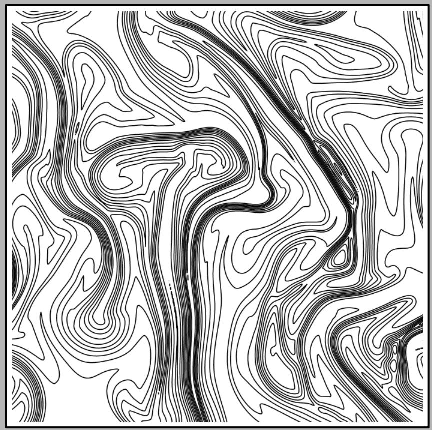
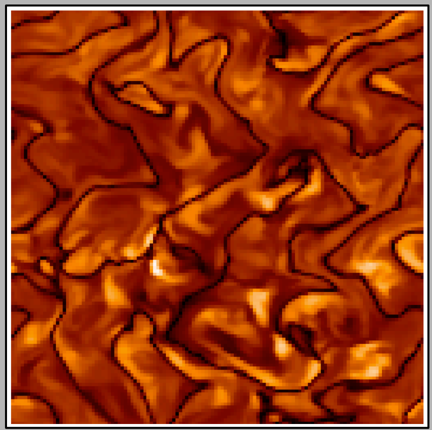
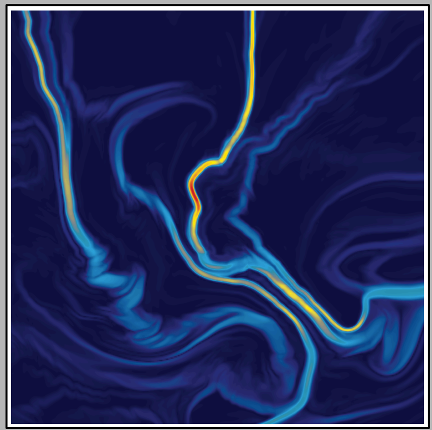
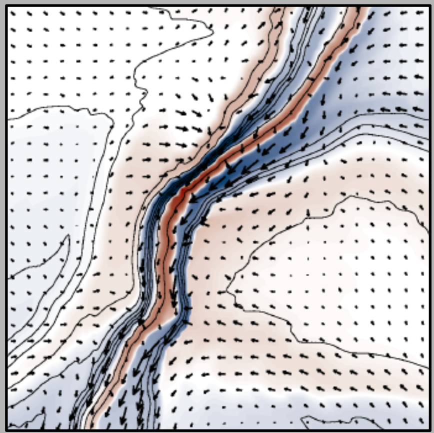
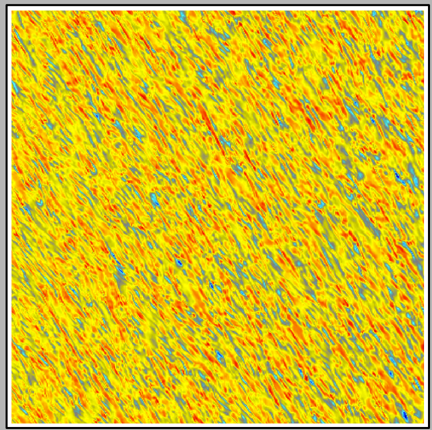
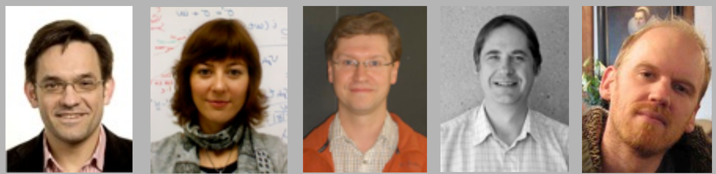
Matthew Kunz

NASA Einstein Fellow – Princeton University



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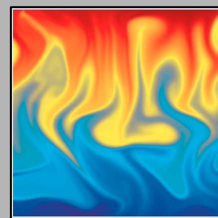
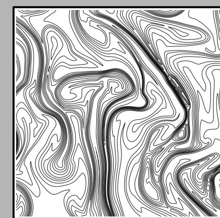
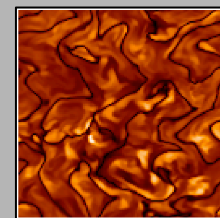
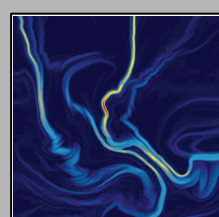
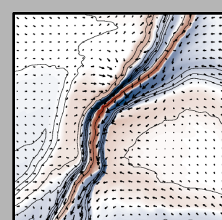
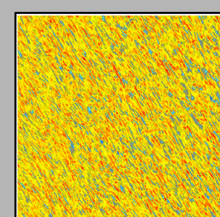
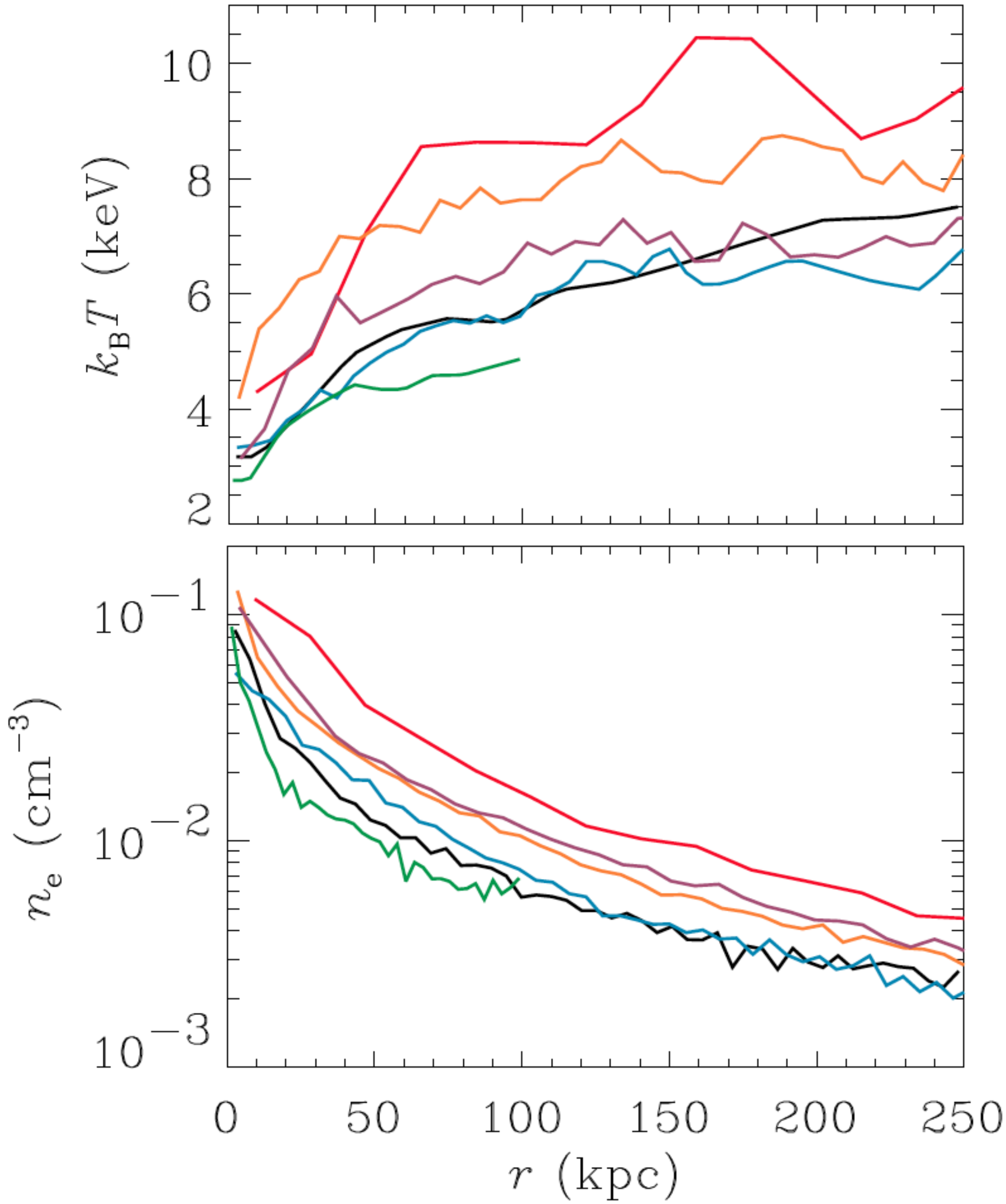




Multiscale Plasma Dynamics & Anisotropic Transport in the Intracluster Medium





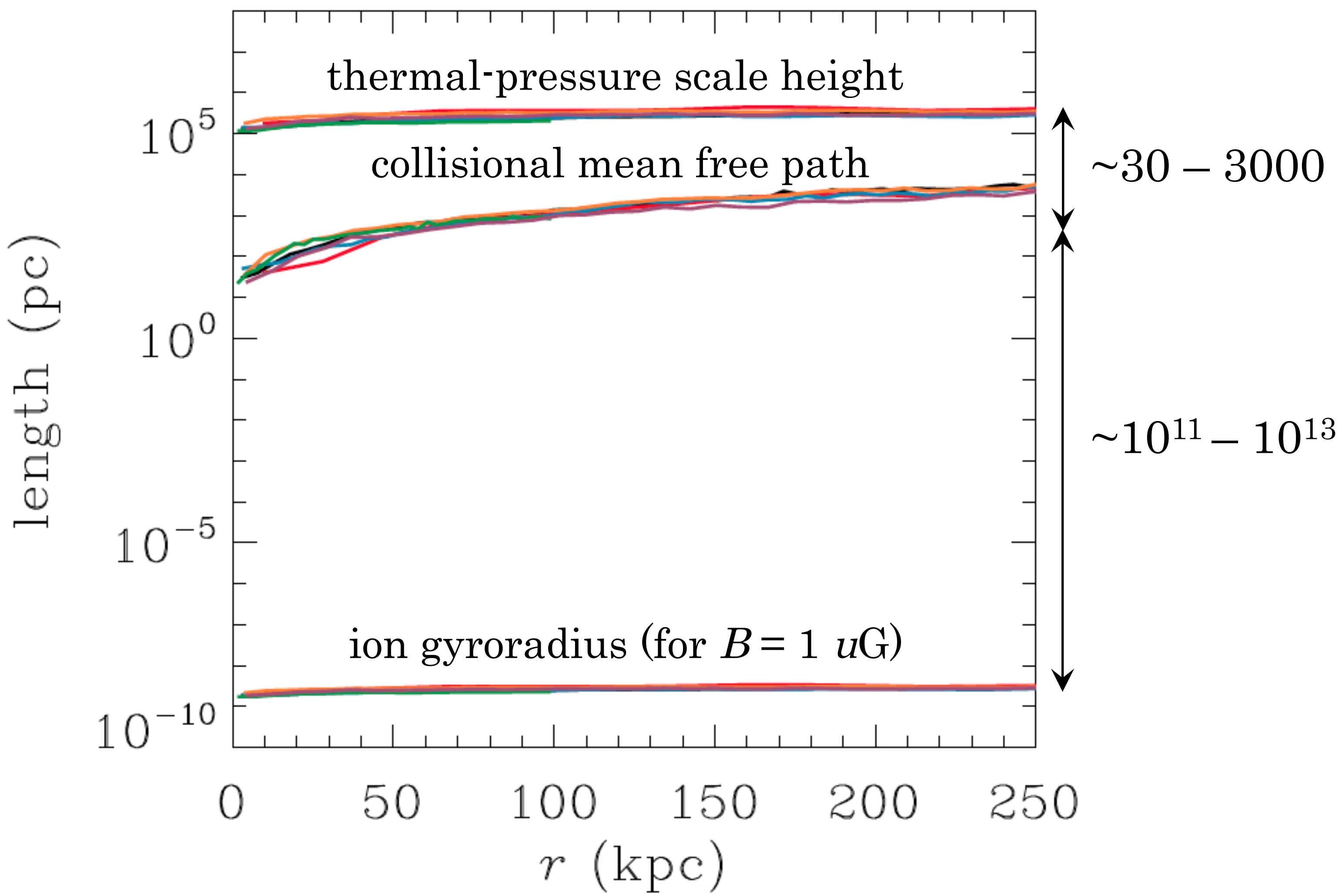


Cool-Core Clusters

from the ACCEPT archive

A85
A1835
A1795
A2029
A2199
A478

ICM Dynamics: A 3-Scale Problem

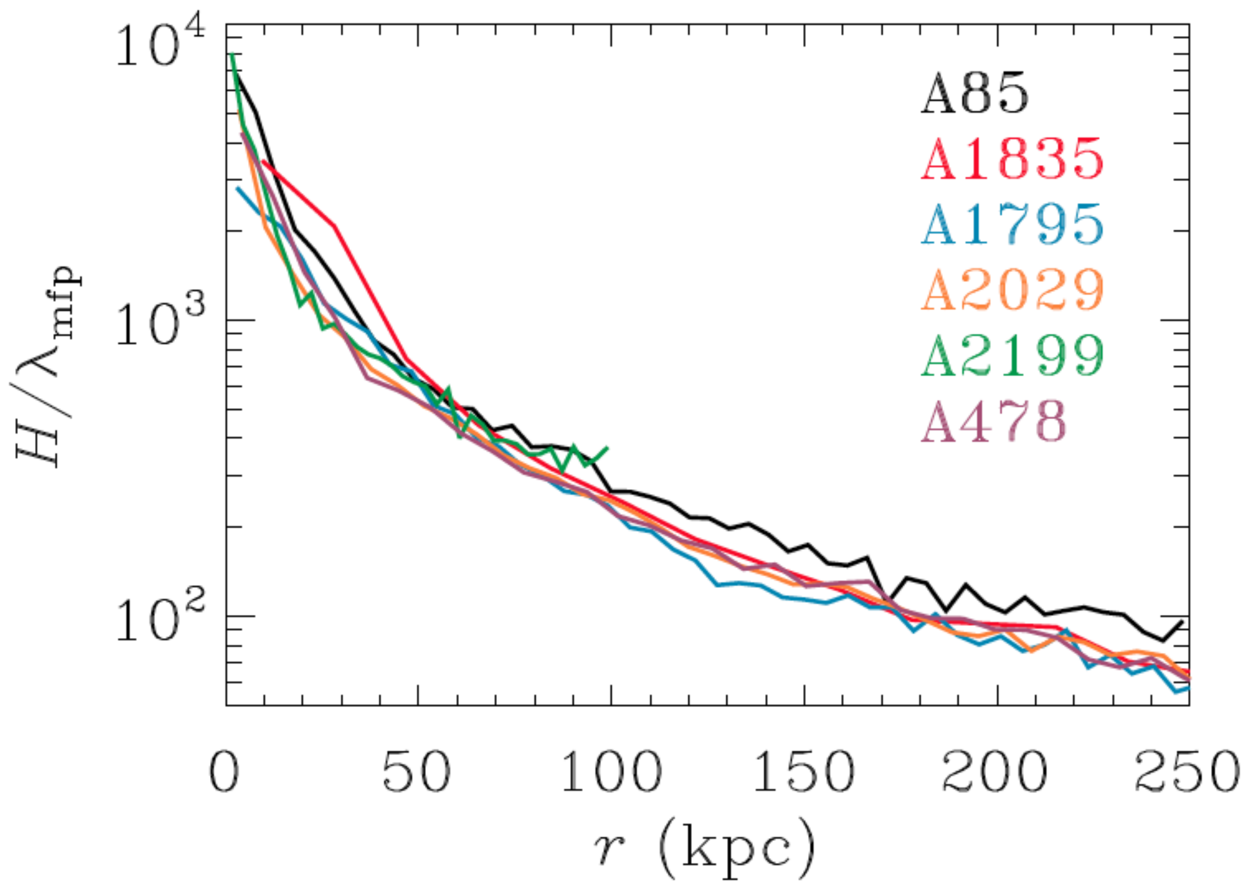


1 npc ~ 1 trip around the Earth ~ 20,000 miles

Fundamental Parameter

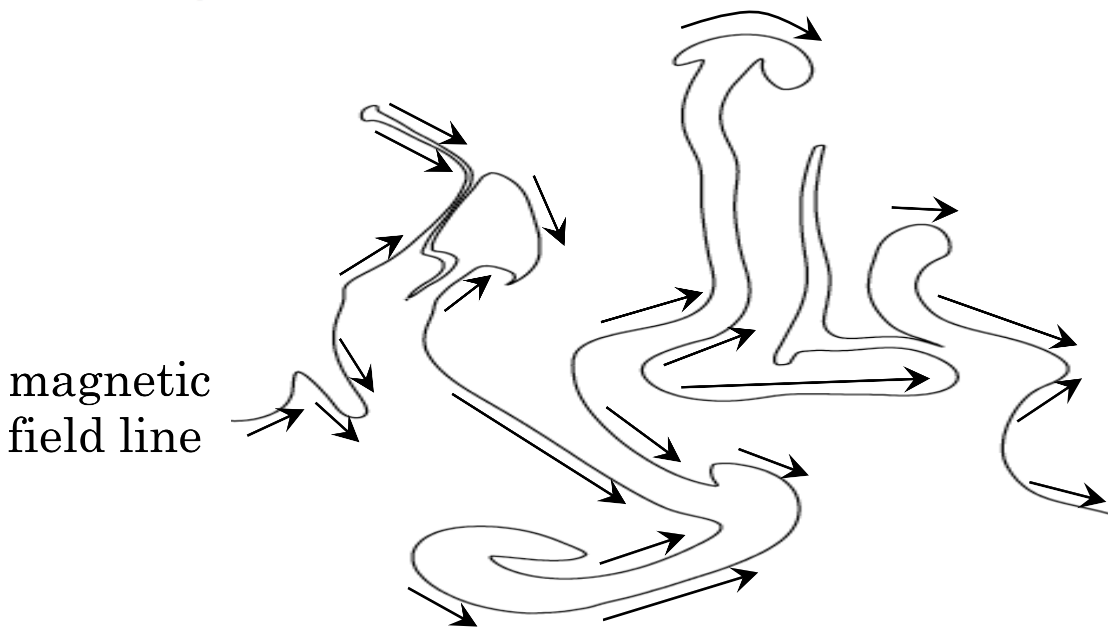
$$\mathrm{Kn}^{-1} \equiv \frac{H}{\lambda_{\mathrm{mfp}}} \simeq 1207 \left(\frac{g}{10^{-8}\ \mathrm{cm\ s^{-2}}}\right)^{-1} \left(\frac{n_{\mathrm{i}}}{0.01\ \mathrm{cm^{-3}}}\right) \left(\frac{k_{\mathrm{B}}T}{2\ \mathrm{keV}}\right)^{-1}$$

Knudsen



What does $H \gg \lambda_{\rm mfp} \gg r_{\rm g,i}$ mean?

1. $\boldsymbol{Q} = -\chi\, \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} T$



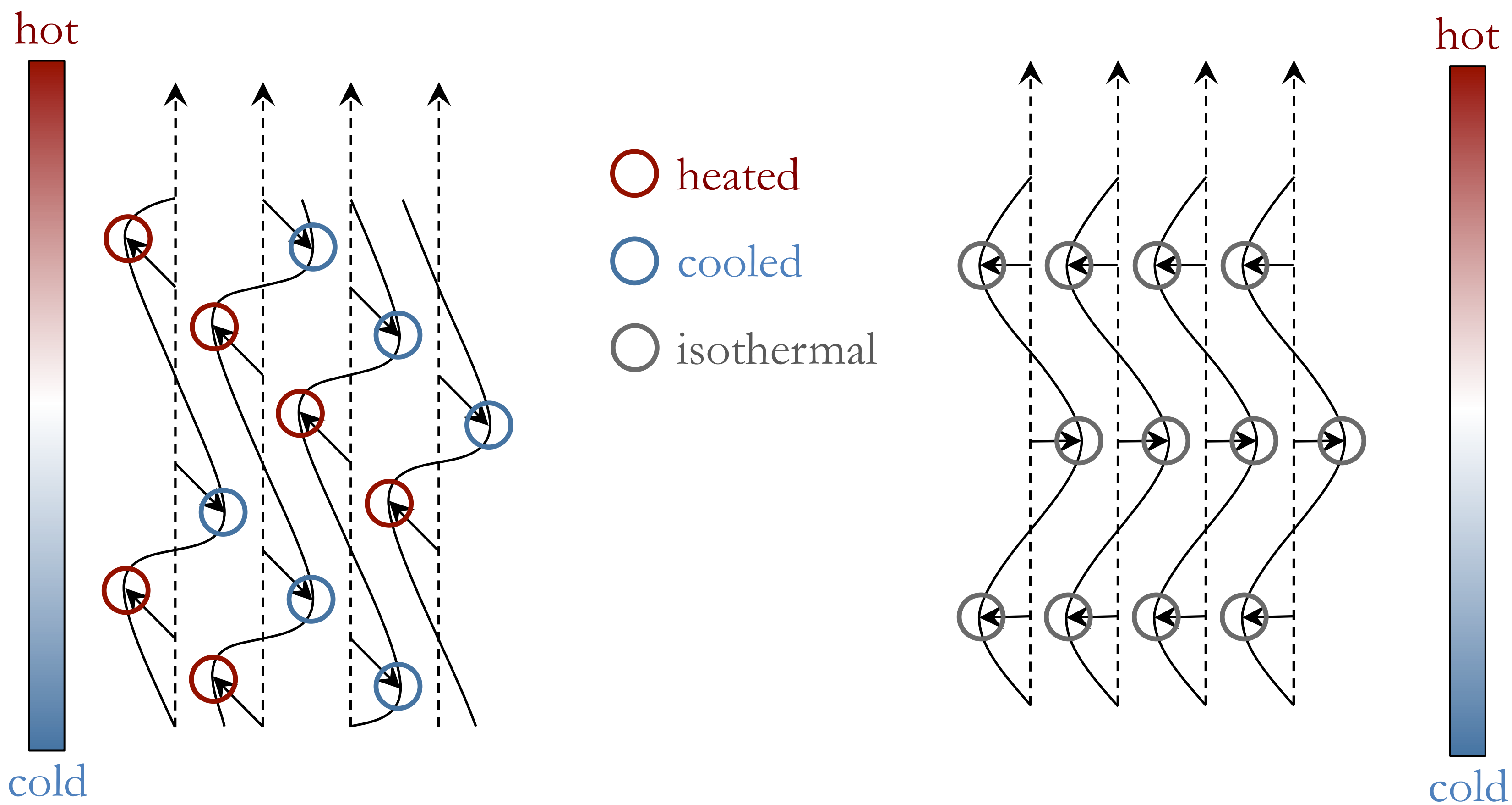
$\delta\boldsymbol{Q} = -\chi\boldsymbol{\nabla}\delta T$ vs. $\delta\boldsymbol{Q} = -\chi\, \hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla}\delta T - \chi\, \hat{\boldsymbol{b}}\, \delta\hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} T - \chi\, \delta\hat{\boldsymbol{b}}\, \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} T$

$\delta\hat{\boldsymbol{b}} = \nabla_{||}\boldsymbol{\xi}_{\perp}$

When conduction is rapid:

$$\Delta T \simeq 2\xi_{||}\nabla_{||}T$$

i.e. compressions/rarefactions in ∇T-oriented field lines lead to heating/cooling



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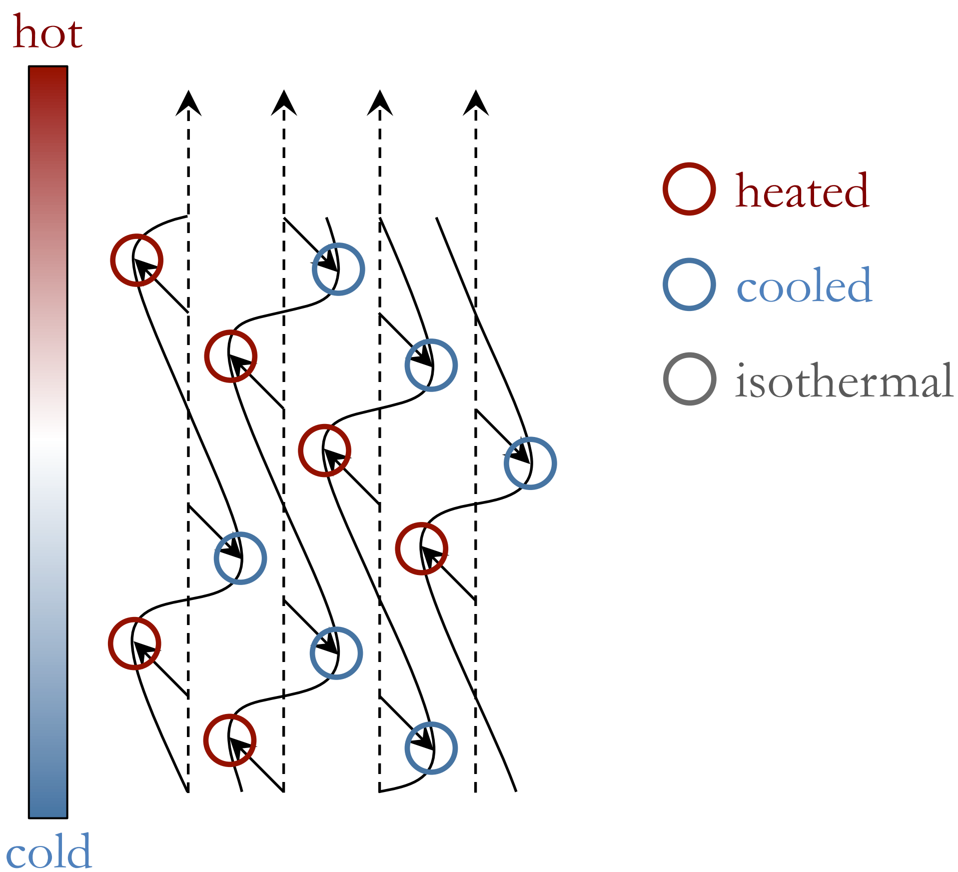


gravity

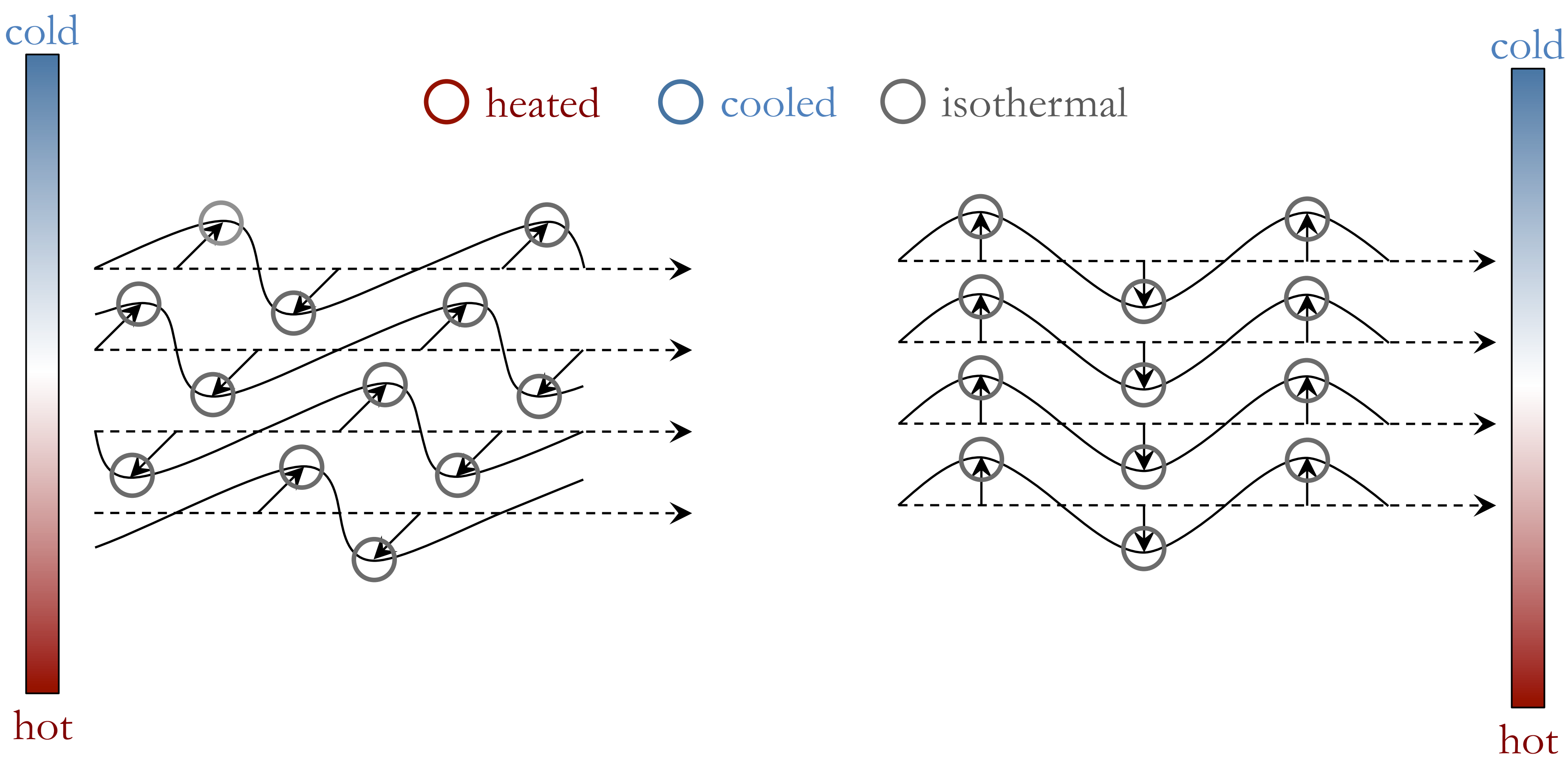
Heat-flux
Buoyancy-driven
Instability
Quataert (2008)

Saturation: $\nabla_{||}T \to 0$

When conduction is rapid:

$$\Delta T \simeq 2\xi_{||}\nabla_{||}T$$

i.e. compressions/rarefactions in ∇T-oriented field lines lead to heating/cooling



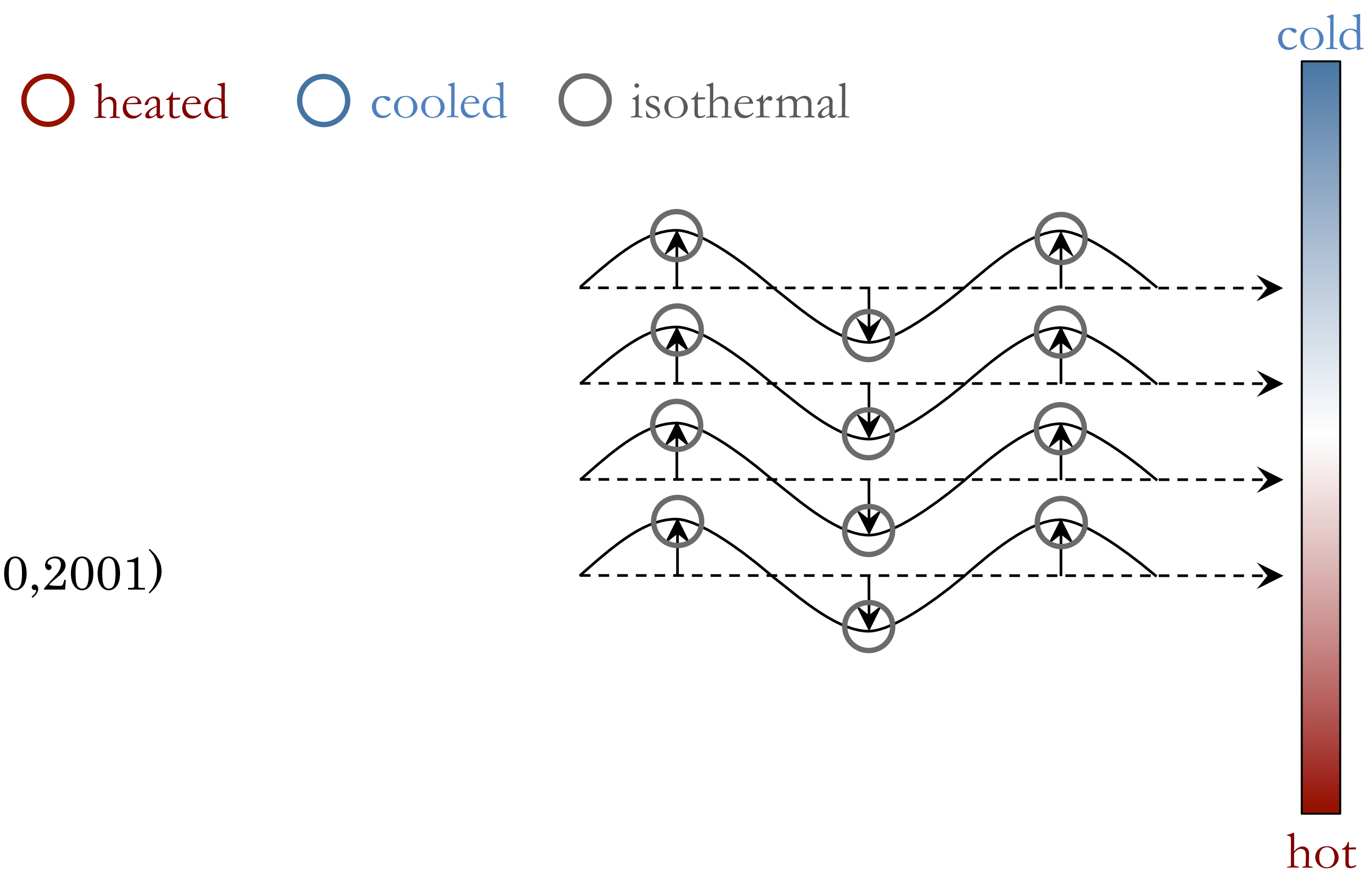
When conduction is rapid:

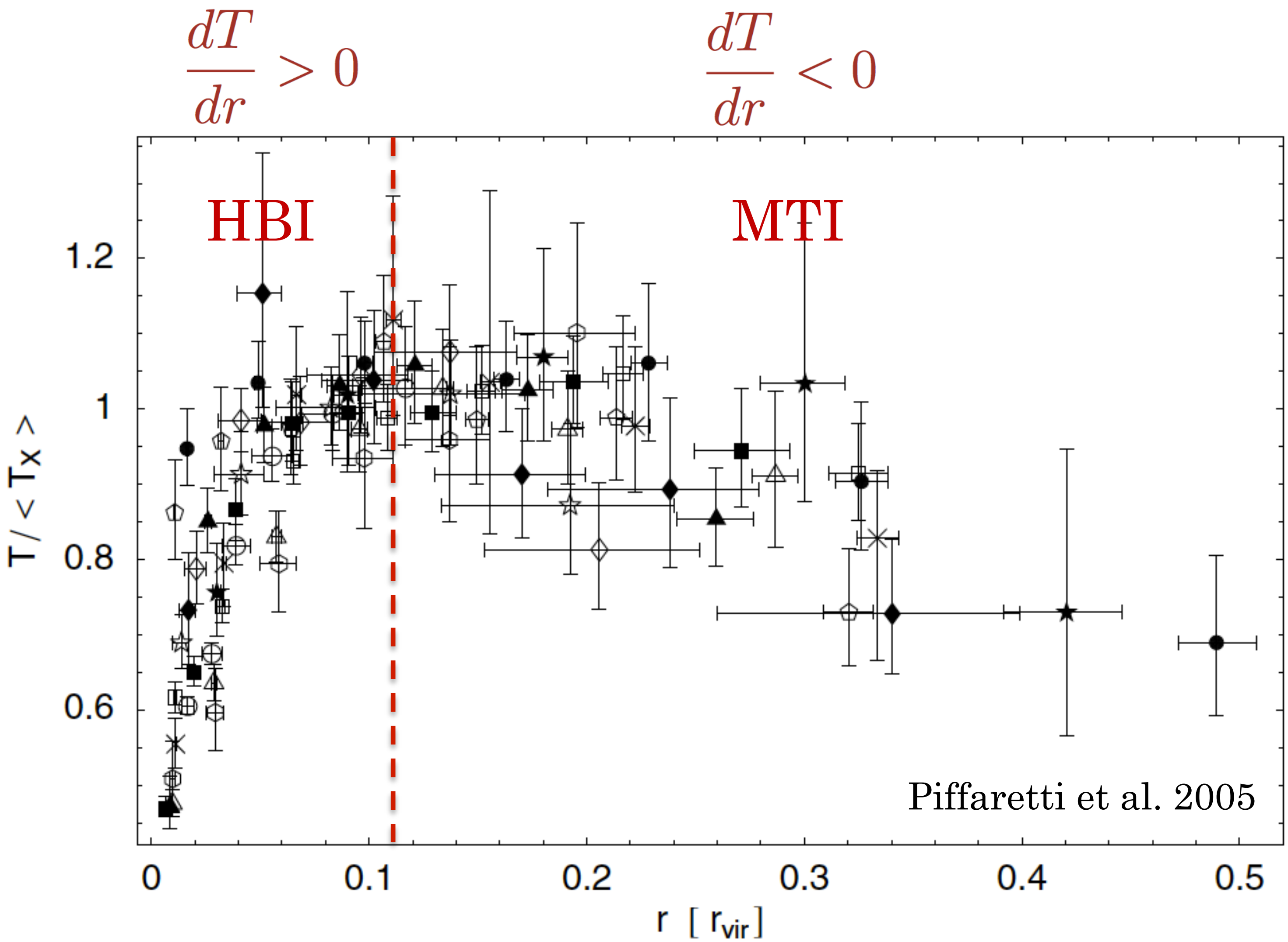
$$\Delta T \simeq 2\xi_{||}\nabla_{||}T$$

i.e. compressions/rarefactions in ∇T-oriented field lines lead to heating/cooling

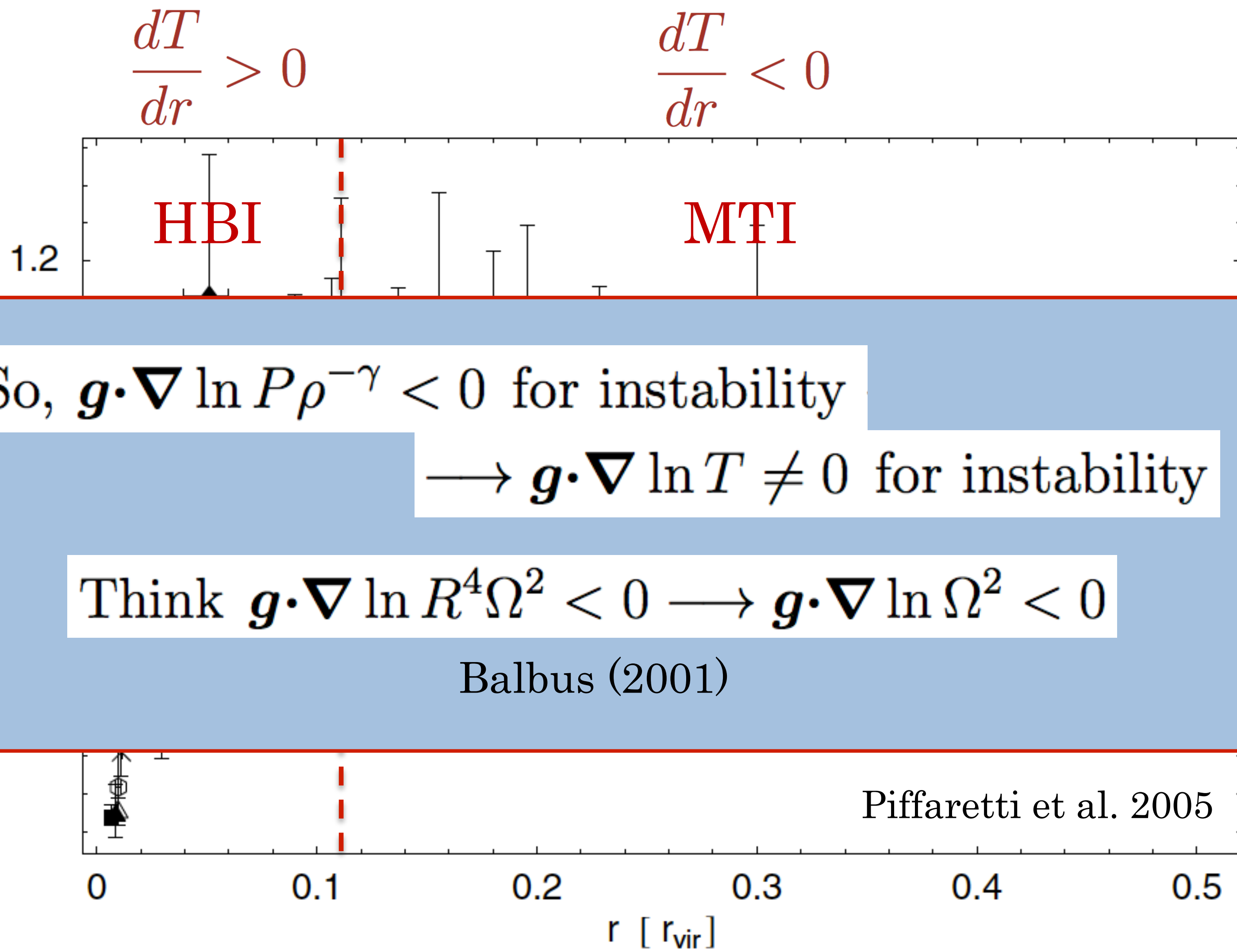
gravity

Magneto-
Thermal
Instability
Balbus (2000,2001)





$\frac{dT}{dr} > 0$
$\frac{dT}{dr} < 0$
HBI
MTI
1.2
1
0.8
0.6
T / < Tx >
0
0.1
0.2
0.3
0.4
0.5
r [r_{vir}]
Piffaretti et al. 2005



So, $\boldsymbol{g} \cdot \boldsymbol{\nabla} \ln P\rho^{-\gamma} < 0$ for instability

$\longrightarrow \boldsymbol{g} \cdot \boldsymbol{\nabla} \ln T \neq 0$ for instability

Think $\boldsymbol{g} \cdot \boldsymbol{\nabla} \ln R^4\Omega^2 < 0 \longrightarrow \boldsymbol{g} \cdot \boldsymbol{\nabla} \ln \Omega^2 < 0$

Balbus (2001)

not the whole story…

(something was missing)

What else does $H \gg \lambda_{\rm mfp} \gg r_{\rm g,i}$ mean?

2. $p \to \mathbf{P} = p_\perp \mathbf{I} - (p_\perp - p_\parallel)\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}$ modifies magnetic tension

First adiabatic invariant $\mu = \frac{m v_\perp^2}{2B}$ conserved if $\Omega_{\rm i}/\nu_{\rm ii} \gg 1$

holds already for $B > 10^{-18}$ G

$$\longrightarrow \sum_{\rm particles} \mu = \frac{p_\perp}{B} = \mathrm{const}$$

(angular momentum conservation of a gyrating particle)

Changes in field strength ⇔ pressure anisotropy

$$\frac{1}{p_\perp}\frac{dp_\perp}{dt} \sim \frac{1}{B}\frac{dB}{dt} - \nu_{ii}\frac{p_\perp - p_\parallel}{p_\perp}$$

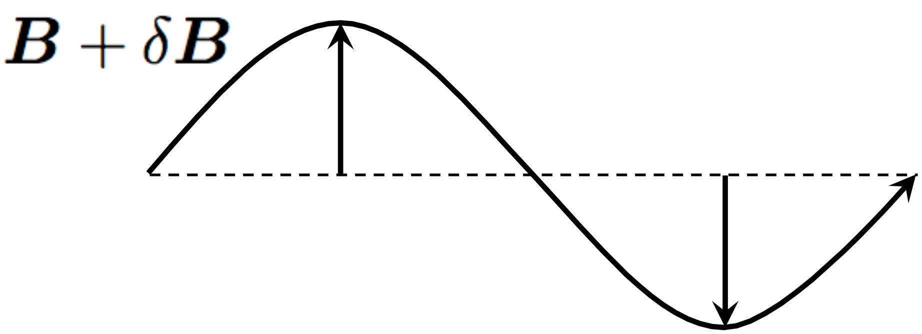
change in B drives anisotropy

anisotropy relaxed by collisions

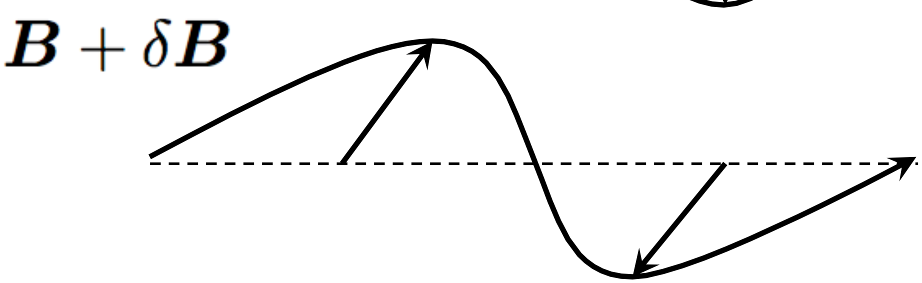
What does $H \gg \lambda_{\rm mfp}$ mean?

$$\frac{p_\perp - p_{||}}{p} \sim \frac{1}{\nu_{\rm ii}} \frac{d \ln B}{dt} = \frac{\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} \boldsymbol{:} \boldsymbol{\nabla v}}{\nu_{\rm ii}}$$

Linearly, this implies:



$\delta B_{||} = 0$ Alfvén mode is undamped

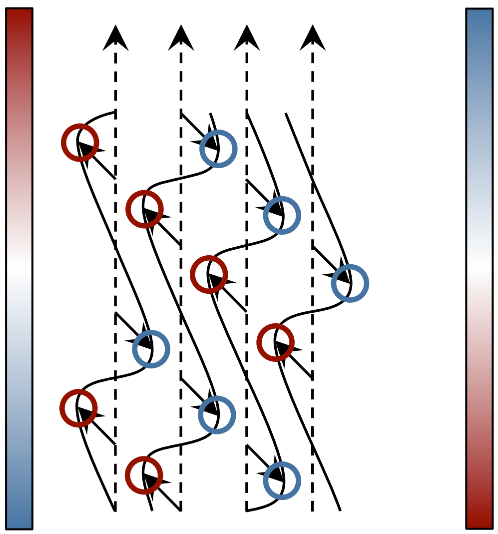


$\delta B_{||} \neq 0$ slow mode is damped

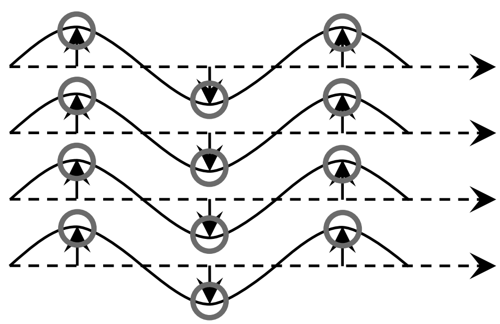
acts as an anisotropic viscosity (“Braginskii viscosity”)
targets motions that change the field strength

How does this affect the ICM?

$$\omega_{\rm cond} \gg \omega_{\rm dyn} \longrightarrow \Delta T \propto \xi_{||} \propto \delta B_{||}$$

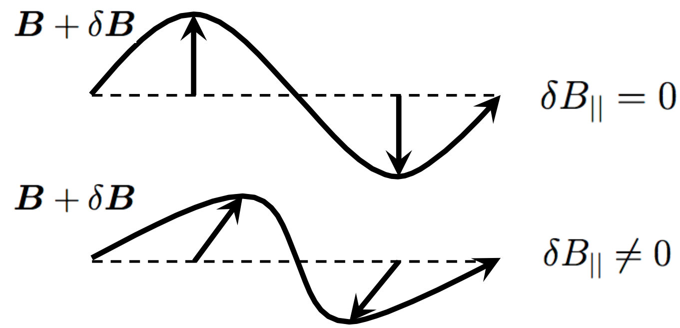


HBI



MTI

Slow modes damped at rate $\omega_{\rm visc}$.

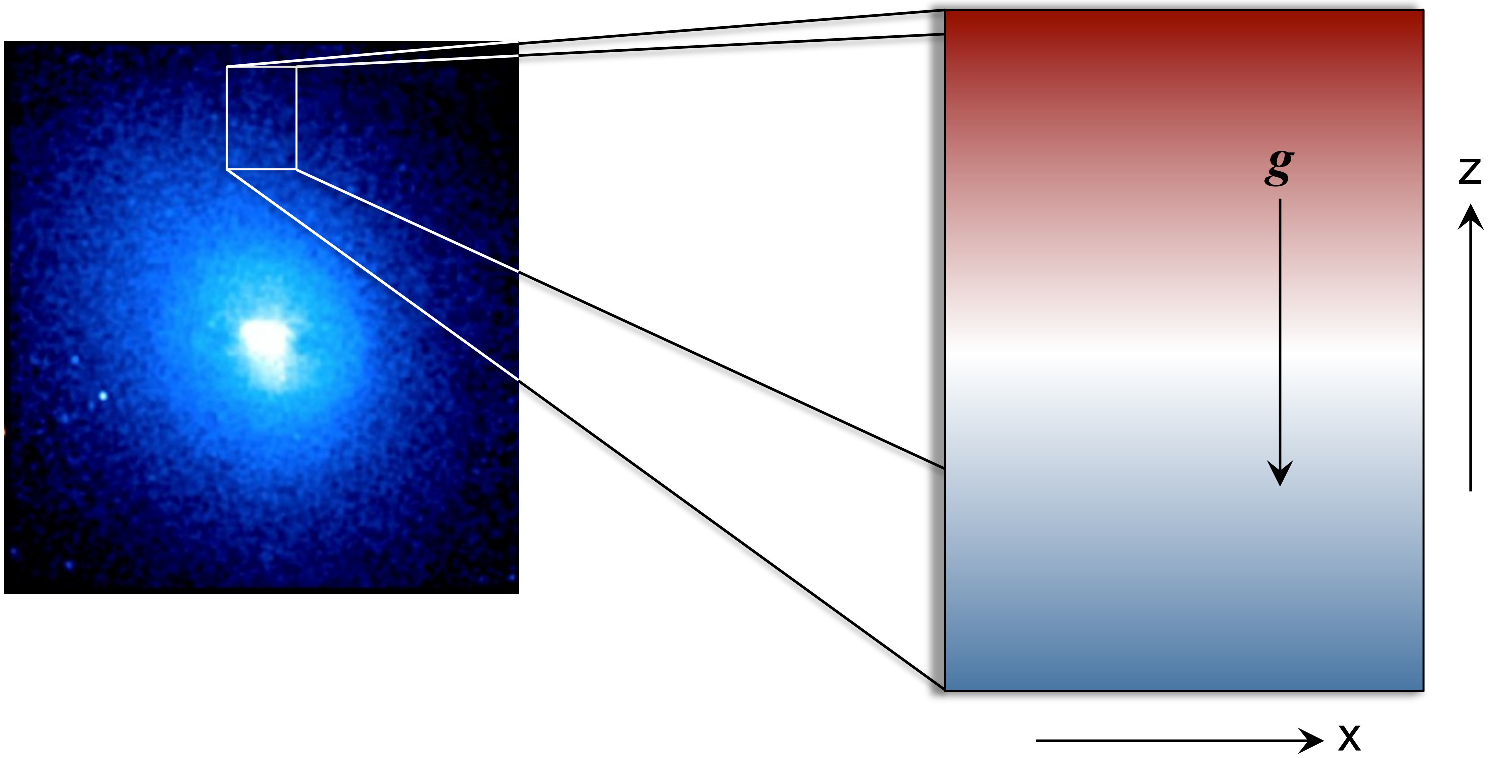


HBI is suppressed. MTI is strengthened.
HBI modes confined to wavelengths satisfying

$$\omega_{\rm cond} \gtrsim \omega_{\rm dyn} \gtrsim \omega_{\rm visc}$$

NOTE: $\omega_{\rm cond} \sim 6\omega_{\rm visc}$ for hydrogenic plasma

put a weakly collisional fluid in a gravitating, thermally stratified atmosphere



$$\boldsymbol{v} = \delta \boldsymbol{v} \qquad \boldsymbol{B} = B_{0,x}\hat{\boldsymbol{x}} + B_{0,z}\hat{\boldsymbol{z}} + \delta \boldsymbol{B} \qquad p = p_0(z) + \delta p \qquad T = T_0(z) + \delta T$$

Dispersion Relation (Kunz 2011)

$$\delta \propto \exp\left(\sigma t + i\boldsymbol{k}\cdot\boldsymbol{r}\right)$$

To leading order in $\omega_{\rm dyn}/\omega_{\rm cond} \ll 1$,

$$\widetilde{\sigma}^2 \left(\widetilde{\sigma}^2 + \sigma\,\omega_{\rm visc} \frac{k_\perp^2}{k^2} + g\,\frac{{\rm d}\ln T}{{\rm d}z}\,\frac{\mathcal{K}}{k^2} \right) \simeq -\sigma\,\omega_{\rm visc}\,g\,\frac{{\rm d}\ln T}{{\rm d}z}\,\frac{b_x^2 k_y^2}{k^2}$$

$$\sigma \simeq -\omega_{\rm cond}$$

↑

entropy mode

Dispersion Relation (Kunz 2011)

$$\delta \propto \exp\left(\sigma t + i\boldsymbol{k}\cdot\boldsymbol{r}\right)$$

To leading order in $\omega_{\rm dyn}/\omega_{\rm cond} \ll 1$,

$$\widetilde{\sigma}^2 \left(\widetilde{\sigma}^2 + \sigma\,\omega_{\rm visc}\frac{k_\perp^2}{k^2} + g\frac{{\rm d}\ln T}{{\rm d}z}\frac{\mathcal{K}}{k^2} \right) \simeq -\sigma\,\omega_{\rm visc}\,g\frac{{\rm d}\ln T}{{\rm d}z}\frac{b_x^2 k_y^2}{k^2}$$

Alfvén mode

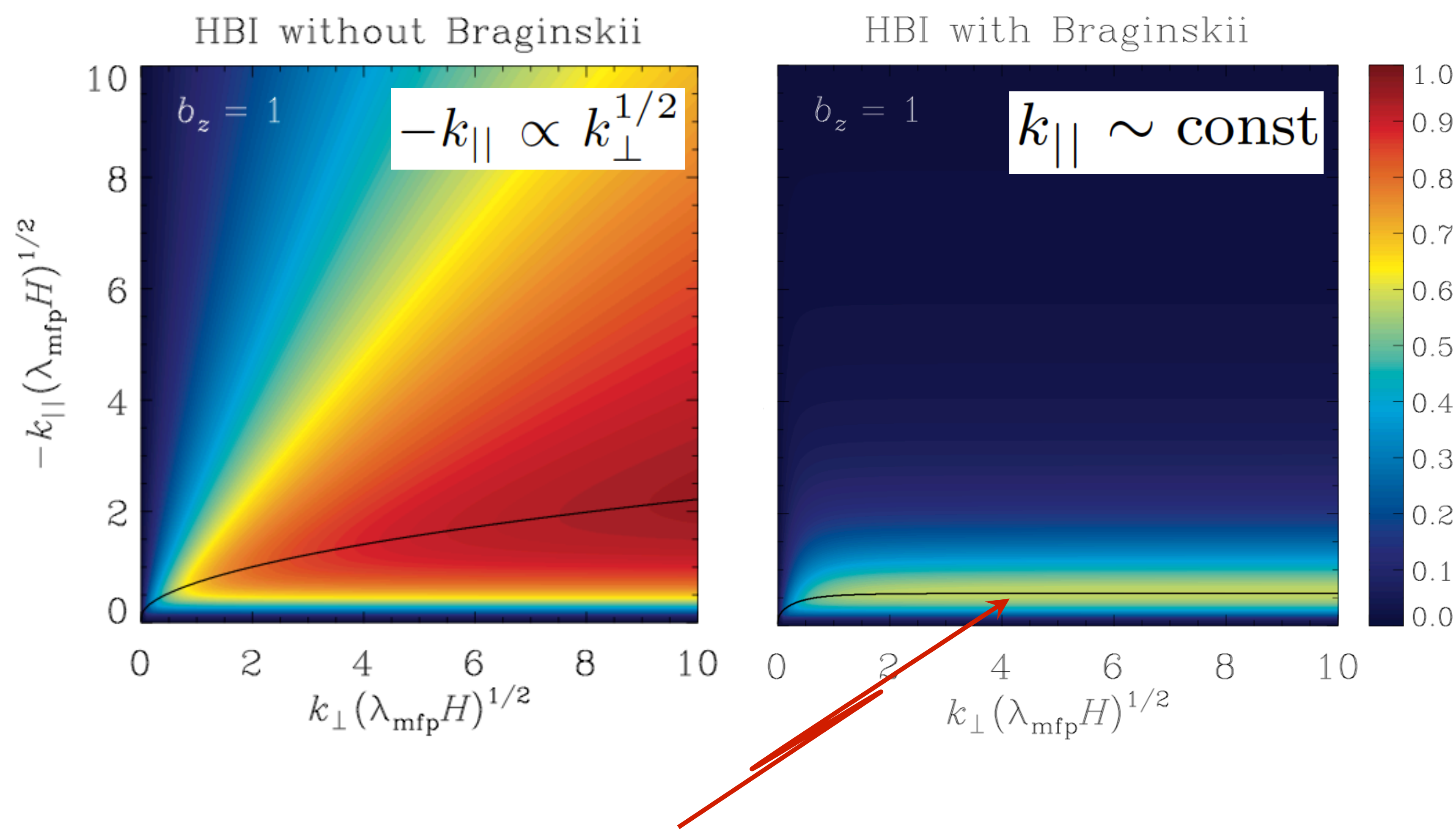
slow mode

Braginskii damping of slow mode

cause of MTI/HBI

coupling of Alfvén and slow modes

$$\widetilde{\sigma}^2 \equiv \sigma^2 + \left(\boldsymbol{k}\cdot\boldsymbol{v}_{\bf A}\right)^2$$

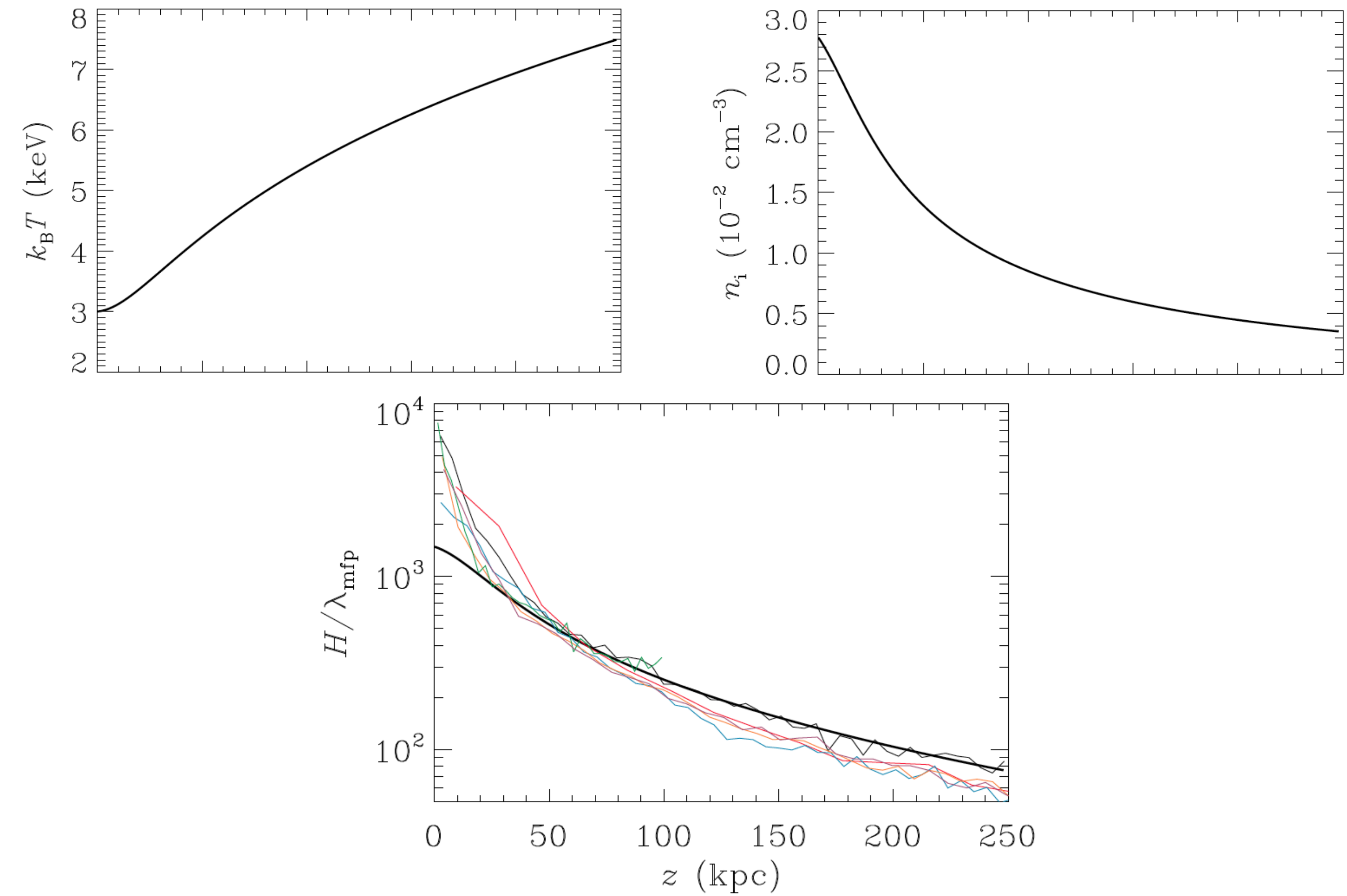


Maximum growth rate reduced by a factor ~1.7;

occurs at $\omega_{\rm visc} \sim 0.6\ \omega_{\rm dyn}$, i.e. $\dfrac{\lambda_{||}}{H} \simeq 10\sqrt{\dfrac{\lambda_{\rm mfp}}{H}}$.

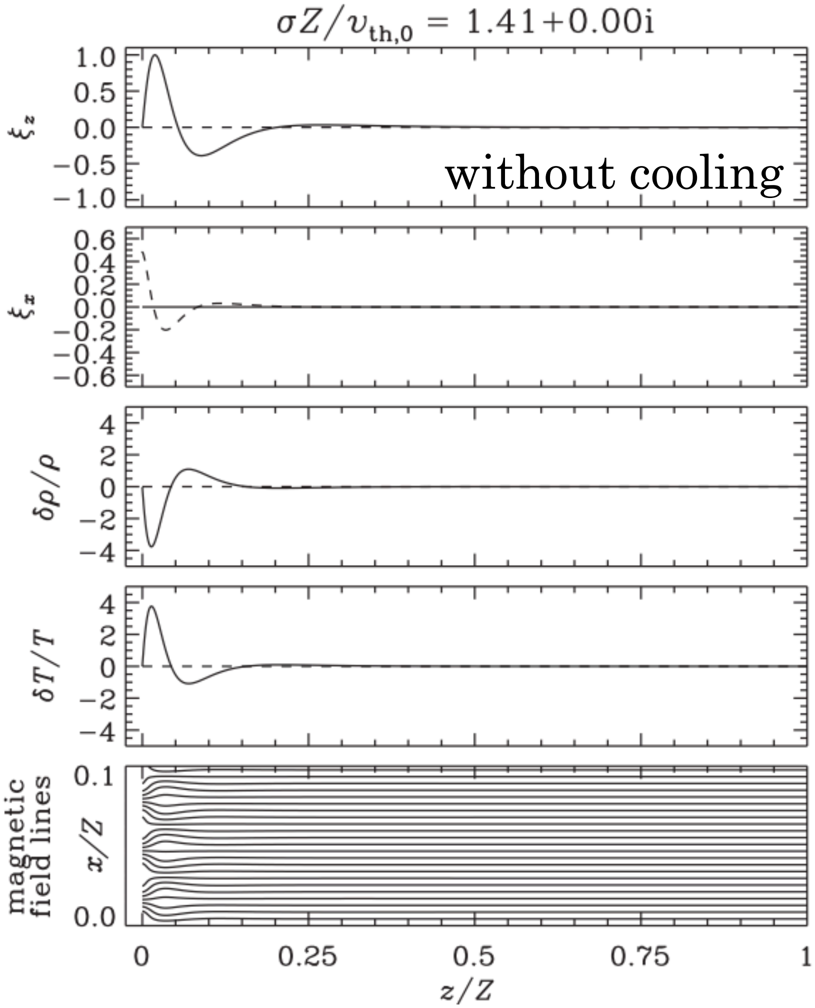
These modes are only “local” within ~10s kpc of clusters.

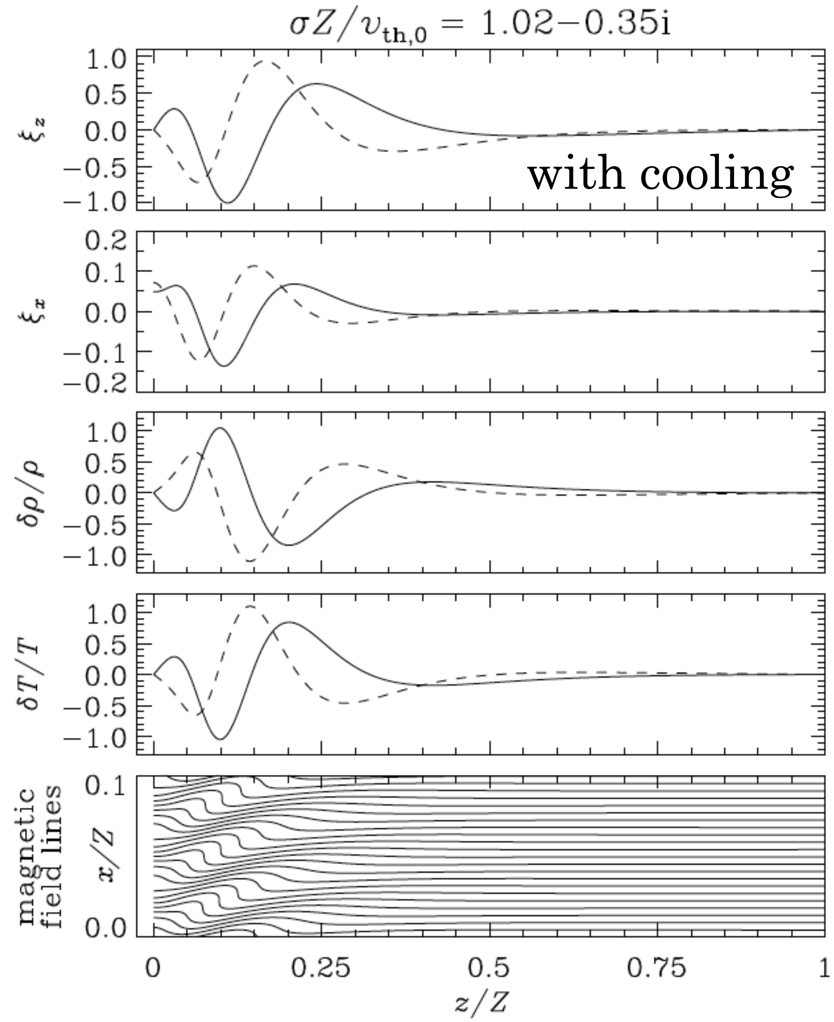
This has been shown rigorously
via a global HBI linear calculation
(Latter & Kunz 2012)



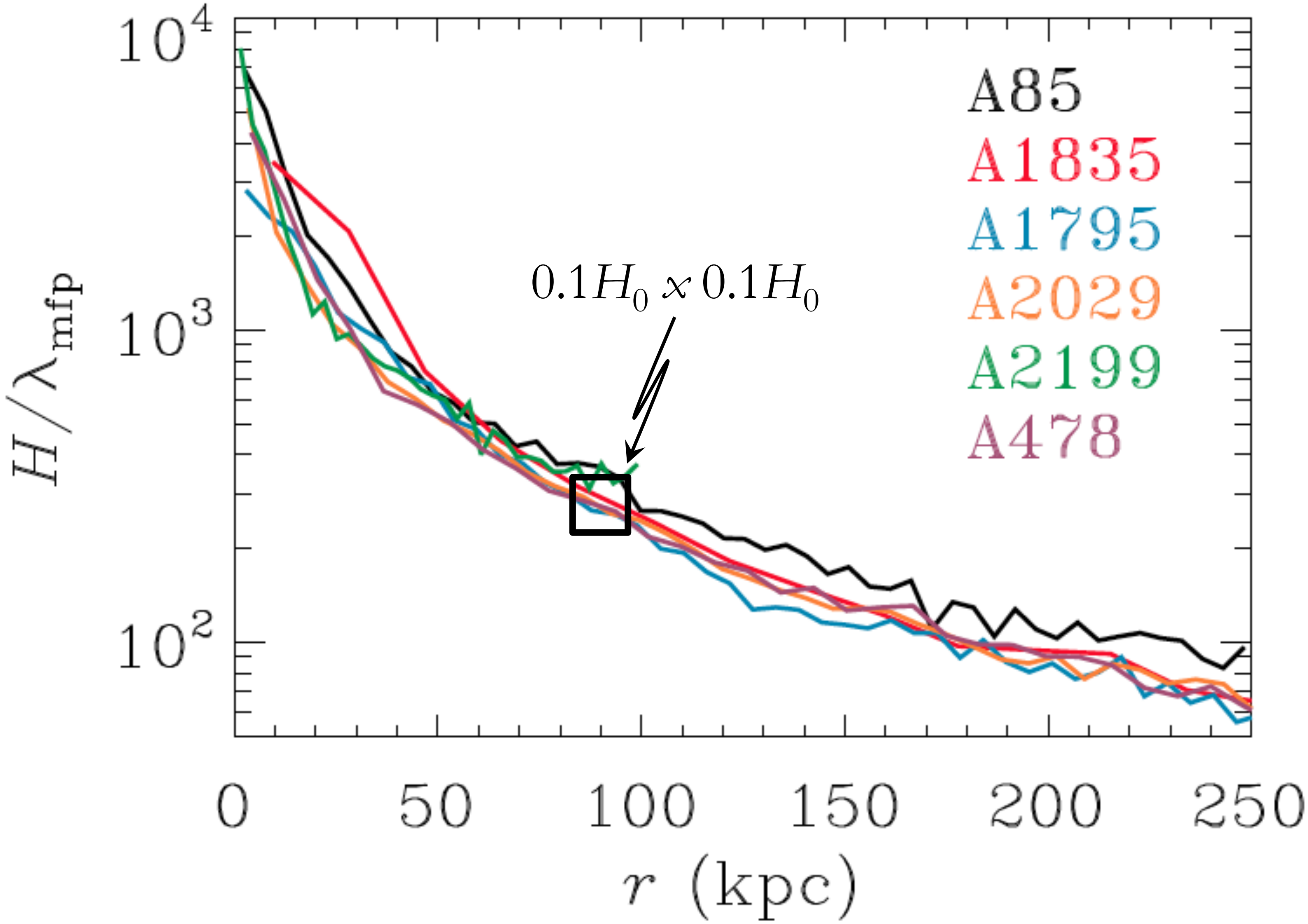
This has been shown rigorously via a global HBI linear calculation (Latter & Kunz 2012)

$\gtrsim 5$ Gyr for significant field-line deformation beyond ~50 kpc





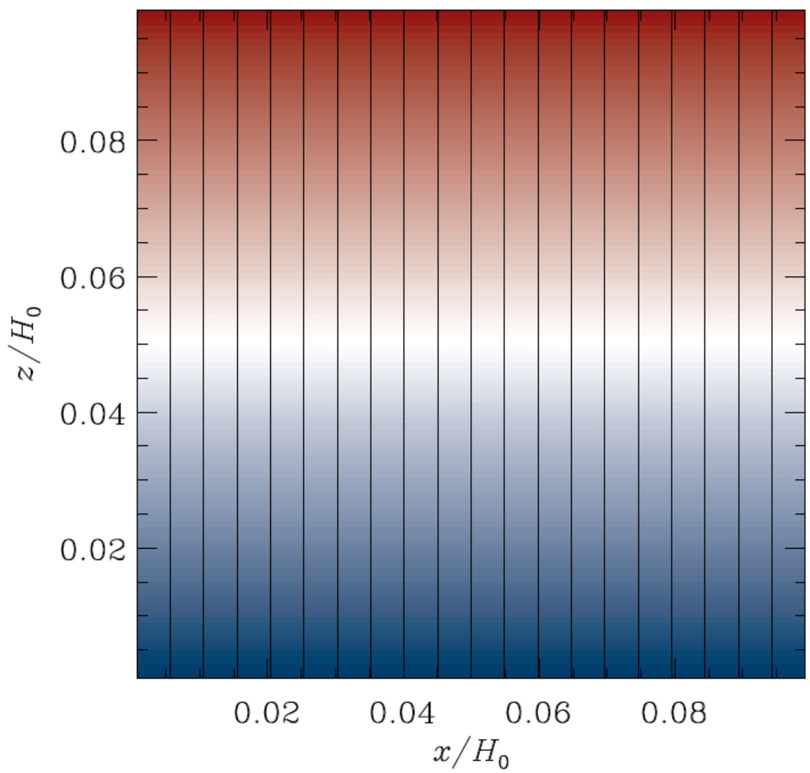
Let's try a local simulation:



Local HBI Simulation:

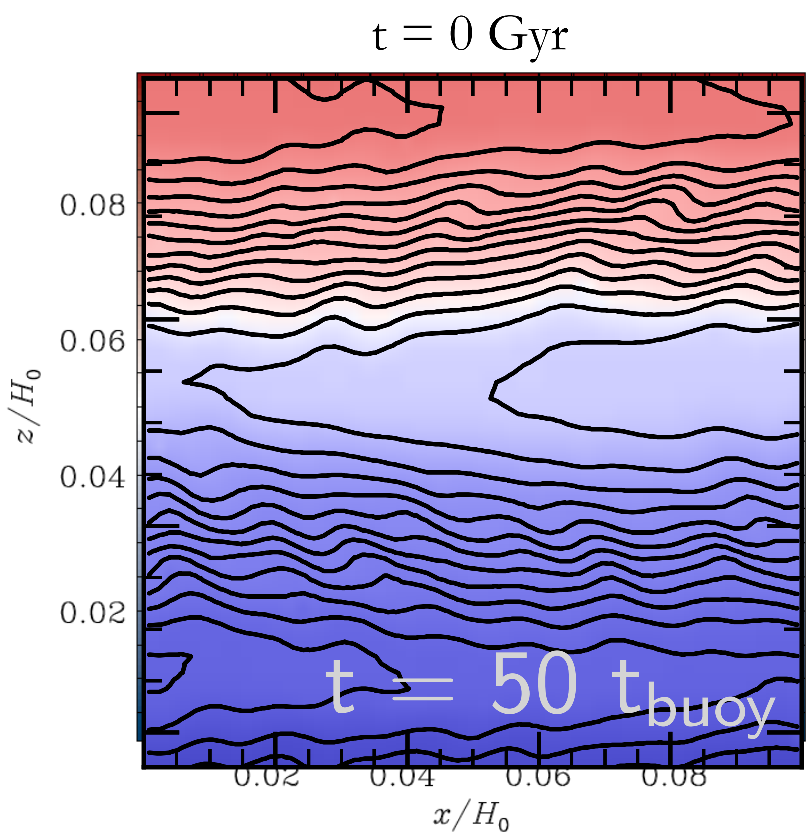
same as McCourt, Parrish, Sharma & Quataert (2011) *but* with Braginskii viscosity

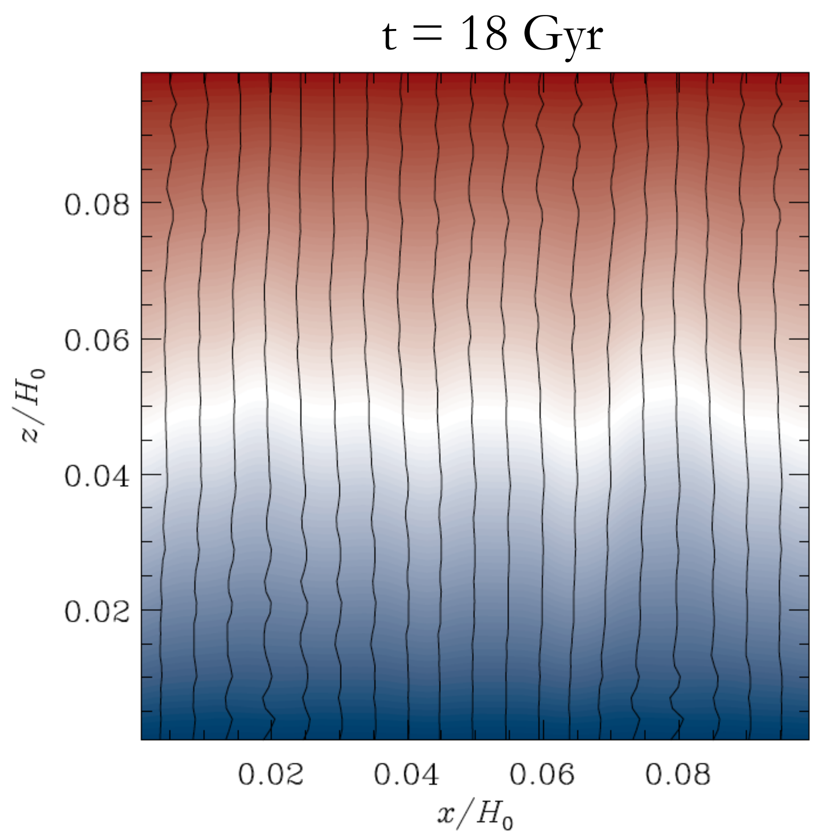
t = 0 Gyr



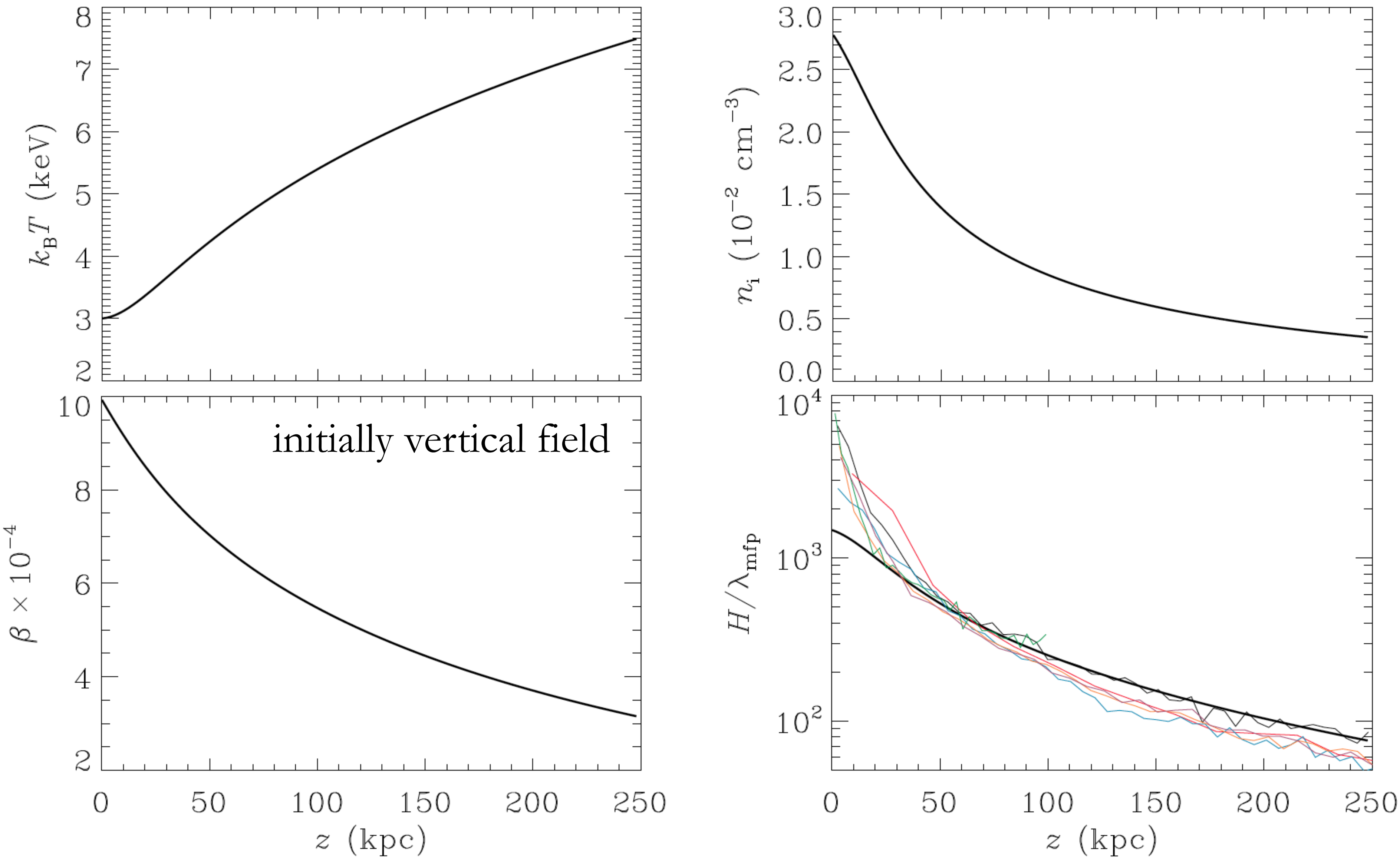
Local HBI Simulation:

same as McCourt, Parrish, Sharma & Quataert (2011) *but* with Braginskii viscosity

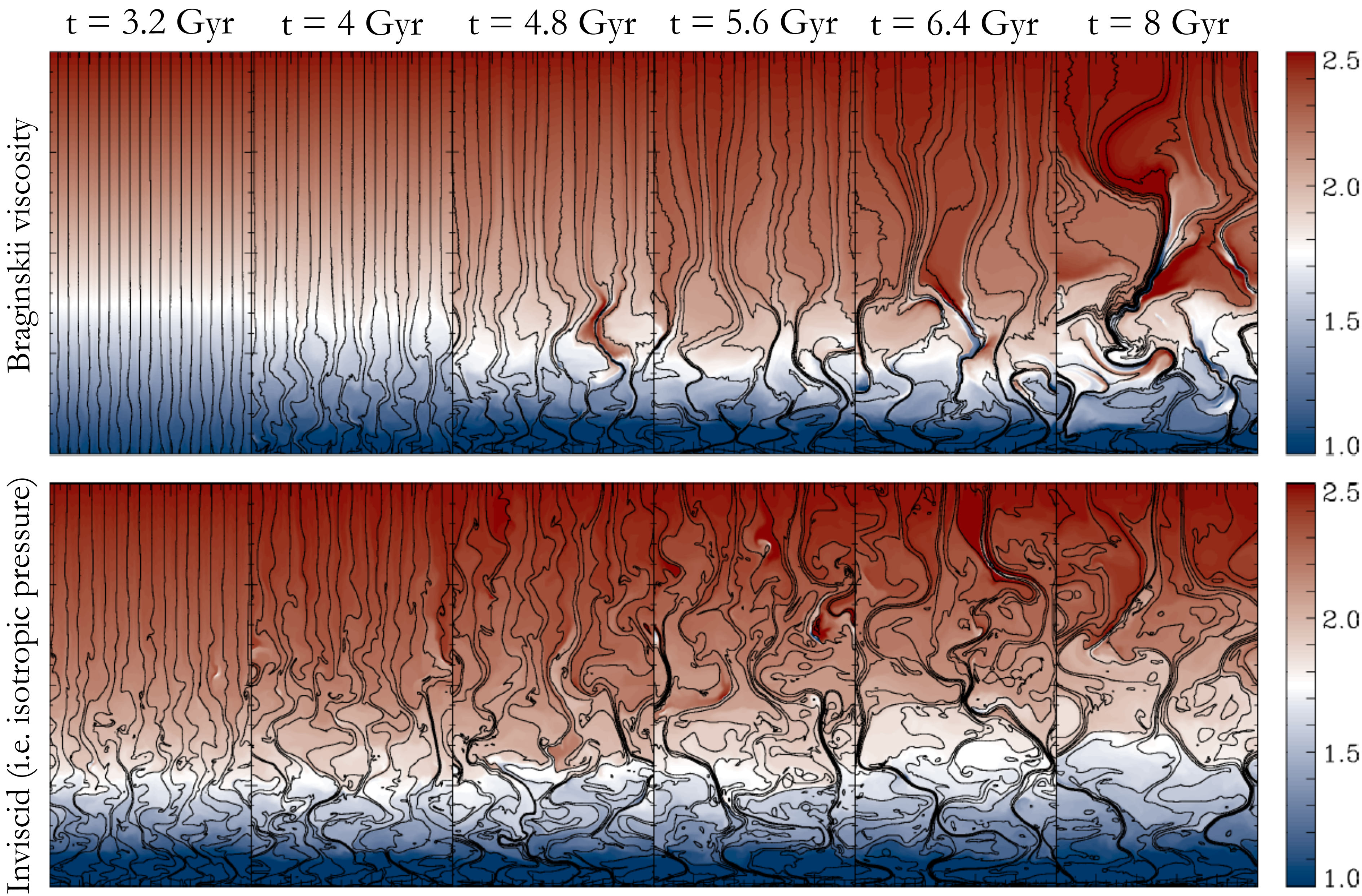




Quasi-global (*Hx2H*) simulations of HBI in weakly collisional, radiative ICM



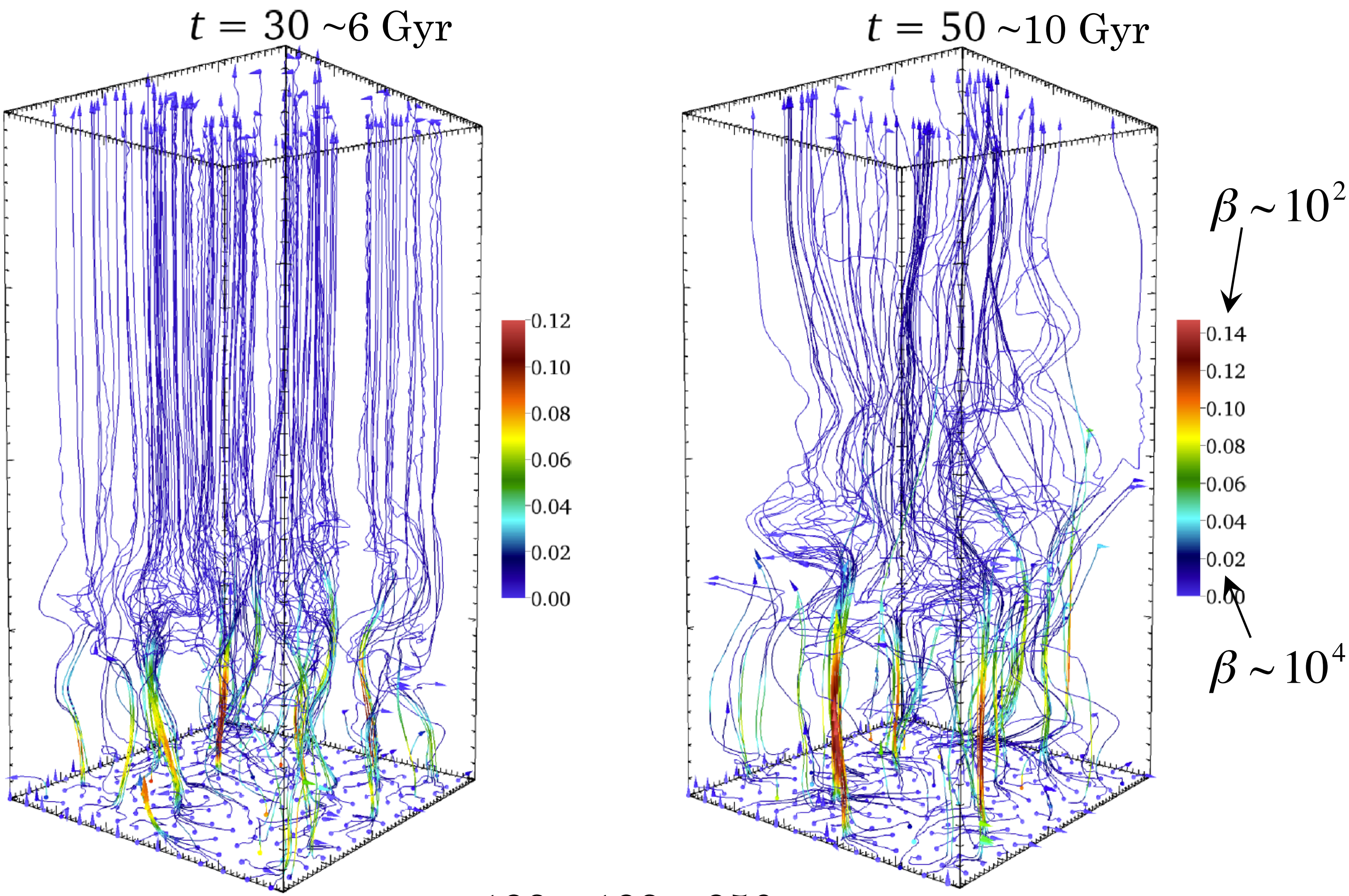

t = 0 Gyr
Braginskii viscosity
Inviscid (i.e. isotropic pressure)
$H_0 \times 2H_0$
512×1024
2.5
2.0
1.5
1.0
2.5
2.0
1.5
1.0



t = 3.2 Gyr
t = 4 Gyr
t = 4.8 Gyr
t = 5.6 Gyr
t = 6.4 Gyr
t = 8 Gyr
Braginskii viscosity
Inviscid (i.e. isotropic pressure)
2.5
2.0
1.5
1.0
2.5
2.0
1.5
1.0

Quasi-global (*HxHx2H)* simulations

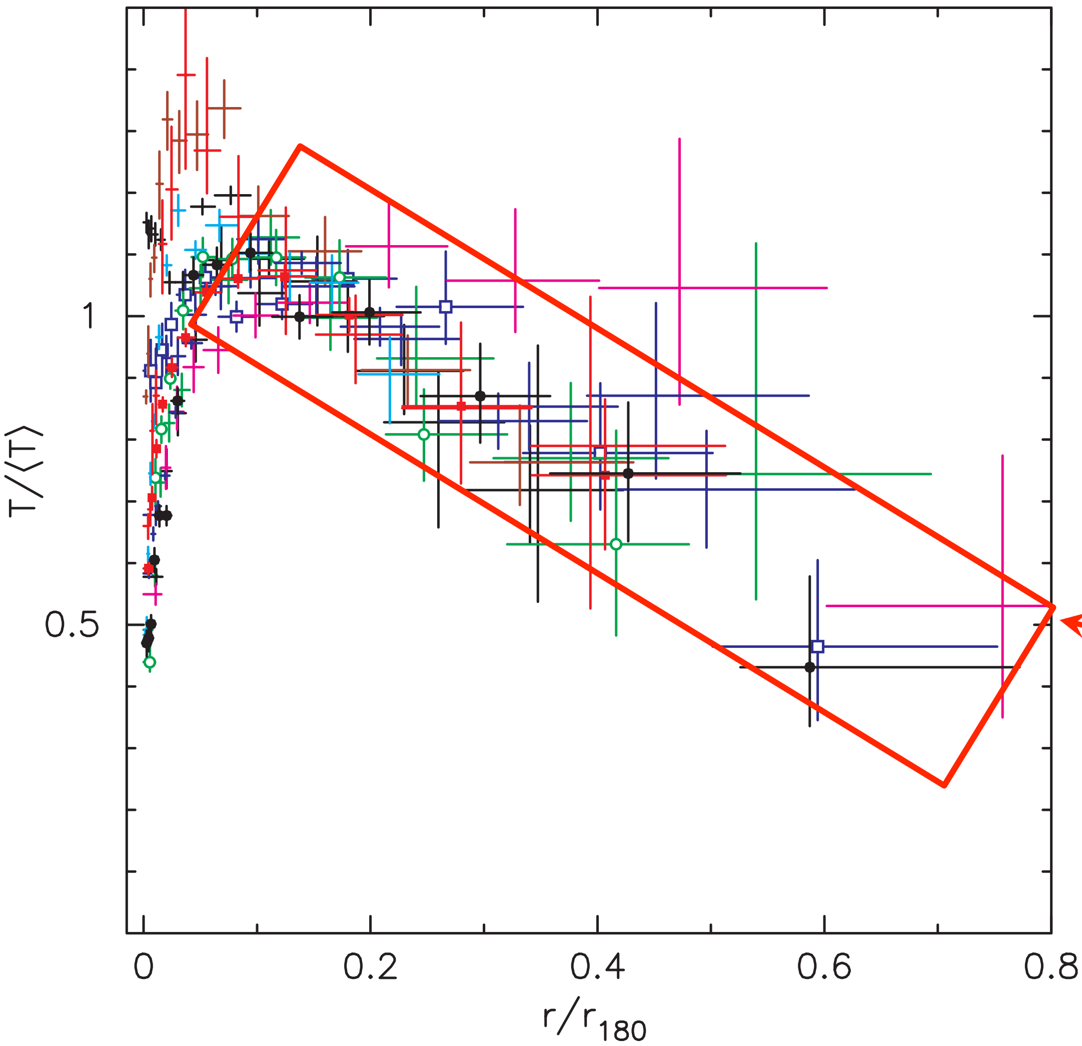
(but without cooling, for now)



128 x 128 x 256

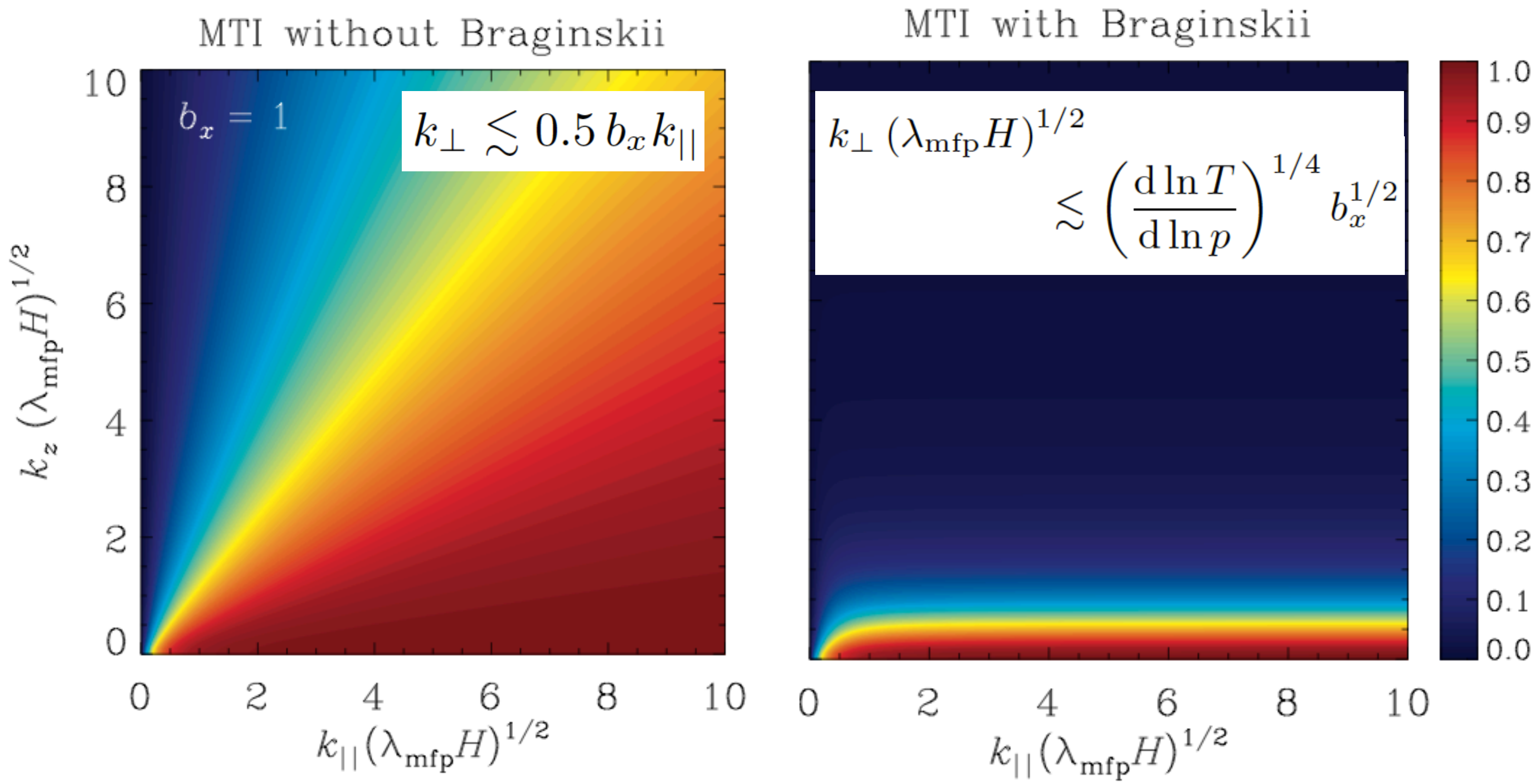
Cluster Outskirts

from Vikhlinin+ 2005



$$\frac{dT}{dr} < 0$$

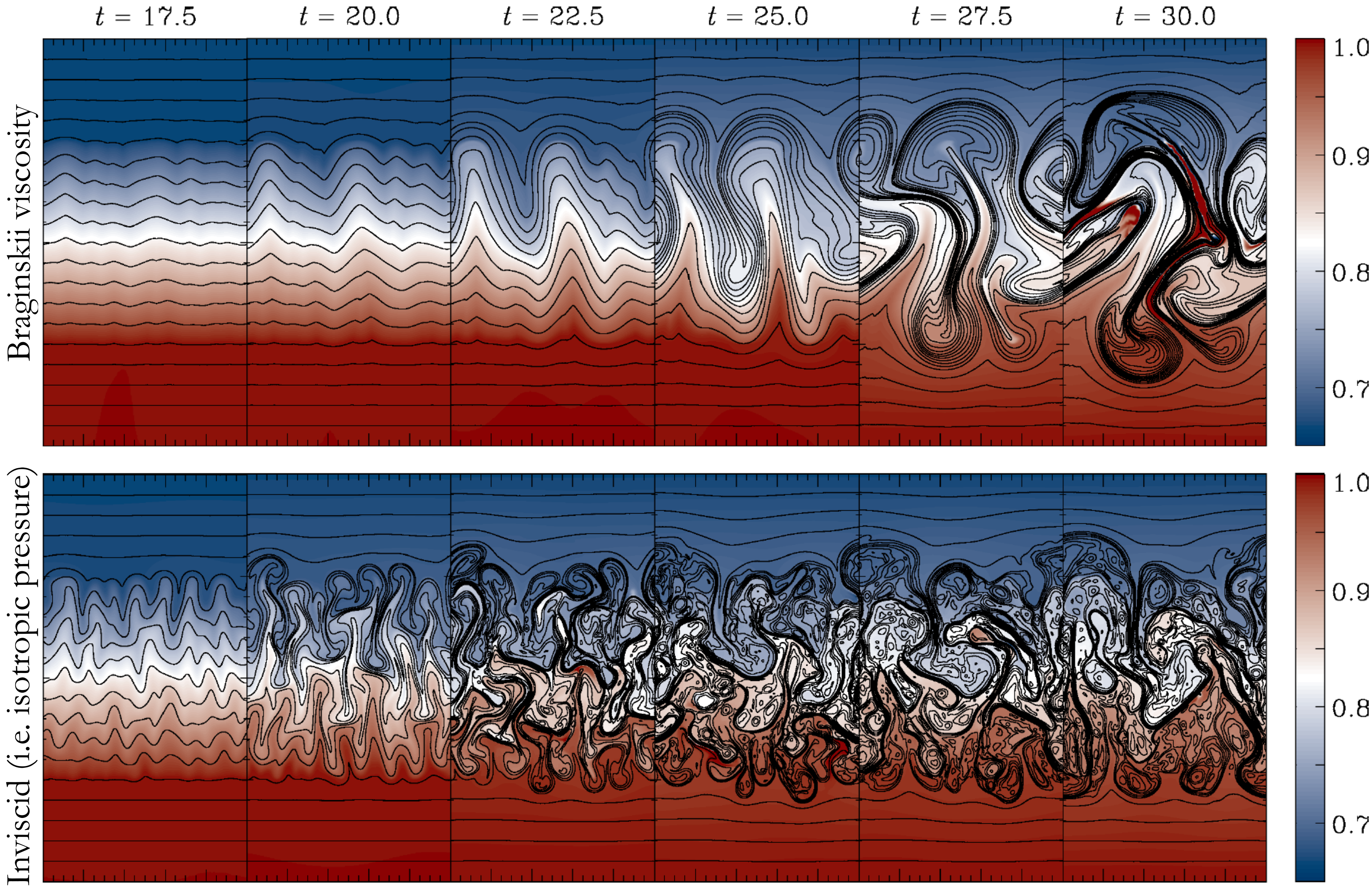
This region is even less collisional, with $H/\lambda_{\rm mfp} \sim 10$.
What happens out here?



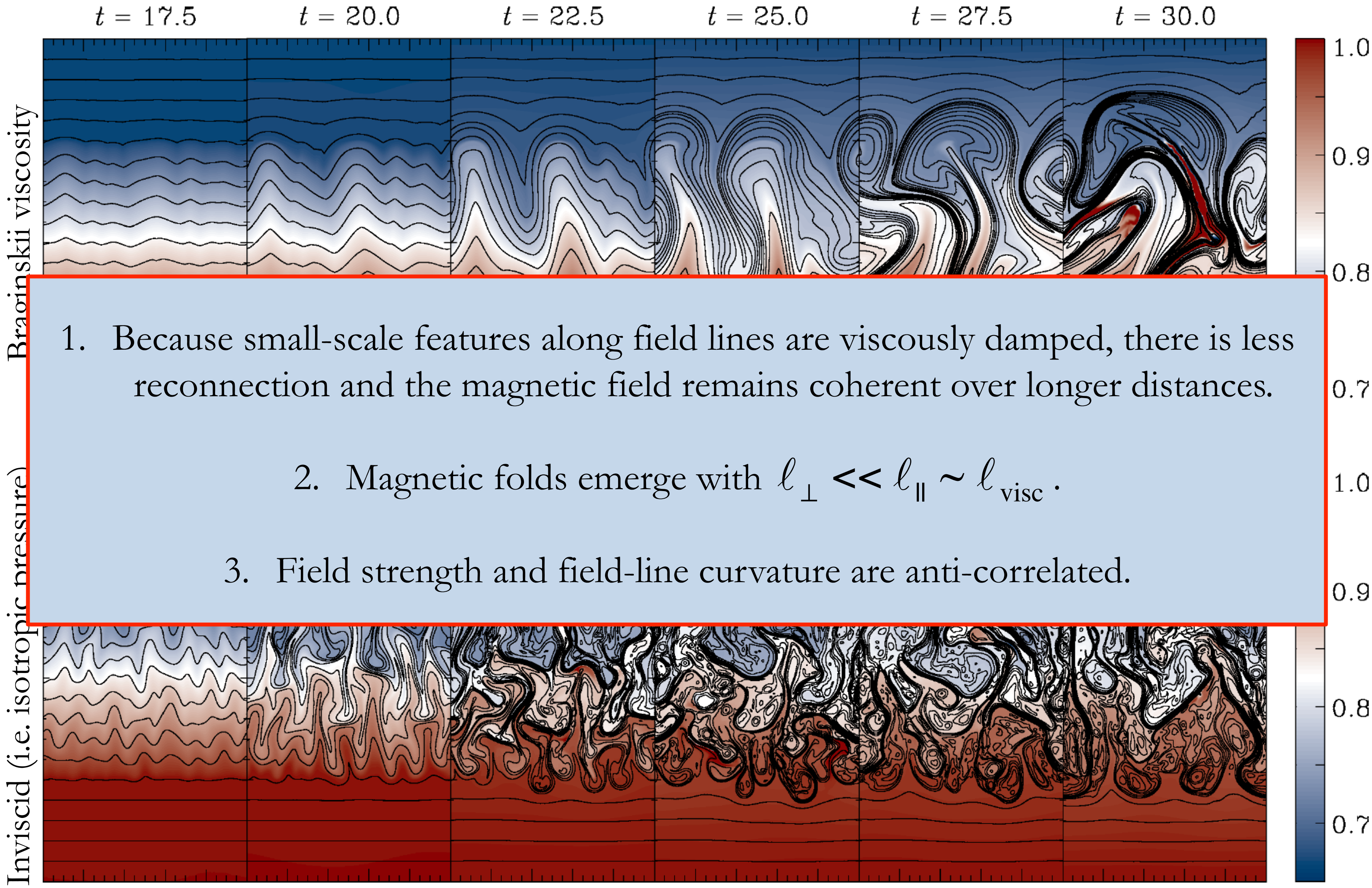
Fastest-growing MTI modes
evade suppression since $\delta B_{||} = 0$.
But the available wavenumber space is
substantially altered.

Quasi-global 2D simulations of MTI

Quasi-global 2D simulations of MTI



Quasi-global 2D simulations of MTI



1. Because small-scale features along field lines are viscously damped, there is less reconnection and the magnetic field remains coherent over longer distances.
2. Magnetic folds emerge with $\ell_\perp << \ell_\parallel \sim \ell_{\rm visc}$.
3. Field strength and field-line curvature are anti-correlated.

Dispersion Relation (Kunz 2011)

$$\delta \propto \exp\left(\sigma t + i\boldsymbol{k}\boldsymbol{\cdot}\boldsymbol{r}\right)$$

To leading order in $\omega_{\rm dyn}/\omega_{\rm cond} \ll 1$,

$$\widetilde{\sigma}^2 \left(\widetilde{\sigma}^2 + \sigma\,\omega_{\rm visc} \frac{k_\perp^2}{k^2} + g \frac{{\rm d}\ln T}{{\rm d}z} \frac{\mathcal{K}}{k^2} \right) \simeq \boxed{-\sigma\,\omega_{\rm visc}\, g \frac{{\rm d}\ln T}{{\rm d}z} \frac{b_x^2 k_y^2}{k^2}}$$

When $\omega_{\rm visc} \gg \omega_{\rm dyn} \sim \sigma$, $\widetilde{\sigma}^2 \simeq -g \frac{{\rm d}\ln T}{{\rm d}z} \frac{b_x^2 k_y^2}{k_\perp^2} > 0$

Just a little bit of k_y will make these maximally unstable!
"Alfvénic MTI"

Dispersion Relation (Kunz 2011)

$$\delta \propto \exp\left(\sigma t + i\boldsymbol{k}\cdot\boldsymbol{r}\right)$$

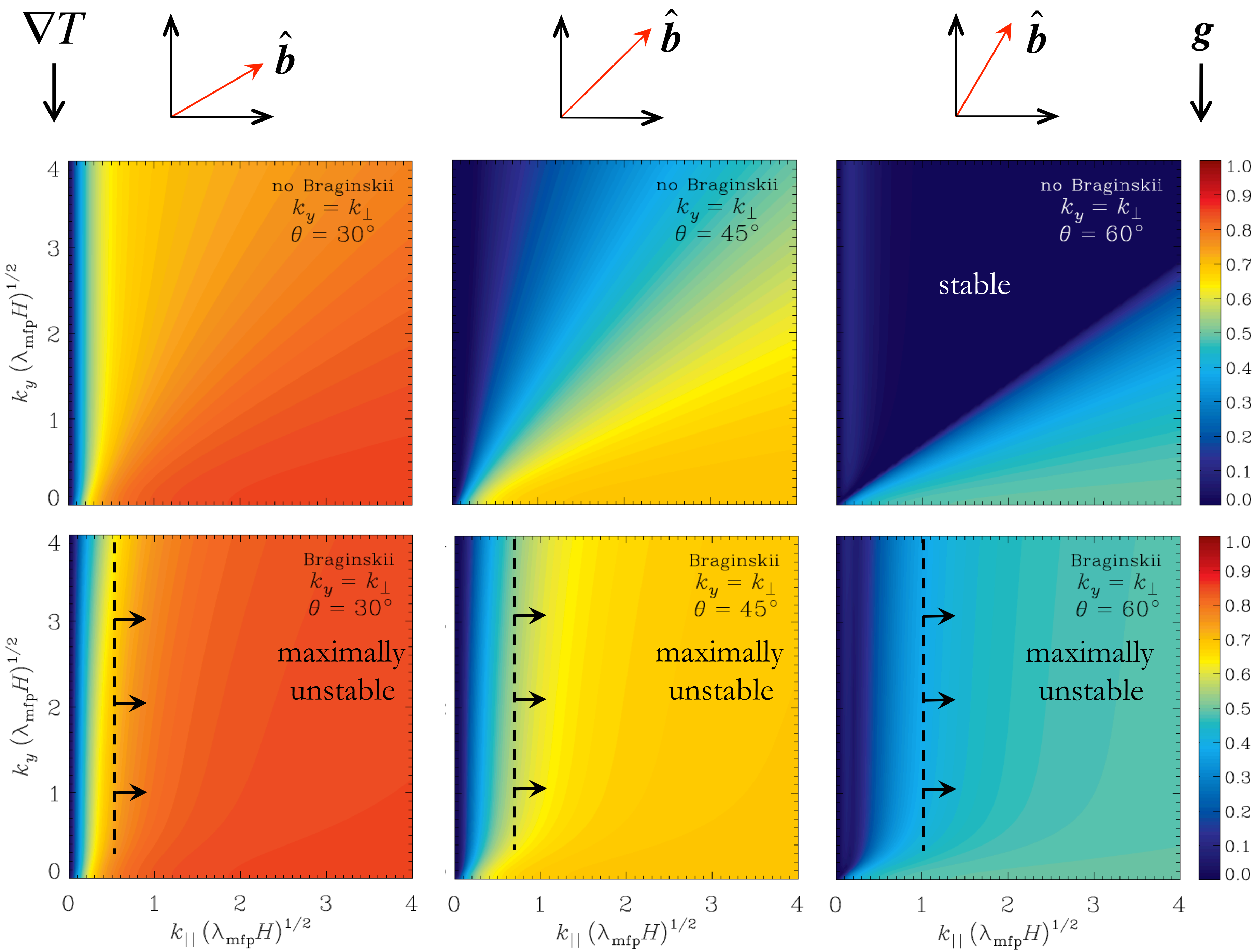
HBI, MTI, Alfvénic MTI:

$$\widetilde{\sigma}^2 \left(\widetilde{\sigma}^2 + \sigma\,\omega_{\mathrm{visc}} \frac{k_\perp^2}{k^2} + g \frac{\mathrm{d}\ln T}{\mathrm{d}z} \frac{\mathcal{K}}{k^2} \right) \simeq -\sigma\,\omega_{\mathrm{visc}}\, g \frac{\mathrm{d}\ln T}{\mathrm{d}z} \frac{b_x^2 k_y^2}{k^2}$$

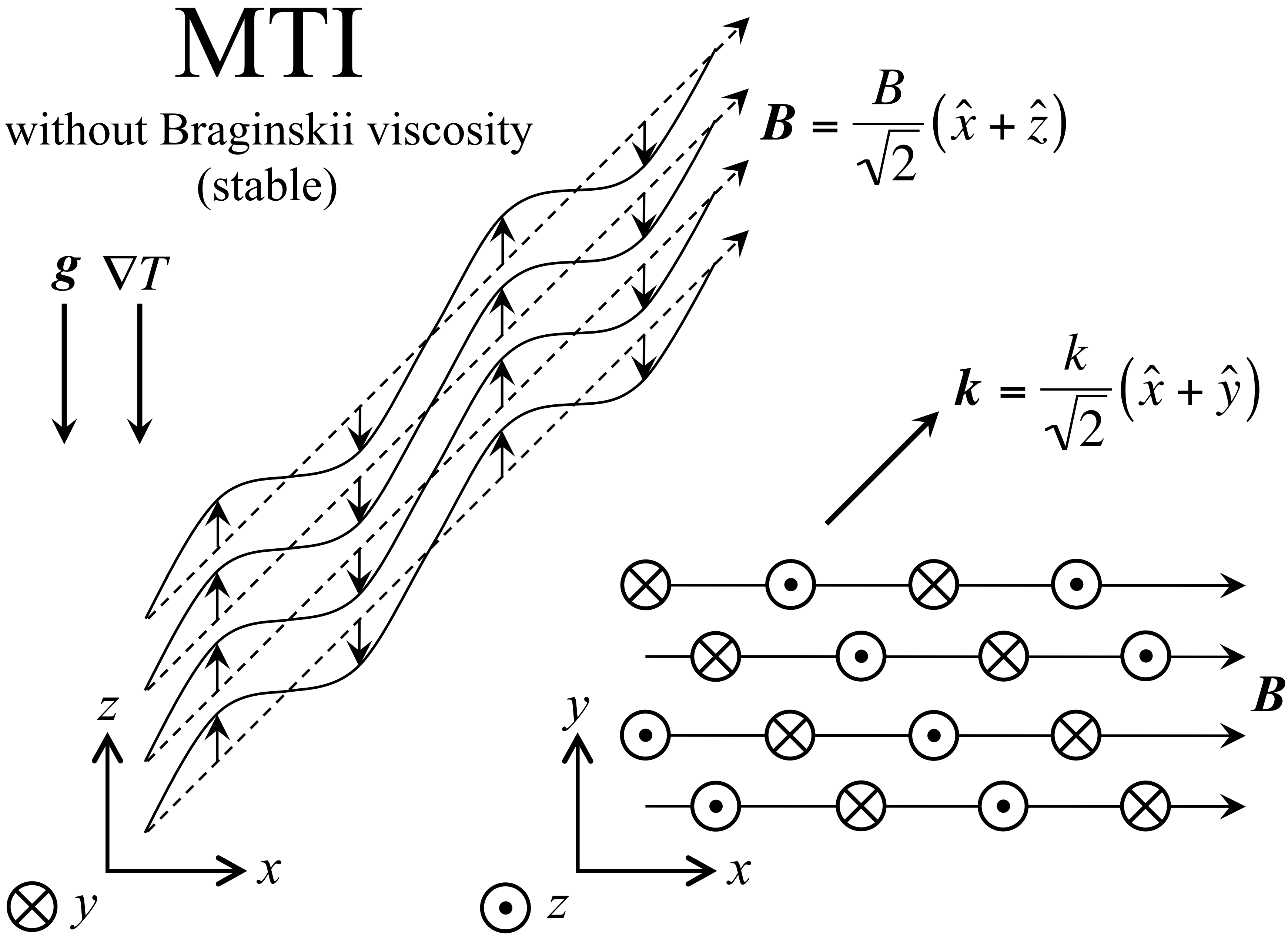
MRI, Alfvénic MRI (i.e. MVI):

$$\widetilde{\sigma}^2 \left(\widetilde{\sigma}^2 + \sigma\omega_{\mathrm{visc}} \frac{k_\perp^2}{k^2} + g \frac{\mathrm{d}\ln \Omega^2}{\mathrm{d}R} \frac{k_Z^2}{k^2} \right) = -\sigma\omega_{\mathrm{visc}}\, g \frac{\mathrm{d}\ln \Omega^2}{\mathrm{d}R} \frac{b_\phi^2 k_Z^2}{k^2} - 4\Omega^2 \frac{k_Z^2}{k^2} \sigma^2$$

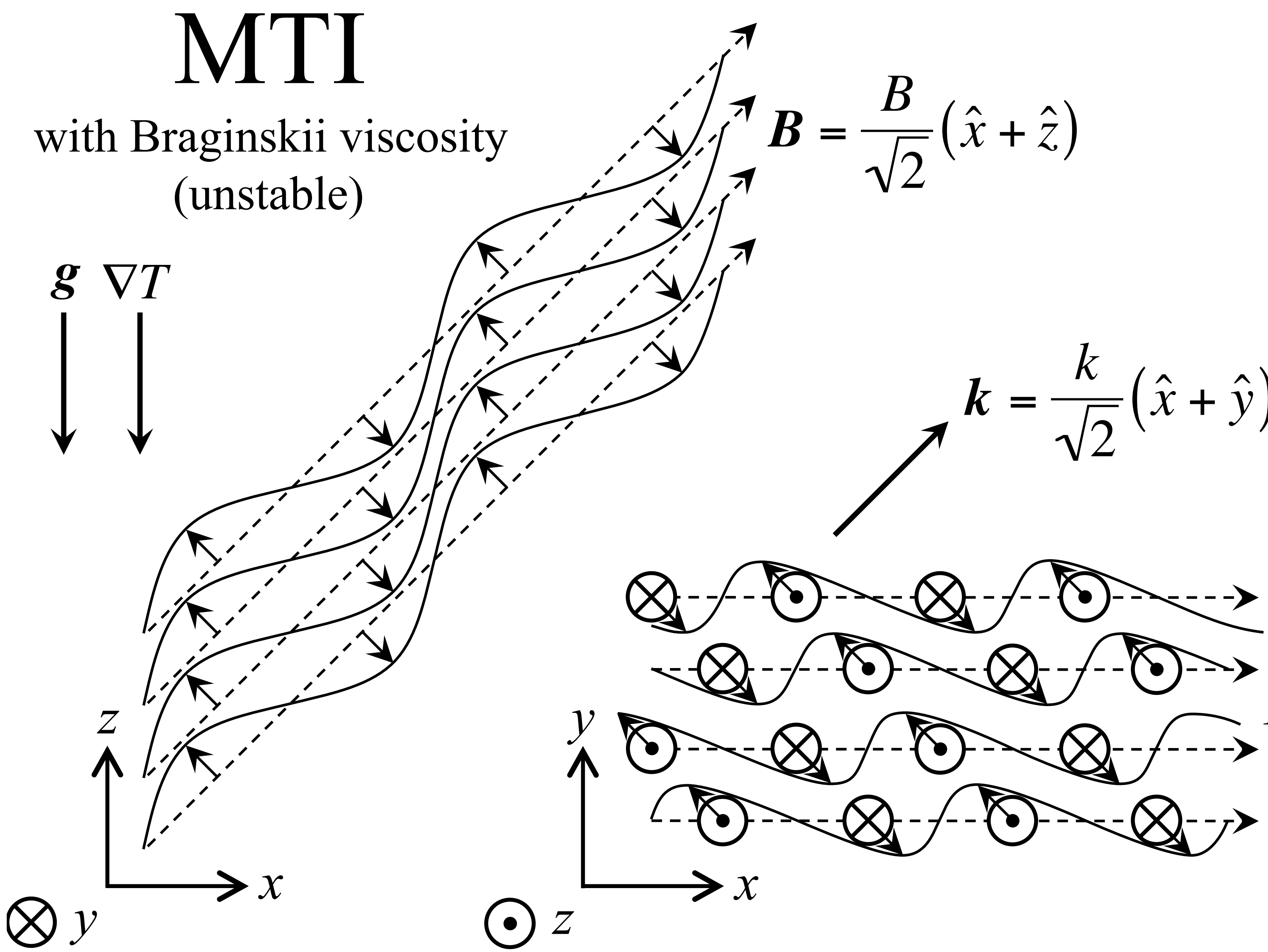
Balbus (2004)

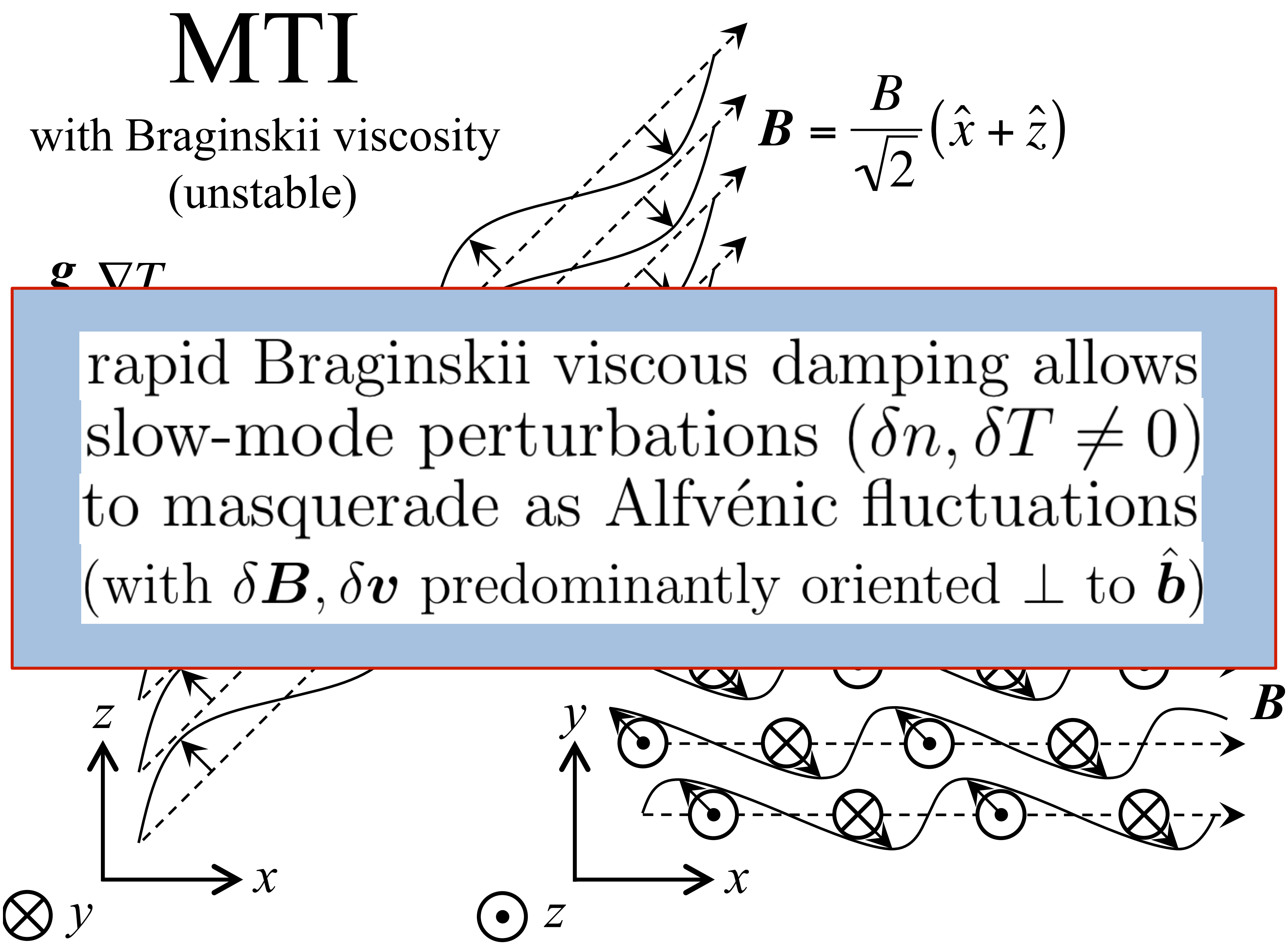


∇T
b̂
b̂
b̂
g
no Braginskii
$k_y = k_\perp$
$\theta = 30°$
no Braginskii
$k_y = k_\perp$
$\theta = 45°$
no Braginskii
$k_y = k_\perp$
$\theta = 60°$
stable
Braginskii
$k_y = k_\perp$
$\theta = 30°$
maximally unstable
Braginskii
$k_y = k_\perp$
$\theta = 45°$
maximally unstable
Braginskii
$k_y = k_\perp$
$\theta = 60°$
maximally unstable
$k_y\,(\lambda_{\rm mfp} H)^{1/2}$
$k_{||}\,(\lambda_{\rm mfp} H)^{1/2}$
0 1 2 3 4
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0



MTI
without Braginskii viscosity
(stable)
$\boldsymbol{g}$ ∇T
$\boldsymbol{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{z})$
$\boldsymbol{k} = \frac{k}{\sqrt{2}}(\hat{x} + \hat{y})$
$\boldsymbol{B}$
z
x
y
y
x
z


MTI
with Braginskii viscosity
(unstable)
$\boldsymbol{g}$ ∇T
$\boldsymbol{B} = \frac{B}{\sqrt{2}}(\hat{x} + \hat{z})$
$\boldsymbol{k} = \frac{k}{\sqrt{2}}(\hat{x} + \hat{y})$
$\boldsymbol{B}$
z
x
y
y
x
z



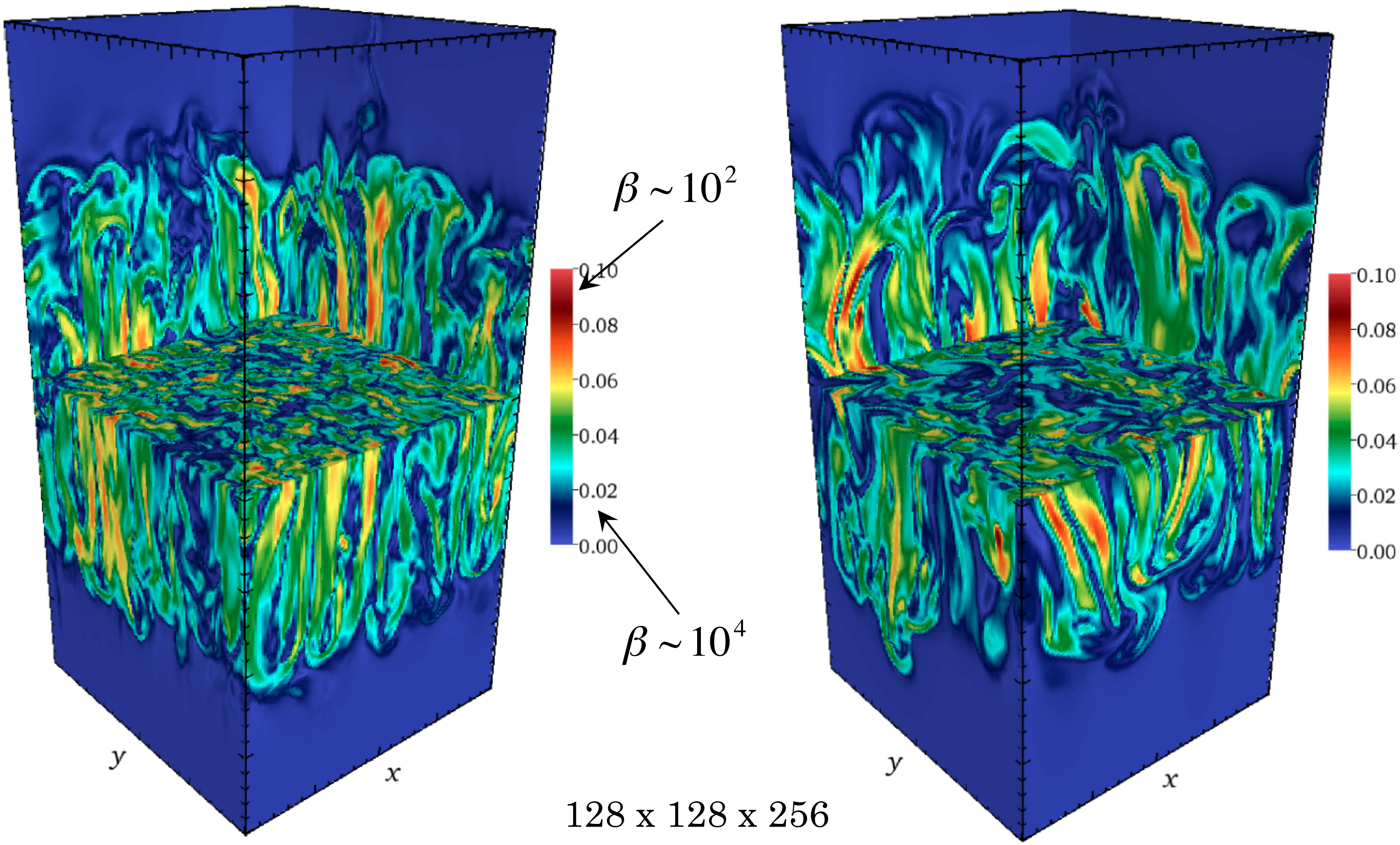
rapid Braginskii viscous damping allows slow-mode perturbations ($\delta n, \delta T \neq 0$) to masquerade as Alfvénic fluctuations (with $\delta \boldsymbol{B}, \delta \boldsymbol{v}$ predominantly oriented $\perp$ to $\hat{\boldsymbol{b}}$)

Quasi-global 3D simulations of MTI

Plotted: Magnetic-Field Strength

Inviscid (i.e. isotropic pressure)

Braginskii viscosity

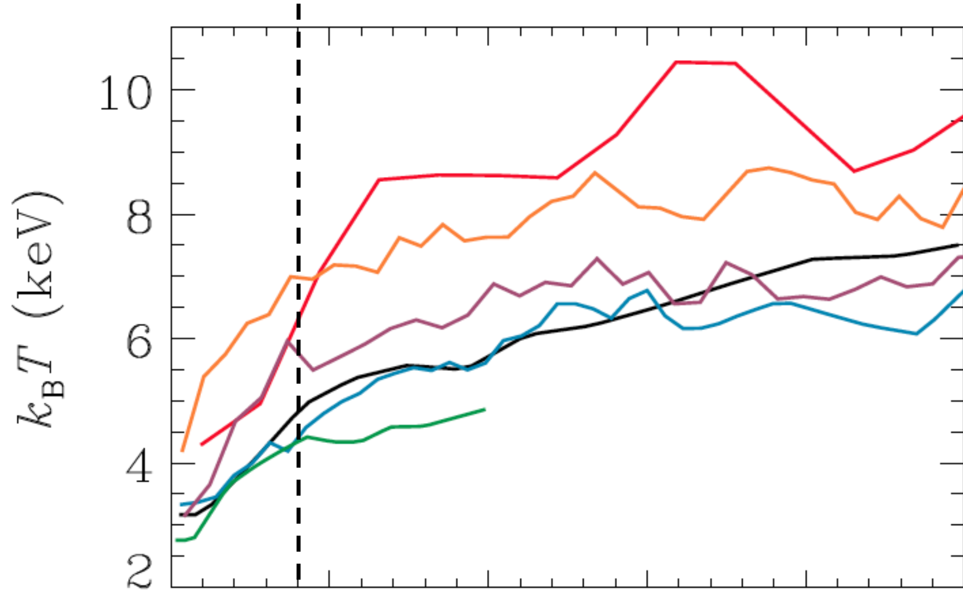


Braginskii viscosity only mildly affects HBI in dense, cold center – may still operate there.

HBI sluggish due to relatively low collisionality and resulting non-local character of unstable modes.

However, radio-mode feedback and magnetic tension may limit its relevance.

Thermal conduction might remain important.



MTI leads to radially biased *B*-field with Alfvénic turbulence ($M \sim< 10$)

Parrish+

Major Caveat

Once

$$\frac{\delta B_{\|}}{B} \gtrsim \frac{\nu_{\rm ii}}{\beta \sigma_{\rm HBI}} \sim \frac{1}{\beta\,{\rm Kn}} \;,$$

rapidly growing microscale instabilities are triggered and Braginskii-MHD is ill-posed.

Cluster cores: ${\rm Kn}^{-1} \sim 10^3 – 10^2$.

Cluster outskirts: ${\rm Kn}^{-1} \sim 10^2 – 10^1$.

We chose

$$\beta \sim 10^4 – 10^5$$

What to do, what to do…

Option #1: Limit pressure anisotropy by increasing collisionality so that no microscale instabilities occur.
(Sharma et al 2006; Schekochihin & Cowley 2006)

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(Schekochihin et al 2008; Rosin et al 2011)

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Option #3: Alex is working on it…

Conclusions

1. Anisotropic viscosity affects how ICM interacts with temperature gradient. One cannot self-consistently take the limit of fast thermal conduction along *B*-field lines while simultaneously neglecting differences in $p_\perp$ and $p_\|$ (singular limit).

2. Anisotropic viscosity (aka pressure anisotropy) significantly impairs the HBI, except in innermost few tens of kpc where collisionality is relatively high. Radio-mode feedback appears necessary there. Elsewhere, HBI becomes global and slow-growing.

3. MTI: anisotropic viscosity maintains coherence of field lines over larger distances than in the inviscid case, providing a natural lower limit for the scale on which the field can fluctuate freely.

4. Choices must be made by a simulator regarding how to limit pressure anisotropy, especially at large plasma beta. Dynamo theories beware.