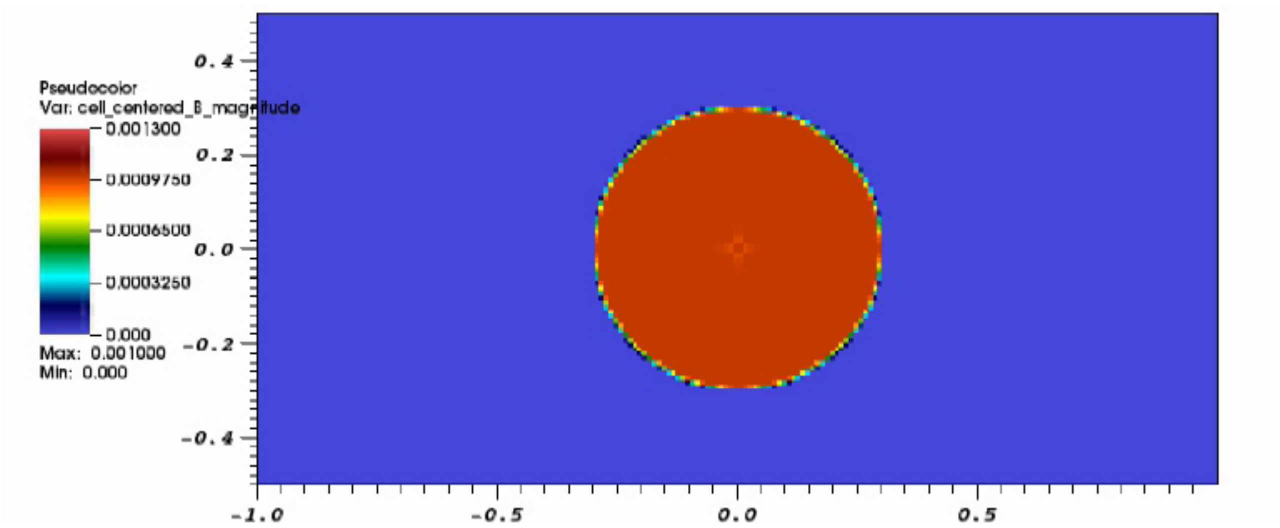


BSCT & GSCT


Pseudocolor
Var: Bmag
0.001300
0.0009750
0.0006500
0.0003250
0.000
Max: 0.001000
Min: 0.000
0.8
0.6
0.4
0.2
0.5
1.0
1.5


Pseudocolor
Var: cell_centered_B_magnitude
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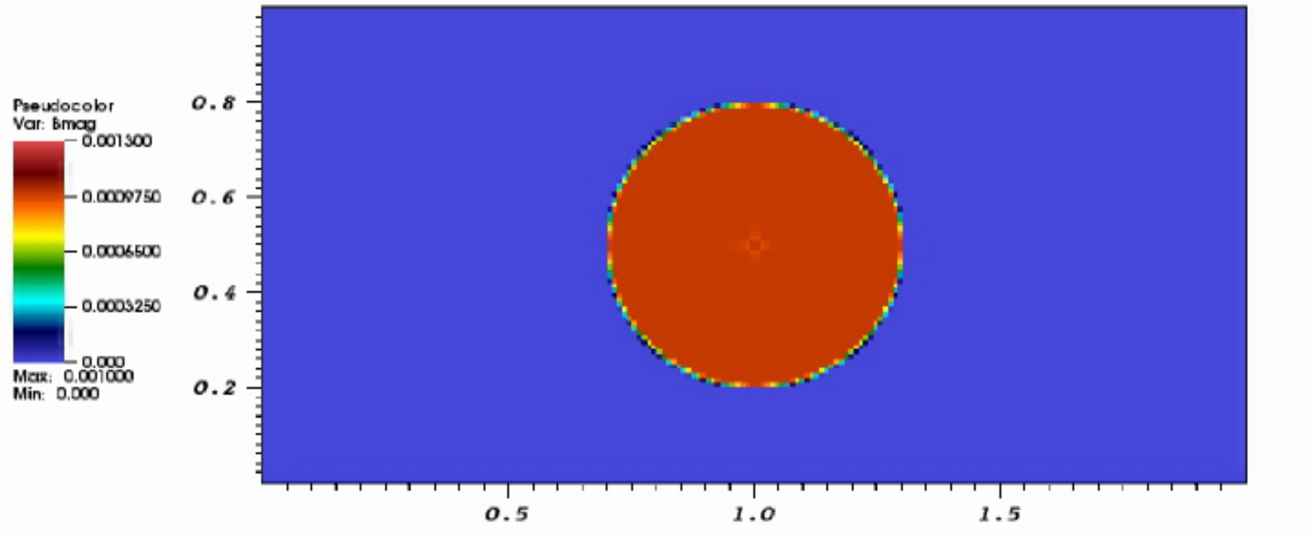
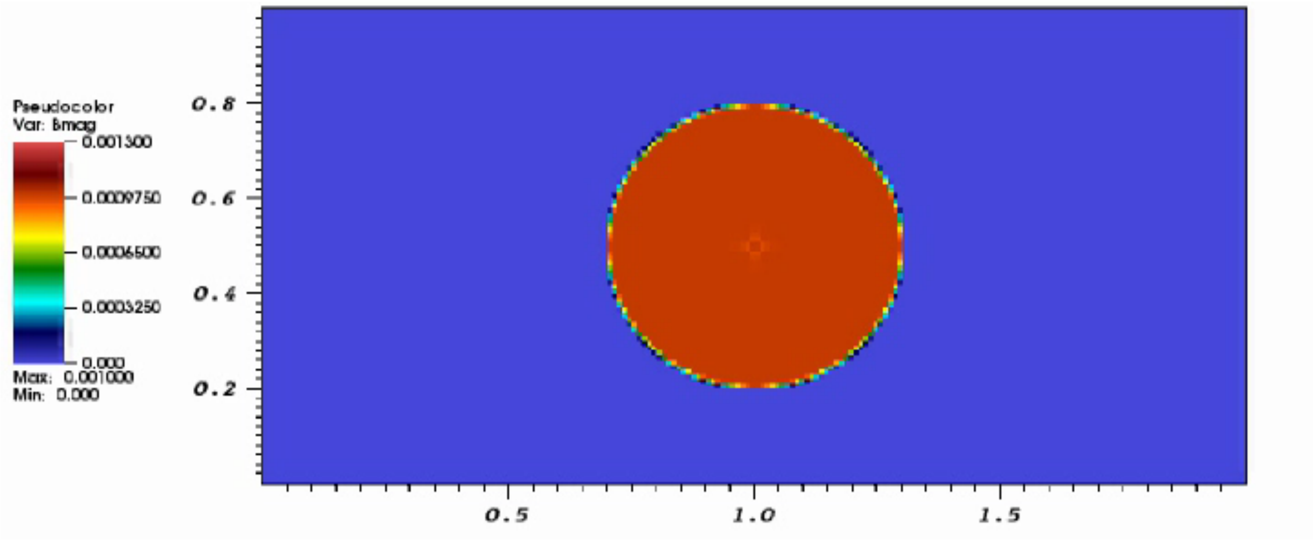
0.0009750
0.0006500
0.0003250
0.000
Max: 0.001000
Min: 0.000
0.4
0.2
0.0
-0.2
-0.4
-1.0
-0.5
0.0
0.5

LD-MEC & BSCT with Upwind

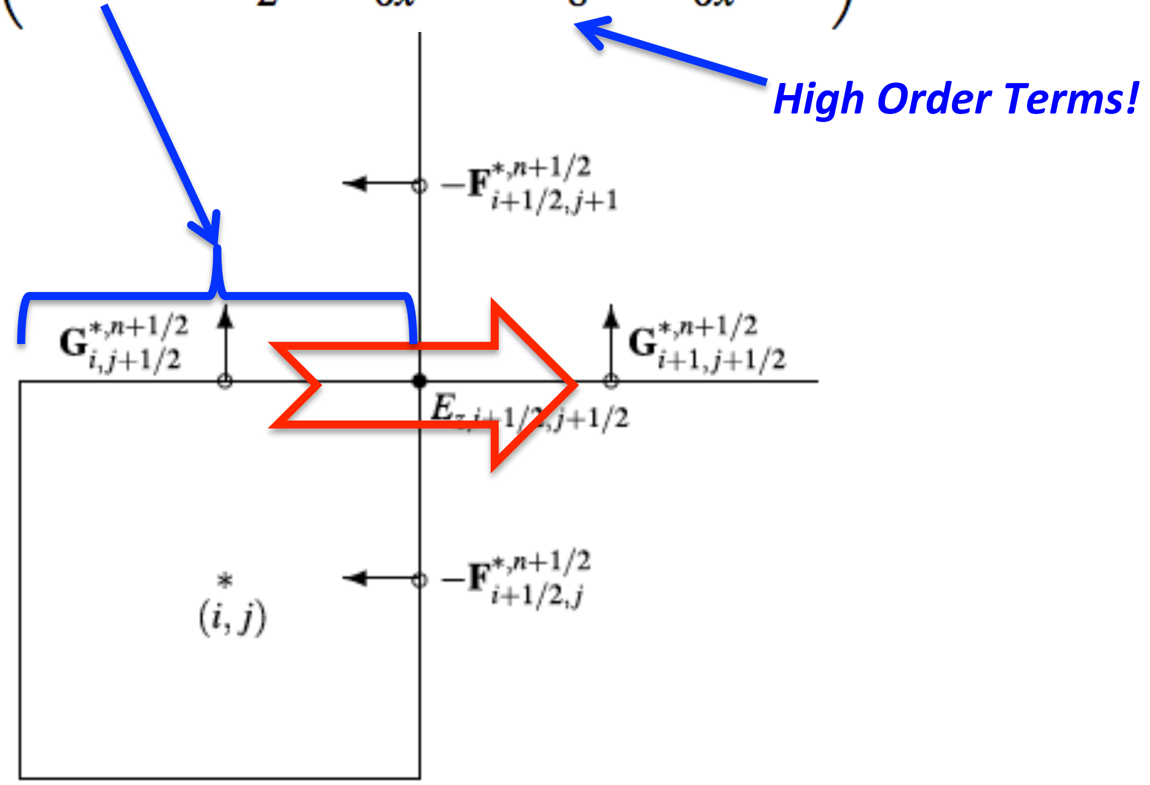




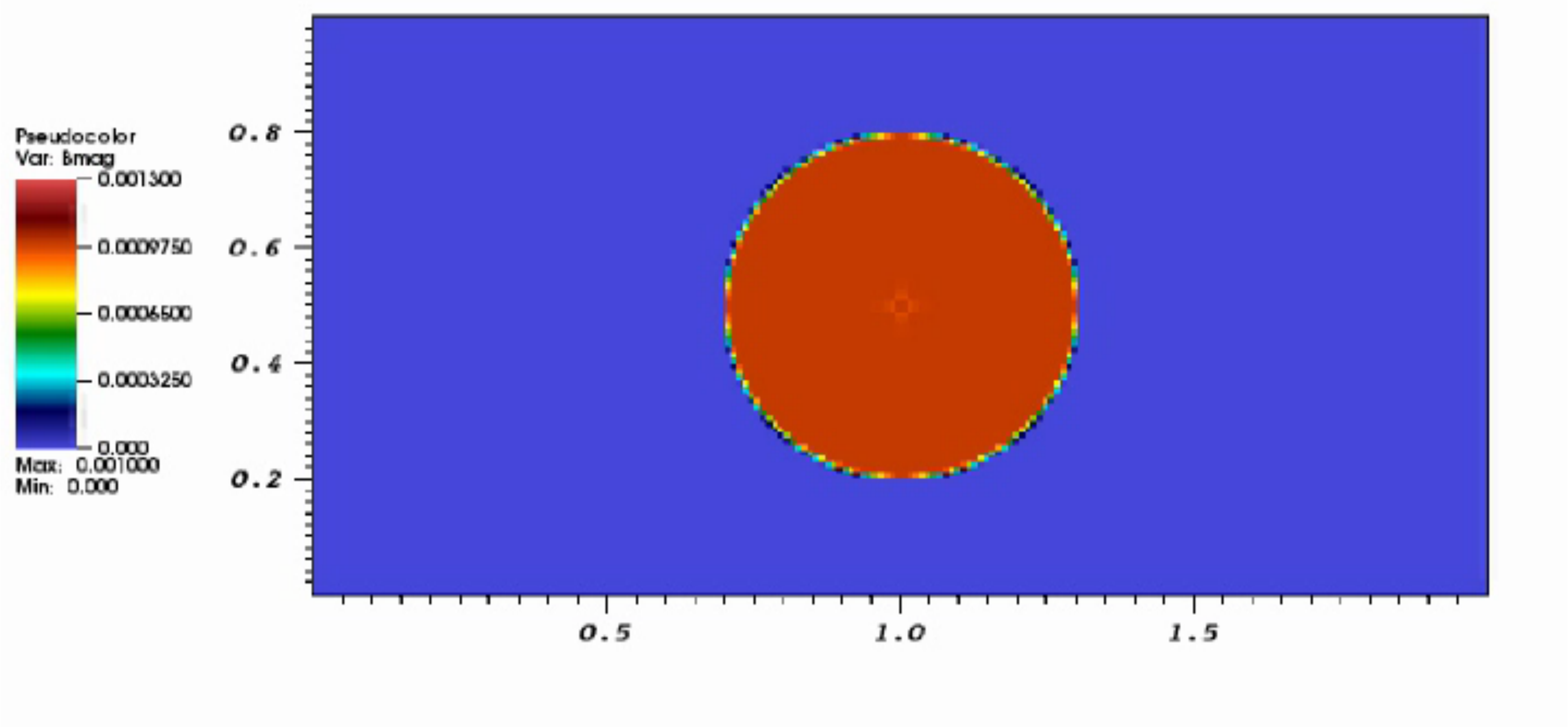
4. New Upwind-MEC (U-MEC)

- New upwind biased modified electric field construction(upwind-MEC), Lee 2012:

$$E^{n+1/2}_{z,i+1/2,j+1/2,k} = \left(E^{*,n+1/2}_{z,i,j+1/2,k} + \frac{\Delta x}{2}\frac{\partial E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x} + \frac{\Delta x^2}{8}\frac{\partial^2 E^{*,n+1/2}_{z,i,j+1/2,k}}{\partial x^2} \right)$$

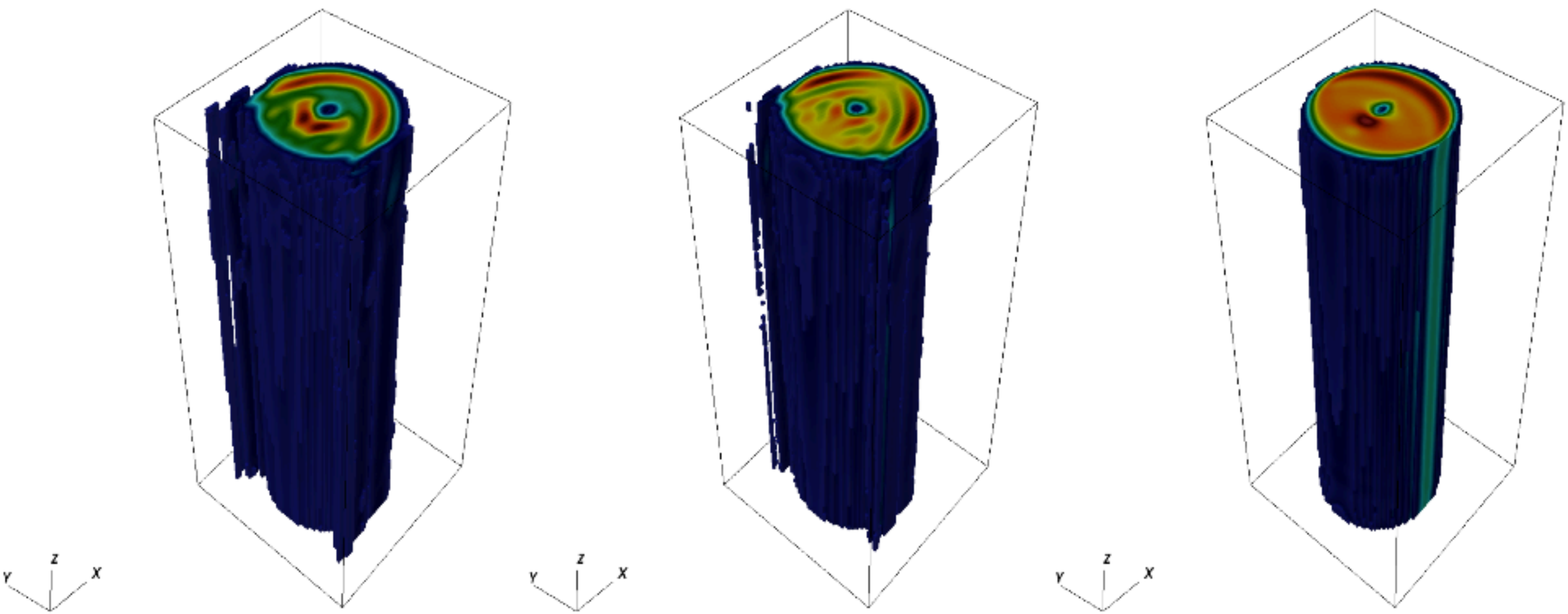


U-MEC


Pseudocolor
Var: Bmag
0.001300
0.0009750
0.0006500
0.0003250
0.000
Max: 0.001000
Min: 0.000
0.8
0.6
0.4
0.2
0.5
1.0
1.5

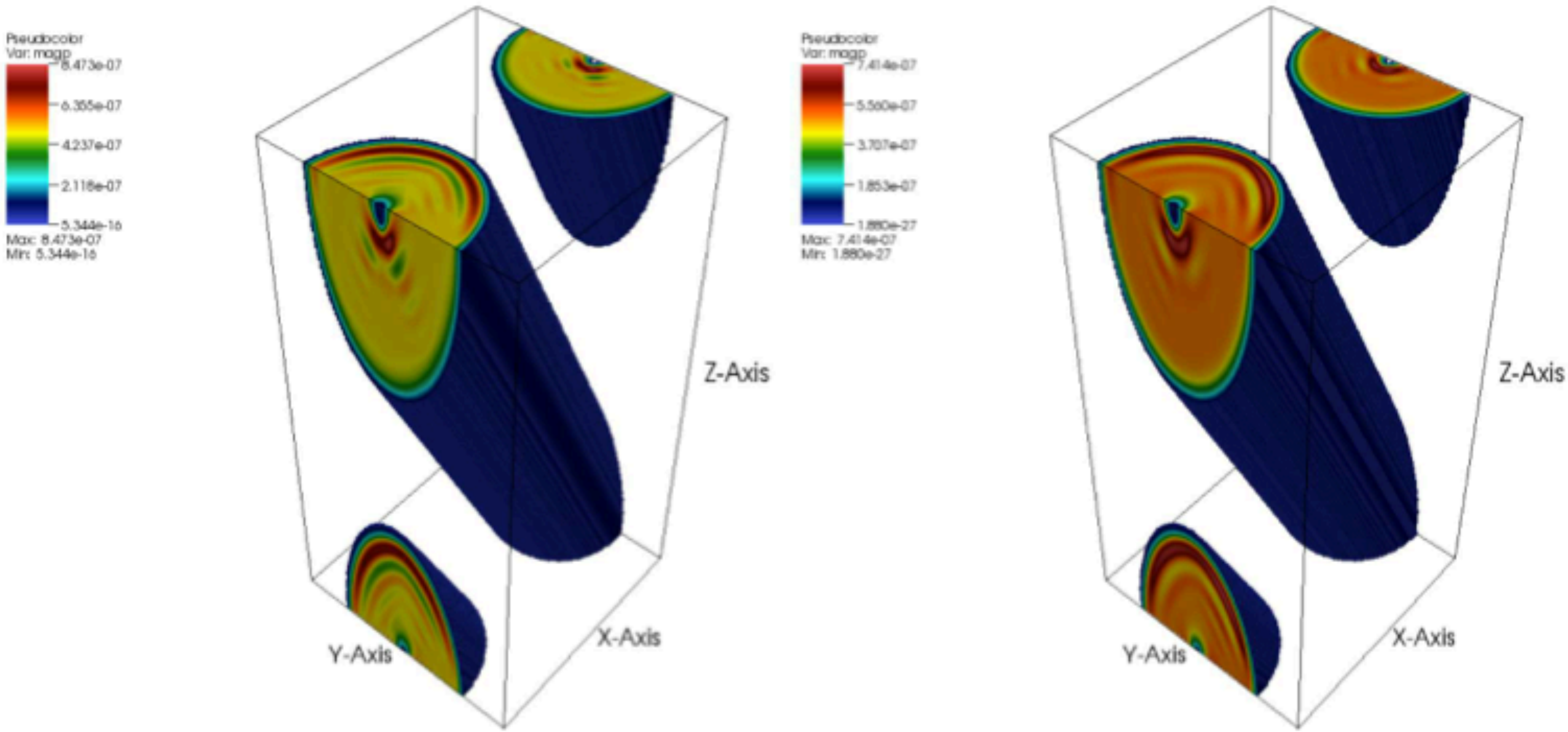
BSCT, LD-MEC & U-MEC

- Small angle advection of the 3D field loop: $\theta = \tan^{-1}(0.01) = 0.573^{\circ}$



(a) B_p using the standard CT at $t = 2$. (b) B_p using the standard MEC at $t = 2$. (c) B_p using the upwind MEC at $t = 2$.

LD-MEC & U-MEC



(a) B_p using the standard-MEC CT at $t = 1$.

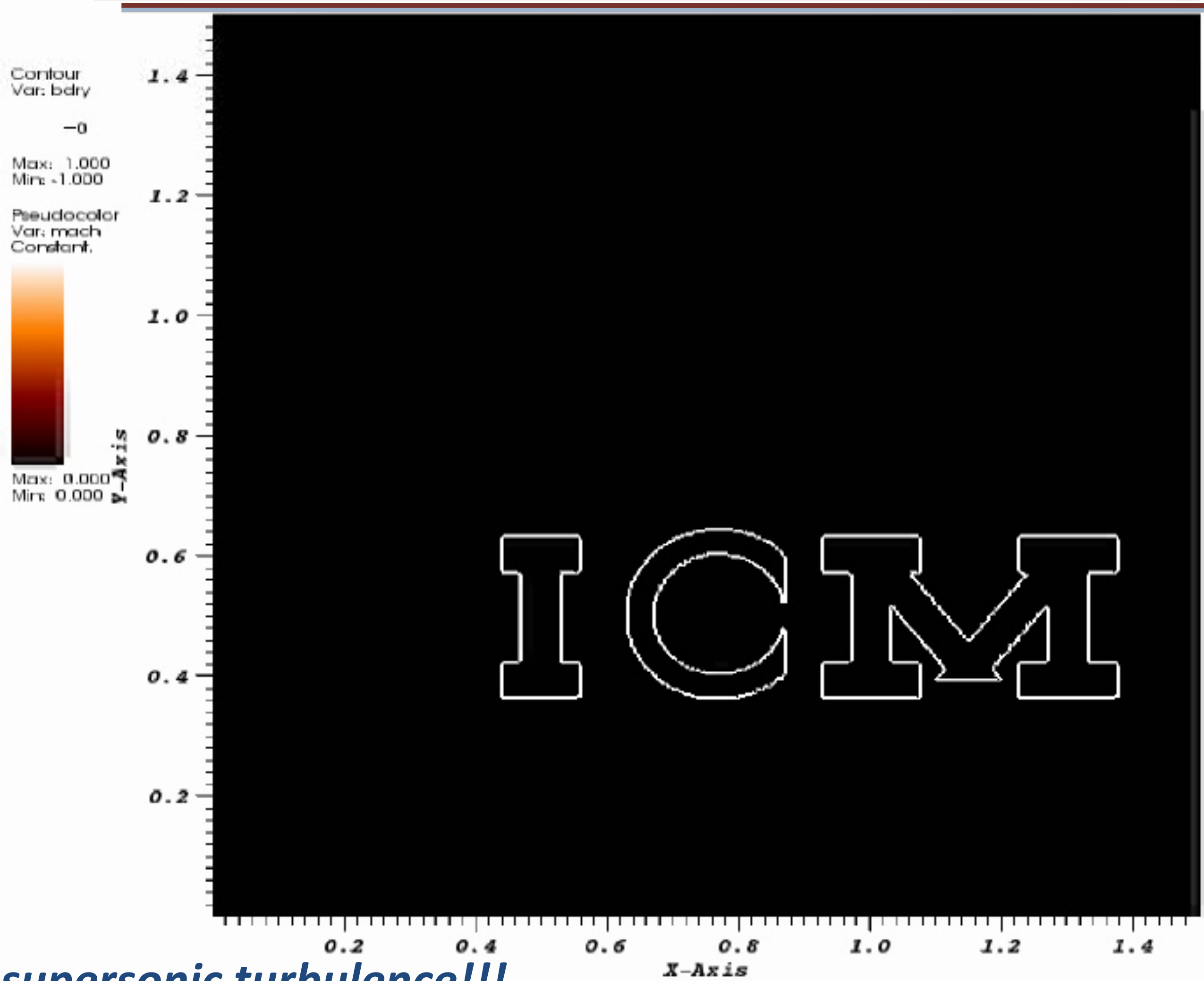
(b) B_p using the upwind-MEC at $t = 1$.

Fig. 6. The standard field loop advection problem at time $t = 1$ using PPM and the Roe Riemann solver. The minmod slope limiter is used for taking slope gradients of characteristic variable in the PPM reconstruction step. All results are resolved on $128 \times 128 \times 256$ grid cells.

Conclusions

- ***Part 1: Applications***
  - FLASH has been extended to simulating HEDP lab experiments
  - Validating lab experiments using simulations is doable but hard
  - If successful, simulations can provide strong support in experimental design, data analysis through extensive diagnostics, and serve as a powerful tool for predictive science!
- ***Part 2: Algorithms***
  - A small angle field loop advection test reveals a special care for CT in conventional Godunov schemes based on 1D Riemann solves
  - An upwinding scheme can improve handling numerical dissipations in preserving circular symmetry for the small angle advection
  - Alternatively, a genuinely multidimensional Riemann solves (Balsara, 2010) can be used

Thanks!



- ❑ ***ICM supersonic turbulence!!!***
  - ❑ Rigid body structures, PPM, MC limiter on characteristic variables with HLLC on 400x400