Sloshing Cold Fronts and Cluster g-modes

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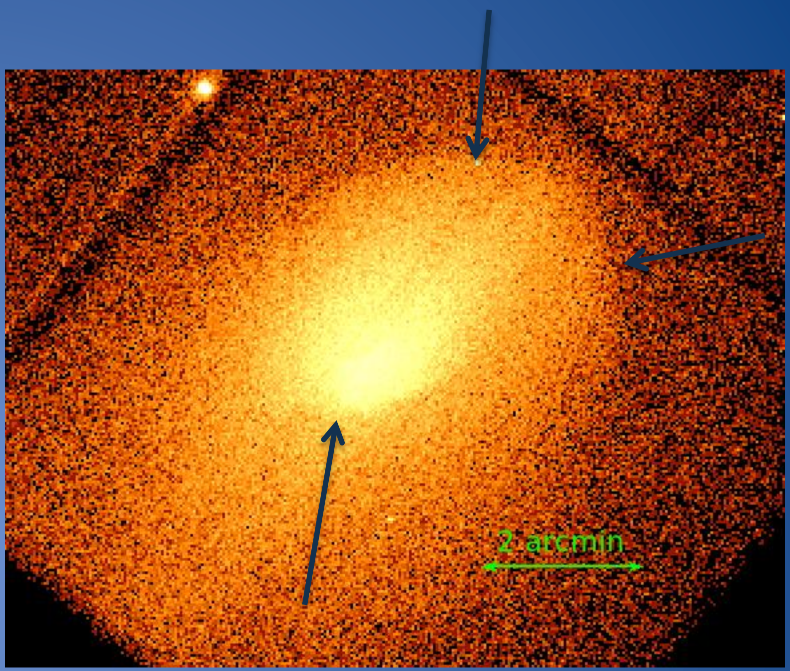
Sloshing Cold Fronts

Contact discontinuities in the intracluster medium appear as “edges” in X-ray images of many clusters – “cold fronts” (Markevitch & Vikhlinin 2007)

Sloshing cold fronts form after core passage of a minor merger remnant (Tittley & Henrikson 2005; Ascasibar & Markevitch 2006)

Sloshing fronts contain information about merger history (Owers et al 2009; Roediger et al 2011)



Sloshing fronts in Abell 2142 (Markevtich et al 2000)

Understanding sloshing cold fronts should make it possible to determine merger properties – potentially without resorting to large numerical simulations

Core Passage Excites Dipolar g-modes

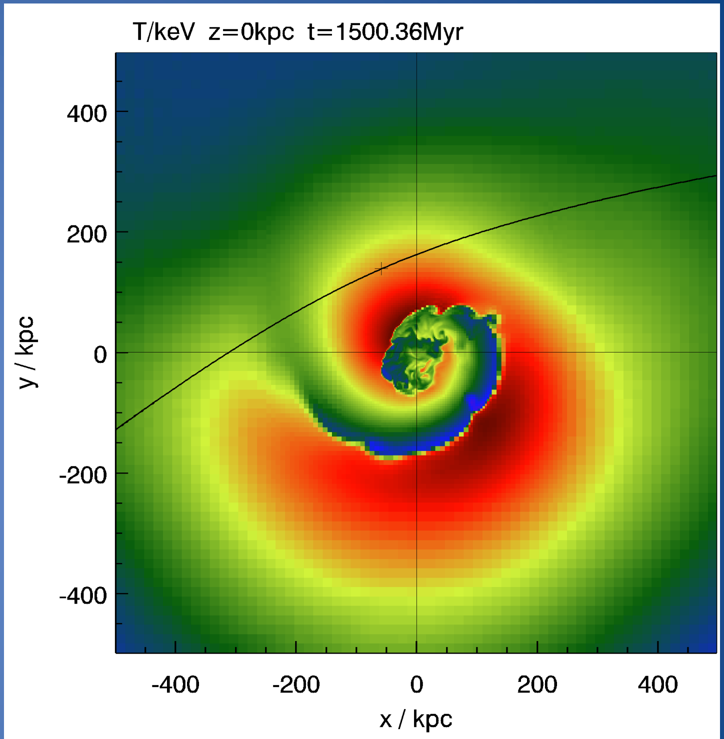
Assume: small peturbations, confined to the gas (Roediger & ZuHone 2012)

– expressible as a superposition of the modes of a cluster atmosphere

– slow => mainly g-modes

– one-armed spiral => mainly dipolar

In a spherical cluster, sloshing is expressible in terms of spherical harmonics, Y_{lm}, with only l = 1



Roediger & ZuHone (2012)

Symmetry: $rY_{1,1} \sim x + i\,y$; $rY_{1,-1} \sim x - i\,y$; $rY_{1,0} \sim z$

=> perturber orbiting in x-y plane ($\theta = \pi/2$) cannot excite m = 0 mode, so only m = ±1

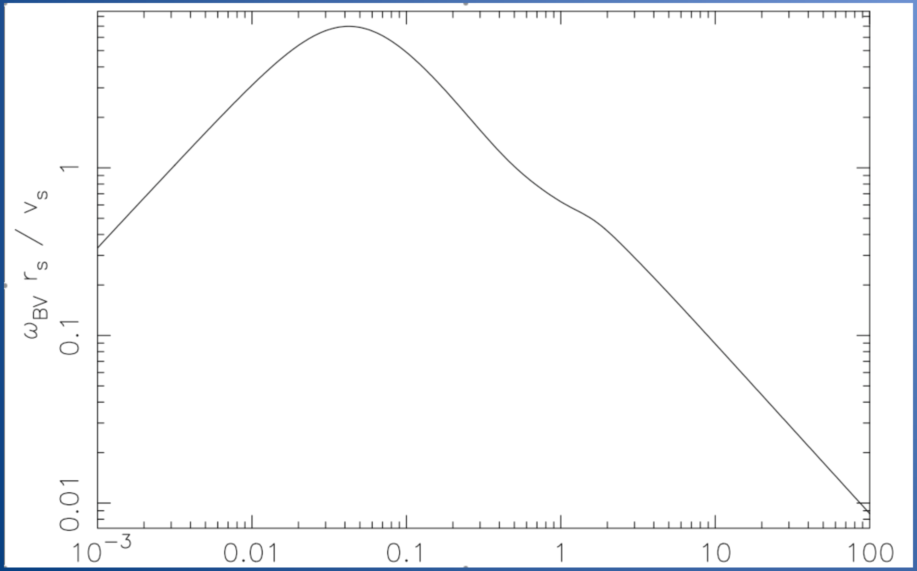
Cluster g-modes

Balbus & Soker (1990)

In a stably stratified atmosphere, when displaced radially, buoyant forces make a gas blob oscillates at the Brunt-Väisälä frequency: $\omega_{\rm BV}^2 = \frac{3g}{5r}\frac{d\ln K}{d\ln r} = \frac{3v_{\rm K}^2}{5r^2}\frac{d\ln K}{d\ln r}$, where $K = \frac{kT}{n_{\rm e}^{2/3}}$

Inclined shearing mode oscillates at $\omega_{\rm BV}\frac{|\mathbf{k}\times\mathbf{g}|}{kg}$

$\omega_{BV}(r)$ for Virgo model of Roediger et al (2011)



r (27.7 kpc)

Roughly: g-modes for $\omega < v_K / r$
sound modes for $\omega > s / r$

g-modes at small r couple to sound at large r => g-modes not fully trapped

Resonant radius: $\omega = \omega_{BV}(r_\omega)$

For g-modes, d ln W / d ln r, with $W = \langle v^2 \rangle$, evaluated at r_ω, shows a pronounced minimum

Cluster g-modes

Flow equations

$$\frac{d\rho}{dt} + \rho\nabla\cdot\mathbf{v} = 0$$

$$\rho\frac{d\mathbf{v}}{dt} = -\nabla p + \rho\mathbf{g}$$

$$\frac{d}{dt}\ln\frac{p}{\rho^{5/3}} = 0$$

Perturbation form ($x = r/r_s$)

density	$\delta\rho(r,\theta,\phi,t) = \rho_0(r) f(x) Y_{lm}(\theta,\phi) e^{-i\omega t}$
pressure	$\delta p(r,\theta,\phi,t) = p_0(r) h(x) Y_{lm}(\theta,\phi) e^{-i\omega t}$
radial velocity	$v_r(r,\theta,\phi,t) = i v_s u(x) Y_{lm}(\theta,\phi) e^{-i\omega t}$
transverse velocity	$\mathbf{v}_t(r,\theta,\phi,t) = i r_s v_s \chi(x) e^{-i\omega t}\nabla_t Y_{lm}(\theta,\phi)$

where $\nabla_t = \mathbf{e}_\theta \frac{1}{r}\frac{\partial}{\partial\theta} + \mathbf{e}_\phi \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}$

Perturbation equations ($\lambda = \omega r_s/v_s$ and $\alpha = c_{\rm sound}/v_s$)

$$0 = -\lambda f + \frac{1}{\rho_0 x^2}\frac{d}{dx}\rho_0 x^2 u - \frac{l(l+1)}{x^2}\chi$$

$$\lambda u = -\frac{3}{5}\alpha^2\left[\frac{dh}{dx} + \frac{1}{p_0}\frac{dp_0}{dx}(h-f)\right]$$

$$-\lambda\chi = \frac{3}{5}\alpha^2 h$$

$$0 = -\lambda\left(h - \frac{5}{3}f\right) + u\frac{d\ln K_0}{dx}$$

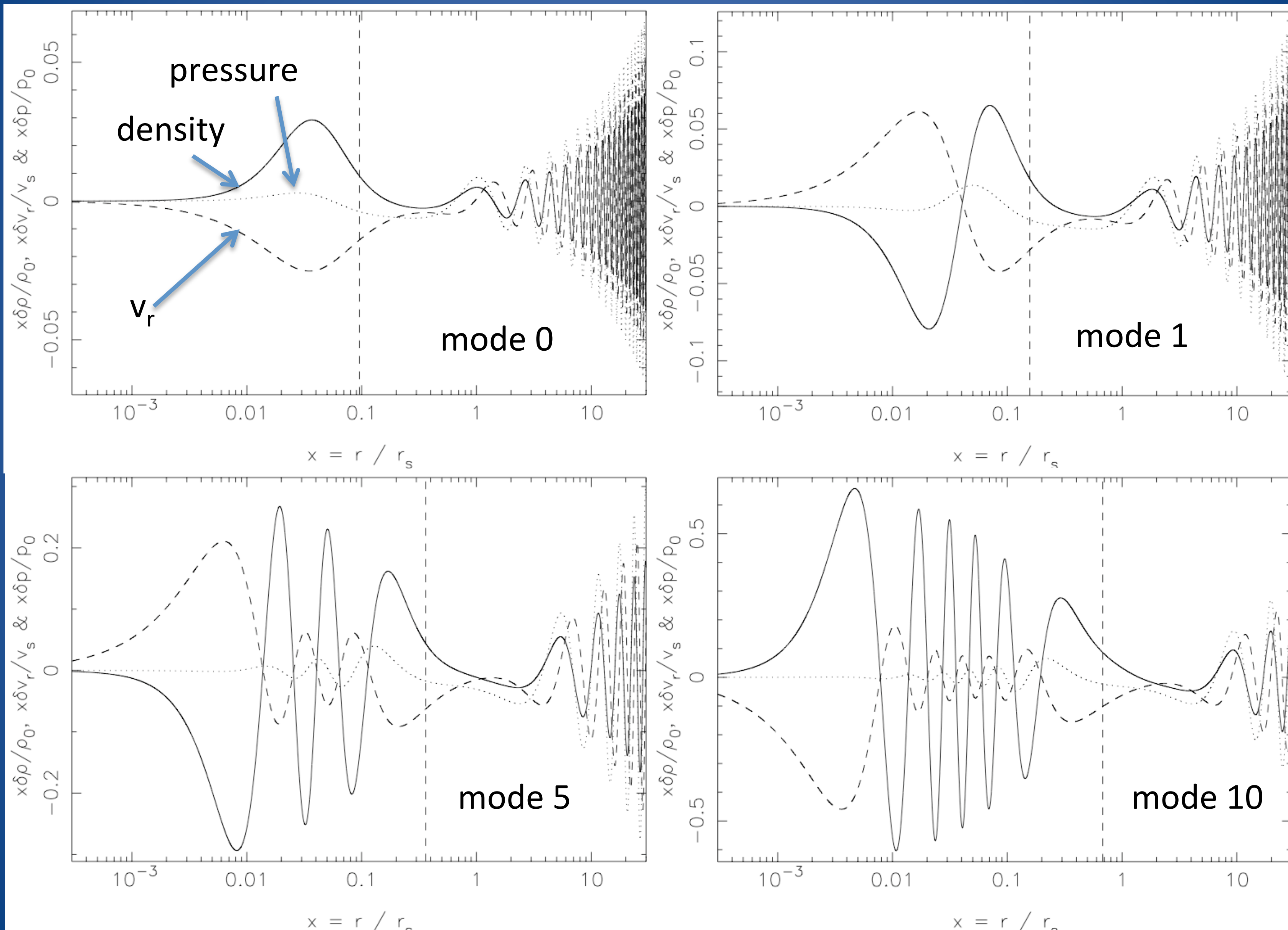
Real parts of the form
q (r, θ) cos (C + m Φ – ω t)

– rigid rotation about polar (z) axis

For m = ±1, rotates with angular frequency ω

All terms real – can use real solutions

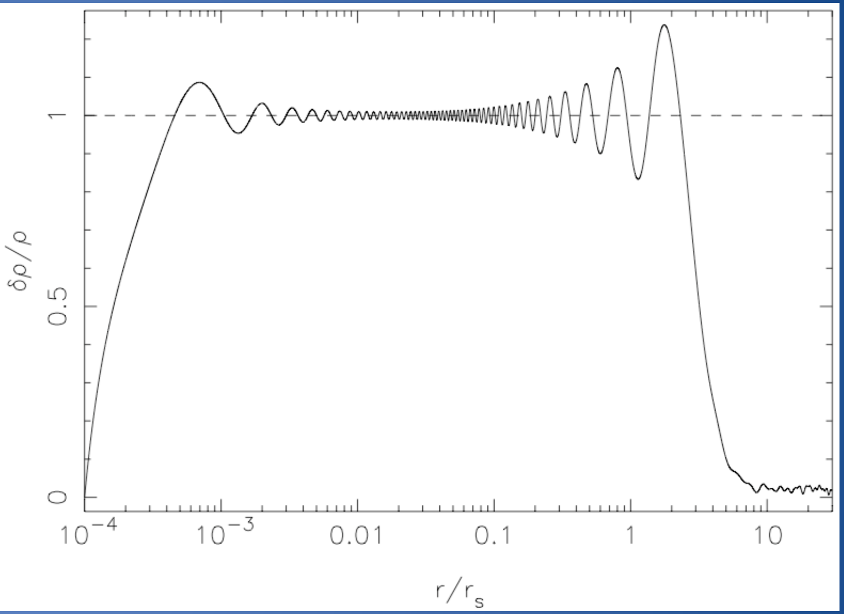
Virgo Cluster g-modes

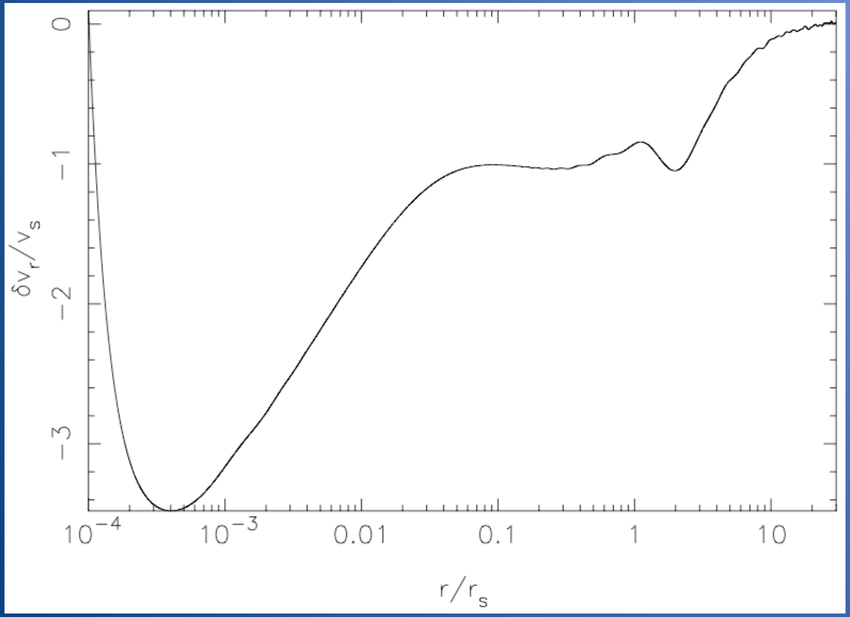


Dipolar Initial Density Perturbation

Assume initial density perturbation is dipolar and uniform in r

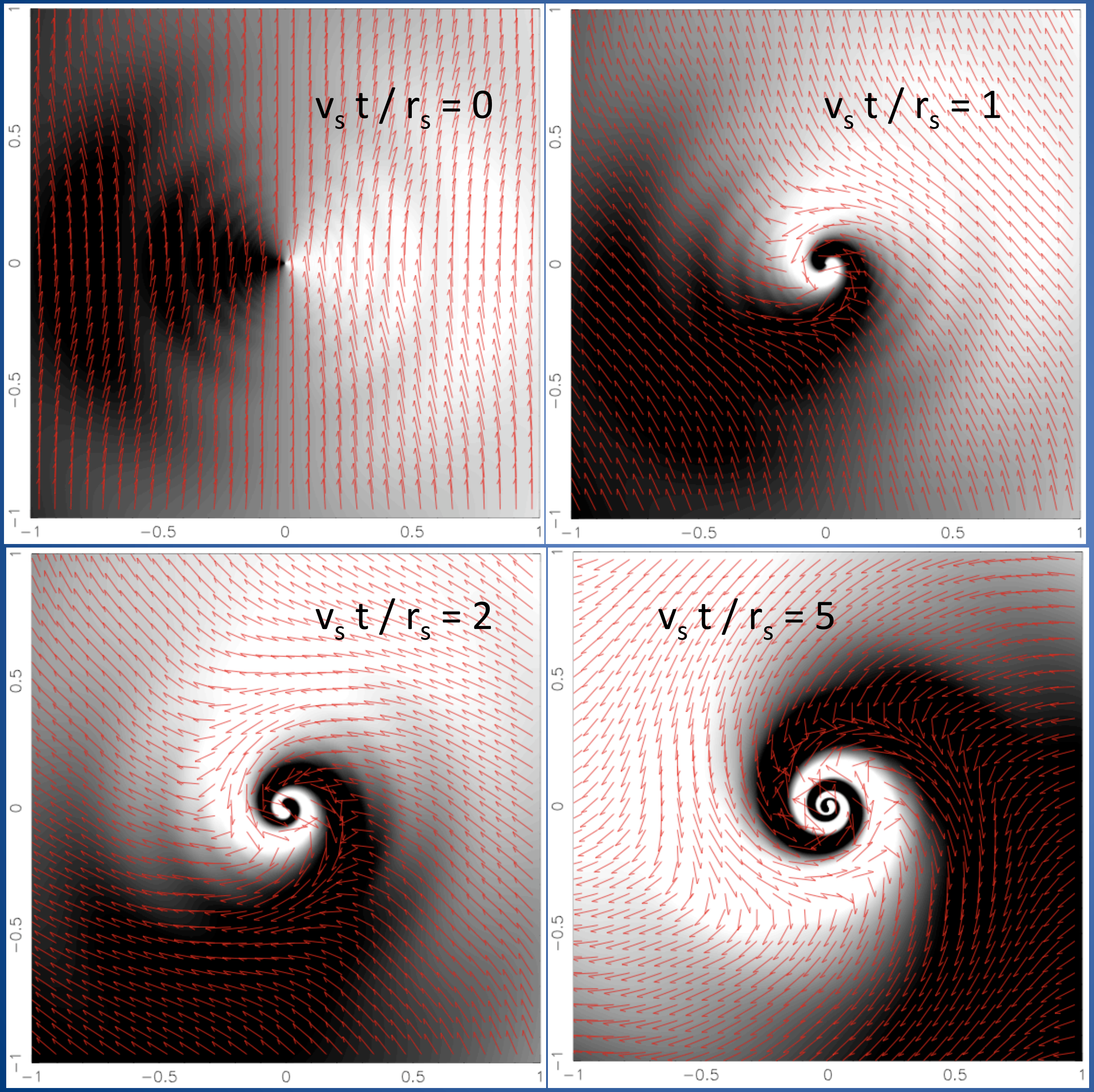
Best fit with modes 0 to 94 (m = +1 only):





Radial velocity

Time development



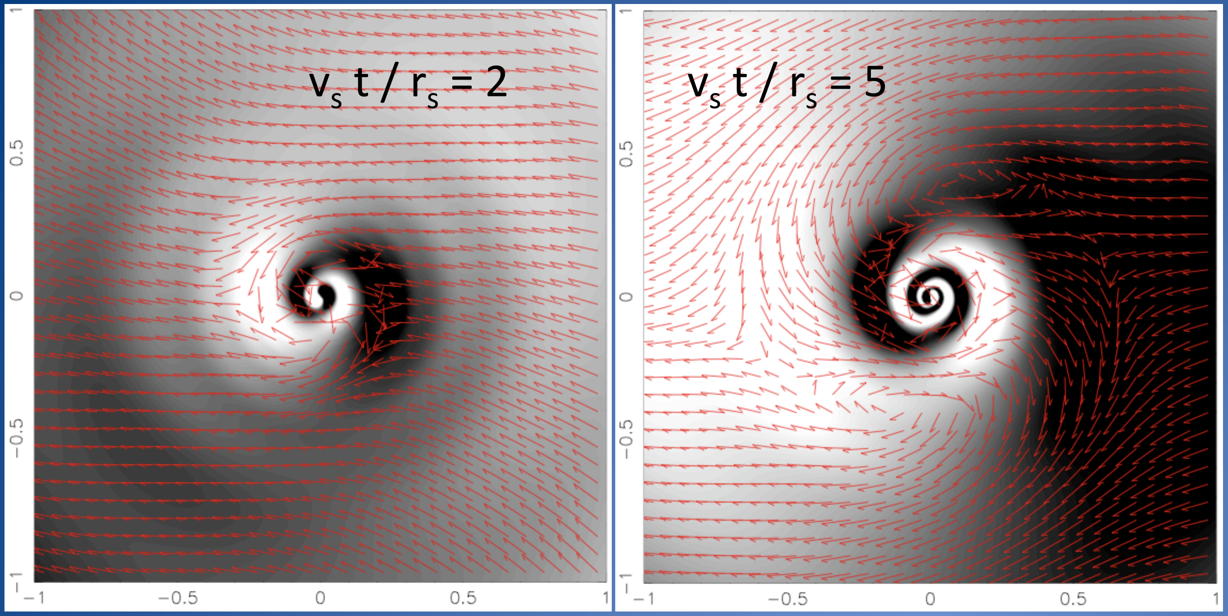
94 mode, m = +1

Higher frequency modes dominate at centre

Mode rotation forms a spiral outward from centre

Amplitude increases as modes separate

Time development



Off centre core passage excites prograde (m = 1) mode preferentially

– not exclusively

e.g., with 30% retrograde (m = -1) modes, smooth spiral is broken into “fronts”

Mode pairs for m = ±1 interfere constructively at fixed Φ

=> enhanced regions move radially

94 modes are insufficient; assembling enough modes for a large scale simulation is not much lest costly than a full simulation

Self-similar Cluster Model

Isothermal cluster atmosphere, with power law initial density profile, ρ (r) ~ $r^{-\kappa}$

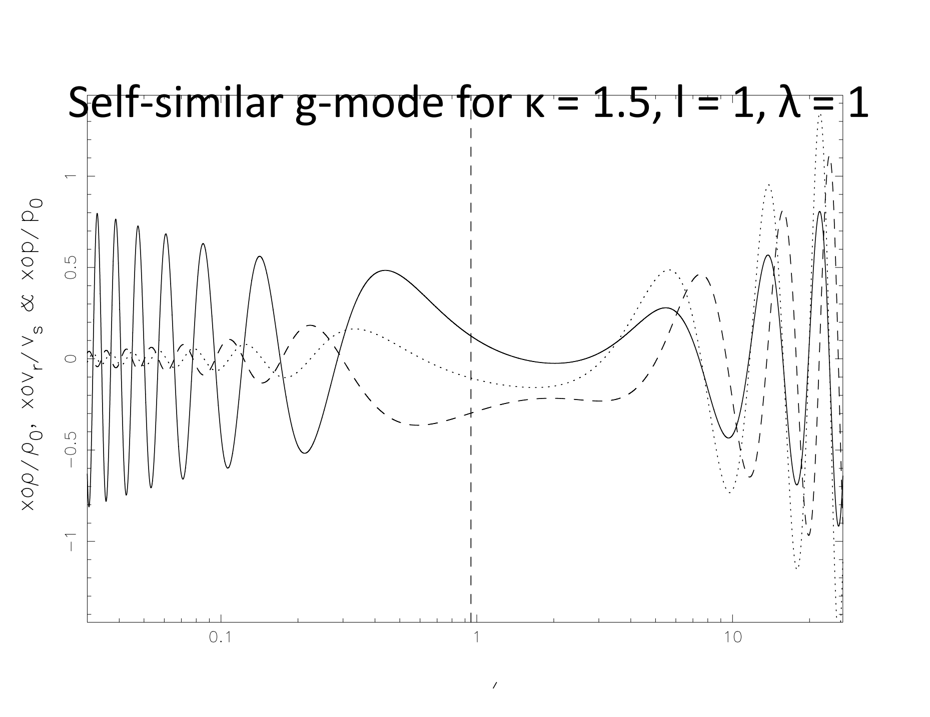
Scaled Brunt-Väisälä frequency: $$\lambda_{\rm BV}^2 = \left(\frac{\omega_{\rm BV} r_{\rm s}}{v_{\rm s}}\right)^2 = \frac{2\kappa^2}{5x^2}$$

Solutions for scaled frequency, λ, related to those for λ = 1 by:

$$f_\lambda(x) = f_1(\lambda x);\ u_\lambda(x) = u_1(\lambda x);\ h_\lambda(x) = h_1(\lambda x);\ \lambda\chi_\lambda(x) = \chi_1(\lambda x)$$

A g-mode of the same form exists for every frequency

General solution is a weighted sum (integral) of these over λ (frequency; also l and m)



Self-similar Cold Front Model

Restricting to the dipolar modes (l = 1, m = ±1), with power law weights, J’ $\lambda^{-(1+\eta)}$, gives self-similar solutions (y = λ x),

$$\delta\rho(r,\theta,\phi,t)/\rho_0(r) = x^\eta e^{im\phi} \sin\theta \int_0^\infty J y^{-(1+\eta)} f_1(y) e^{-iys} dy$$

with similarity variable s = v_s t / r

(similar forms for other parts of the solution)

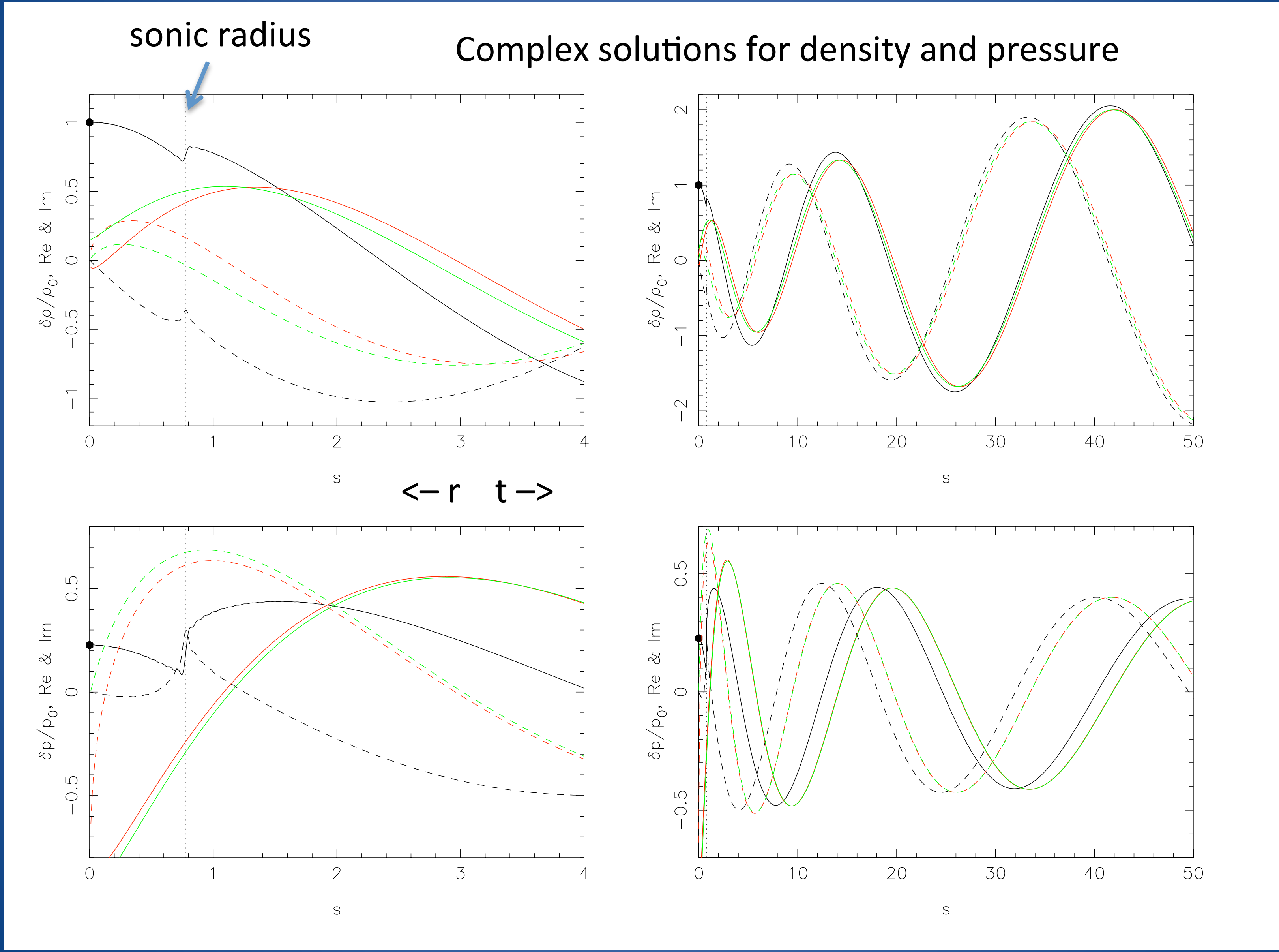
Restricting to η = 0 (constant initial perturbation), gives $\delta\rho(r,\theta,\phi,t)/\rho_0(r) = q(s)e^{im\phi}\sin\theta$

Asymptotic form for y –> 0, $f_1(y) \to By^{-w}\sin(A/y+\delta)$, where $A = 2\kappa/\sqrt{5}$ and $w = 2-\kappa/2$ gives real solution, for s –> ∞ (r –> 0 or t –> ∞), $\delta\rho/\rho_0 \propto s^{w/2-1/4}\sin\theta\cos(m\phi - 2\sqrt{As} - \delta')$

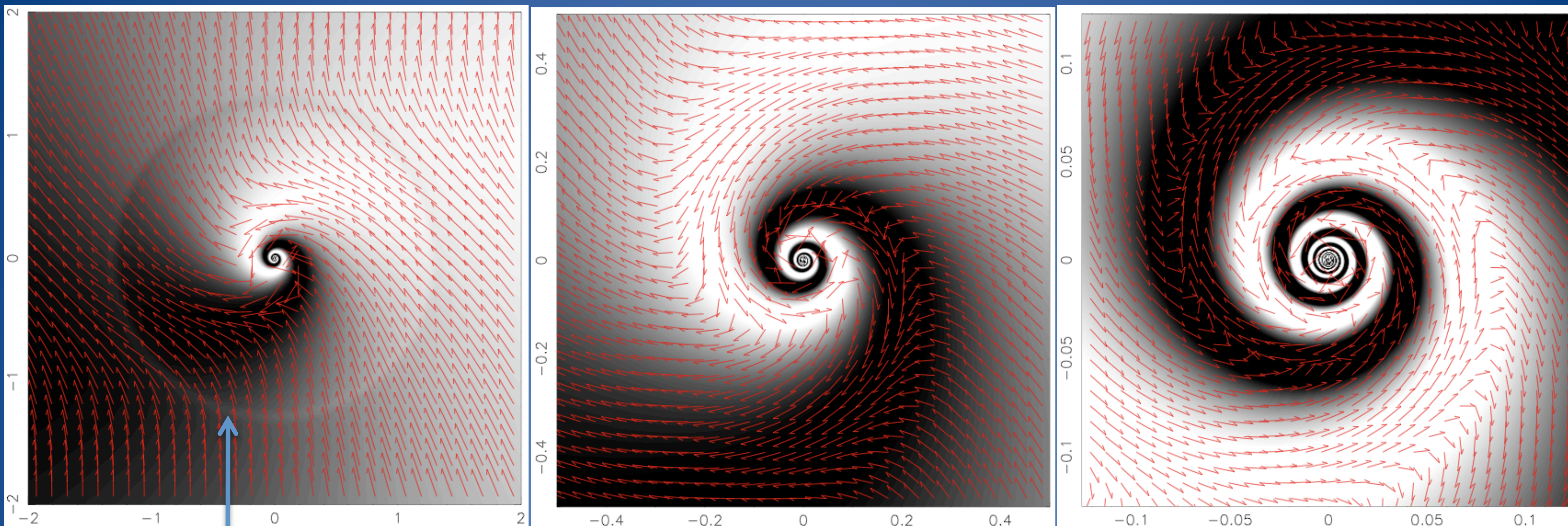
Fixing the phase of the cosine gives $m\phi = 2\sqrt{Av_s t/r} + \text{constant},$ defining a spiral that wraps ever more tightly at small r

Fixed points in the self-similar pattern (fixed s) have speed $\left(\frac{dr}{dt}\right)_s = \frac{r}{t} = \frac{v_s}{s}$ – constant

Self-similar Cold Front Model



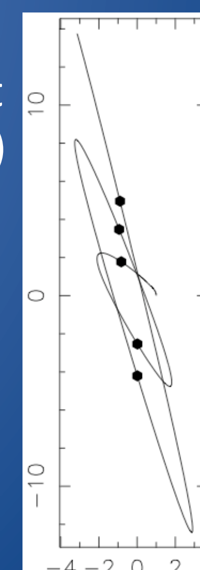
Time Development of Self-similar Model



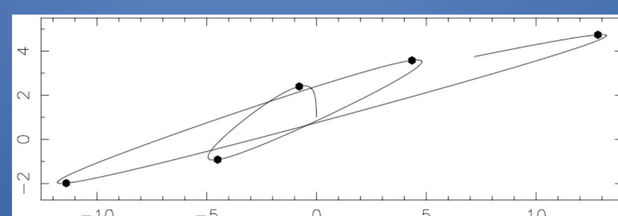
t × 4 –> t × 4 –>

Sound wave starting transient

Path of fluid element starting from (1, 0, 0)



$\kappa = 1.5$, $\eta = 0$, $m = 1$



Path of fluid element starting from (0, 1, 0)

Sloshing involves local oscillation, not large-scale flows (cf. Keshet 2012)

Front Formation

If $A\delta\mathbf{r}_\mathrm{f}(\mathbf{r}_0,t)$ is the displacement of a fluid element from its initial position, the separation between two close fluid elements evolves as

$$d\mathbf{r} \approx d\mathbf{r}_0 + A\sum_{i=1}^{3}\frac{\partial\delta\mathbf{r}_\mathrm{f}}{\partial r_{0,i}}dr_{0,i} = (\mathbf{1} + A\mathbf{M})d\mathbf{r}_0, \text{ where } \mathbf{M} = (\partial\delta\mathbf{r}_\mathrm{f}/\partial\mathbf{r}_0)$$

Here, $\delta\mathbf{r}_\mathrm{f}(\mathbf{r}_\mathbf{0},t)$ is the displacement for a fixed disturbance and A is the amplitude relative to it

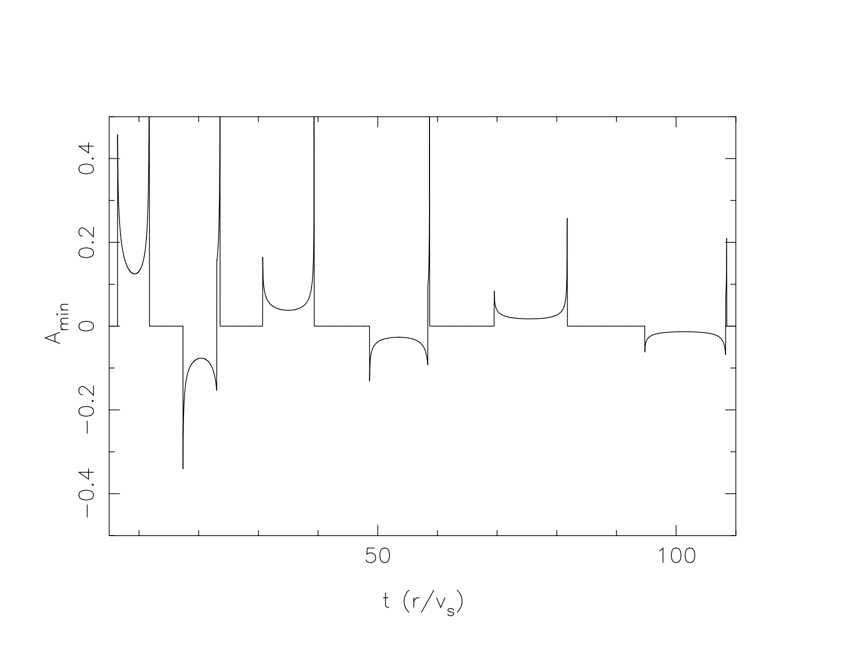
Where the flow is converging (downward zero crossings of the density perturbation vs r), if the amplitude is too large, fluid elements need to pass through one another

At a fixed time and place in the flow, the minimum A > 0 that makes **1** + A **M** singular, A_{min}, gives the amplitude for nonlinearity

Local minima in A_{min} are sites where fronts form first (and the linear model must fail)

Inversion symmetry => $A_{min} < 0$ corresponds to $A_{min} > 0$ at the inverted point

Front Formation



A_{min} vs s for self-similar model (κ = 1.5, η = 0, m = 1) on +x axis

A_{min} = 0 means no root;

A_{min} < 0 means $-A_{min}$ is the minimum A at –s

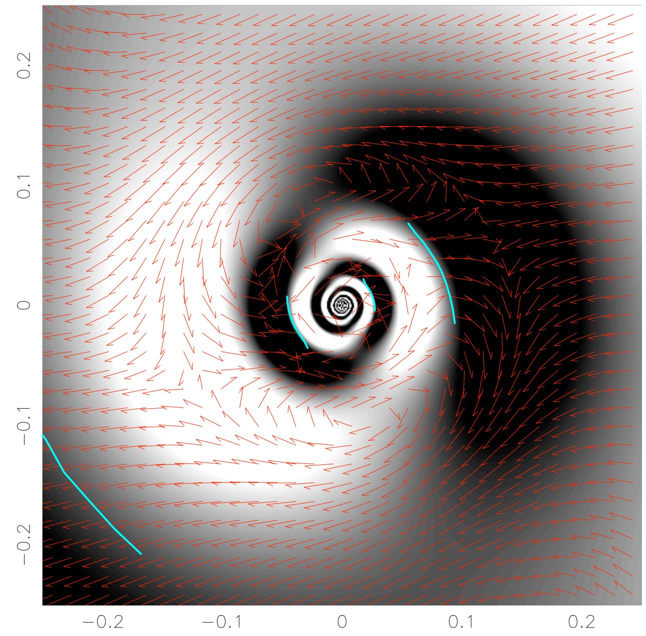
Local minima in $|A_{min}|$ are sites of potential fronts

Amplitude needed for front formation decreases for later (inner) fronts

Initial density perturbations < 10% will cause fronts to form quickly

Front Formation

Self-similar model (κ = 1.5, η = 0, combined m = ±1), showing where cold fronts are expected to form in cyan



Feature Propagation

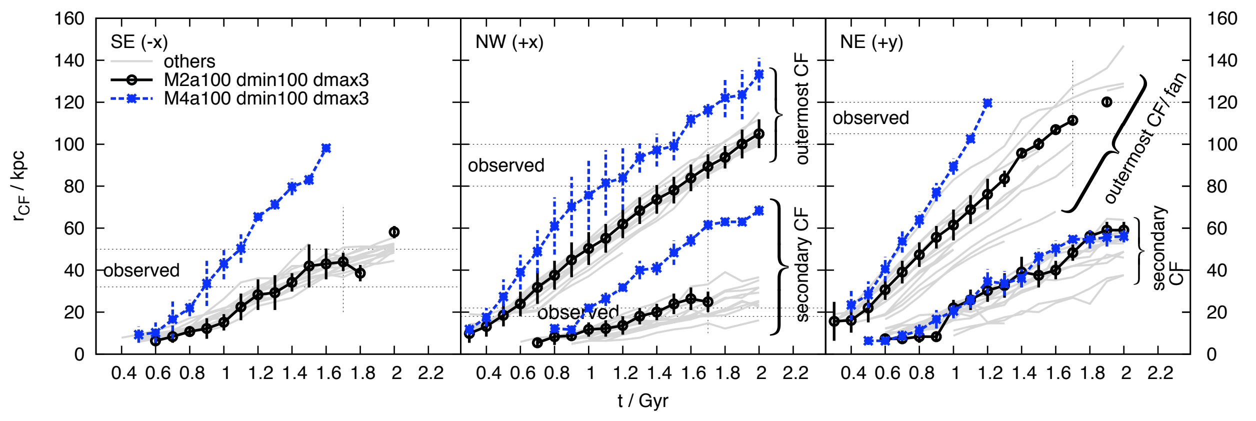
Features (constant $s = v_s\, t / r$) move at constant speed, v_s / s, in the self-similar model

More generally, WKB approximation (Balbus & Soker 1990) gives r and t dependence for g-modes:

$$Q(r)e^{i(\Phi-\omega t)}, \text{ with } \Phi \approx \int_r^{\text{resonance}} k(r')dr' \text{ and } k(r) = \pm\sqrt{(\omega_{\mathrm{BV}}^2/\omega^2 - 1)l(l+1)/r^2}$$

At radius r and time t, the mode sum $\sum_{\text{modes } j} H(\omega_j)Q_j(r)e^{i(\Phi_j - \omega_j t)}$, is dominated by modes near where $\frac{\partial \Phi}{\partial \omega} = \pm t$

Where $\omega_{BV}(r) \sim r^{-1}$, this condition gives $\omega / k = r / t$ => homologous expansion of the pattern – as found by Roediger et al. (2011):



Conclusions

- Sloshing is well-modelled by superpositions of dipolar ($l = 1$) g-modes for $m = \pm 1$
- Core passage of a remnant excites a broad range of g-modes, favouring prograde modes, but also exciting retrograde modes
- Spiral patterns develop outward from the center because the Brunt-Väisälä frequency decreases with radius
- Interference between counter-rotating modes forms higher amplitude front regions that expand radially
- Fronts form where, in linear models, fluid elements would pass through one another
- Initial density perturbations of ~10% are sufficient to cause this nonlinearity
- Since $\omega_{BV}(r) \sim r^{-1}$, fronts expand at nearly constant speed, $v = r / t$