On the Dynamical Stability of the ICM
with Thermal and Composition Gradients

(arXiv:1111.3372 + ongoing work)

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ICM in Galaxy Clusters

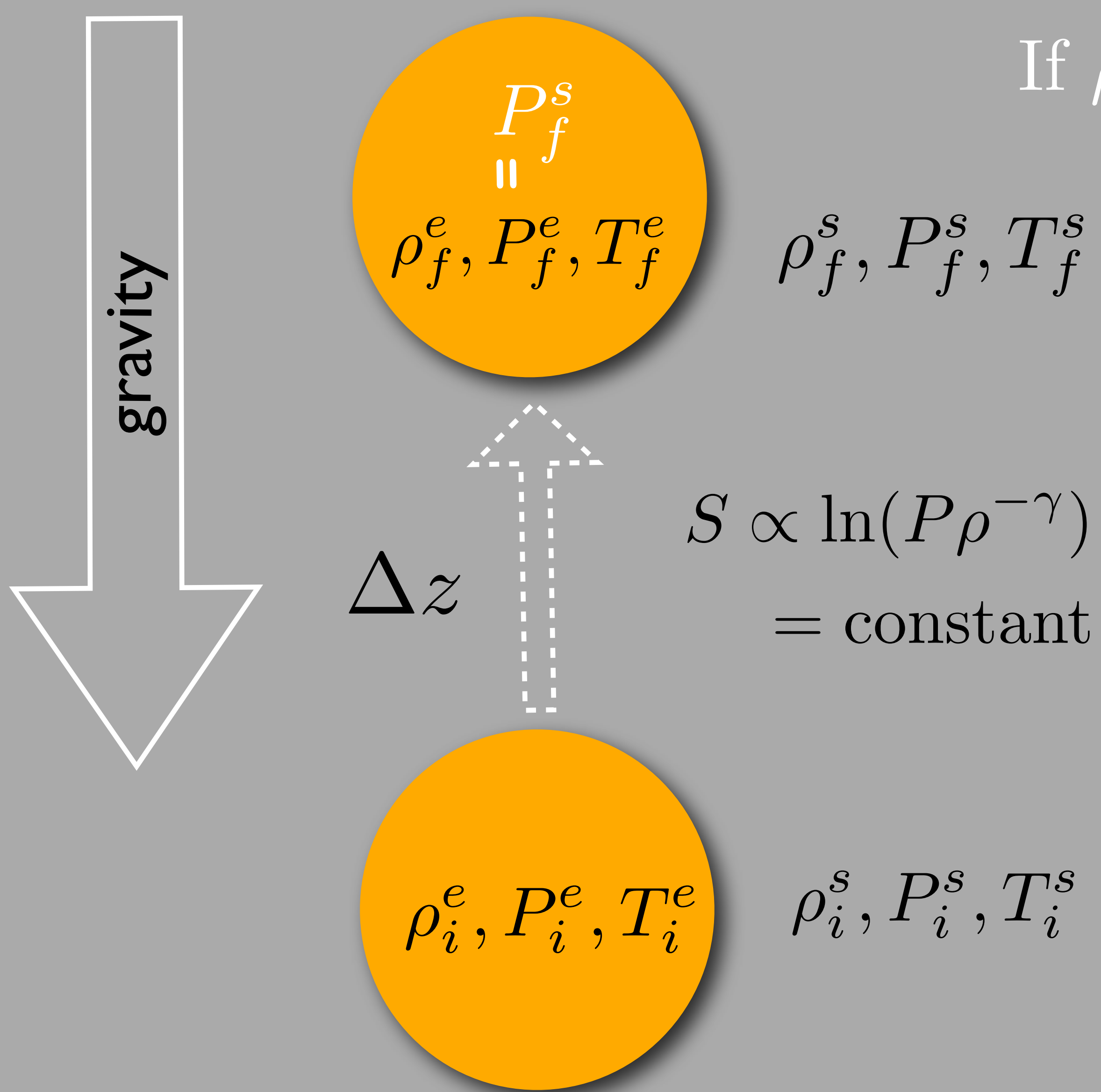
- Low density ~1-$10^{-3}cm^{-3}$
- Hot gas ~10^6 - 10^8 K
- Magnetized ~ few μG
- Size ~ Mpc
- Age ~ Gyr

Cluster gas dynamics & thermodynamics depend of physical processes at small scales

Galaxy Cluster Abell 1689. Credit: X-ray: NASA/CXC/MIT/E.-H Peng et al; Optical: NASA/STScI

Schwarzschild Criterion

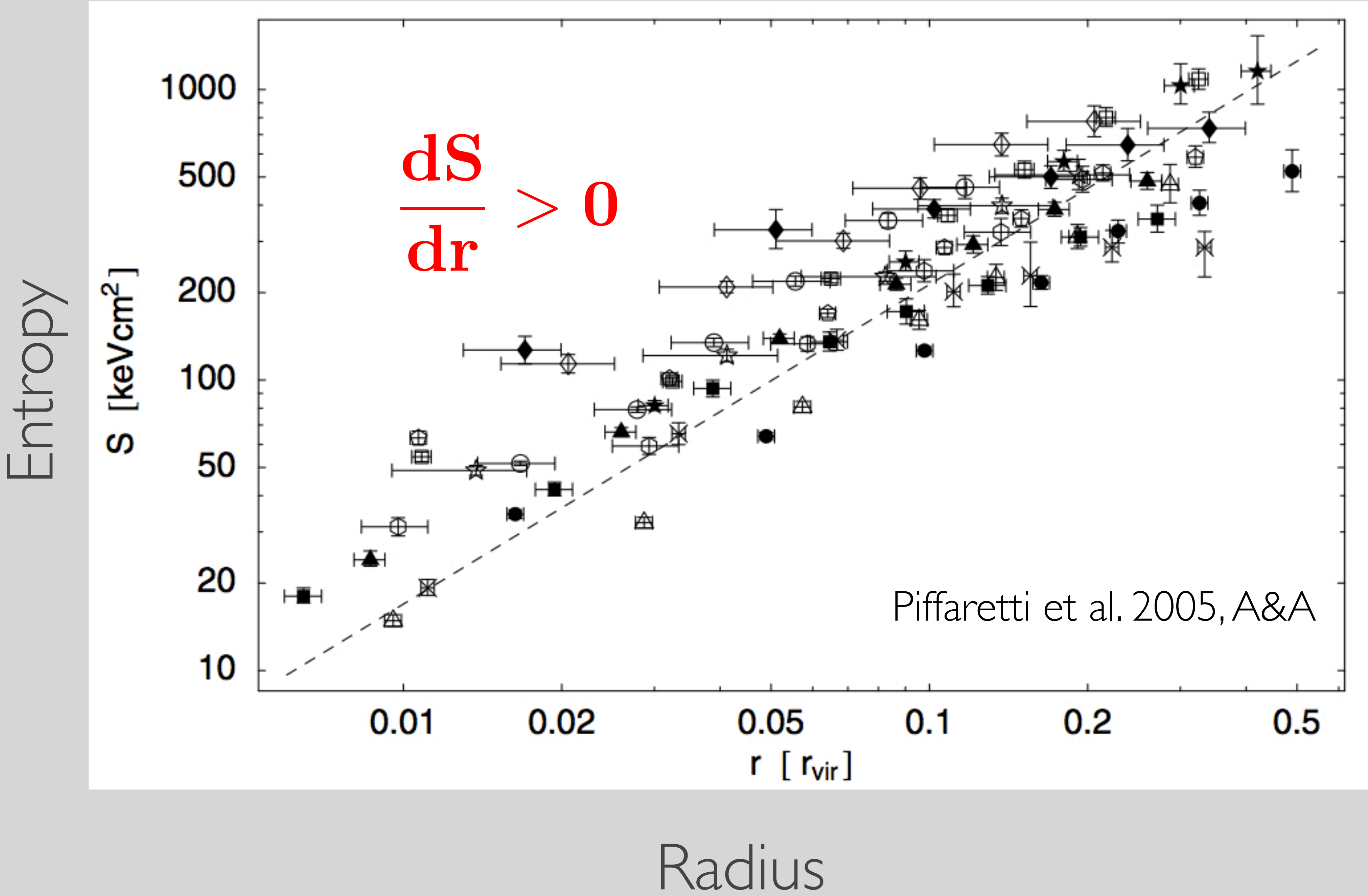
If $\rho_f^e - \rho_f^s > 0 \Rightarrow$ Blob sinks: Stability



Schwarzschild Criterion
Convective Stability

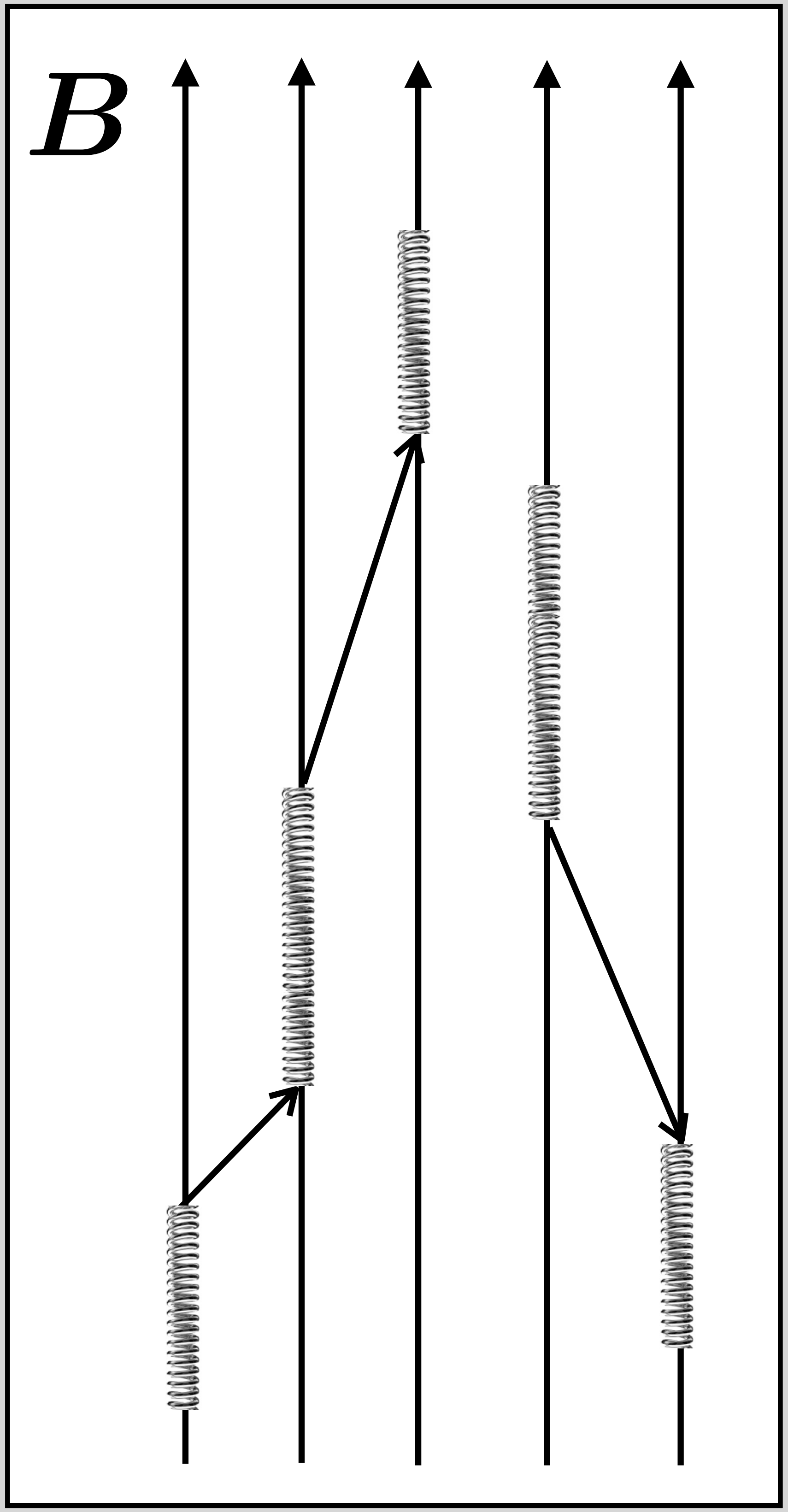
$$\frac{\mathbf{dS}}{\mathbf{dz}} > \mathbf{0}$$

Entropy Profile in ICM



ICM stable according to Schwarzschild criterion

Anisotropic Transport in ICM



$$\rho_{\rm gyro} \sim 10^4 \rm km \ll \lambda_{\rm mfp} \sim 10\,kpc$$

- Ions and electrons follow field lines

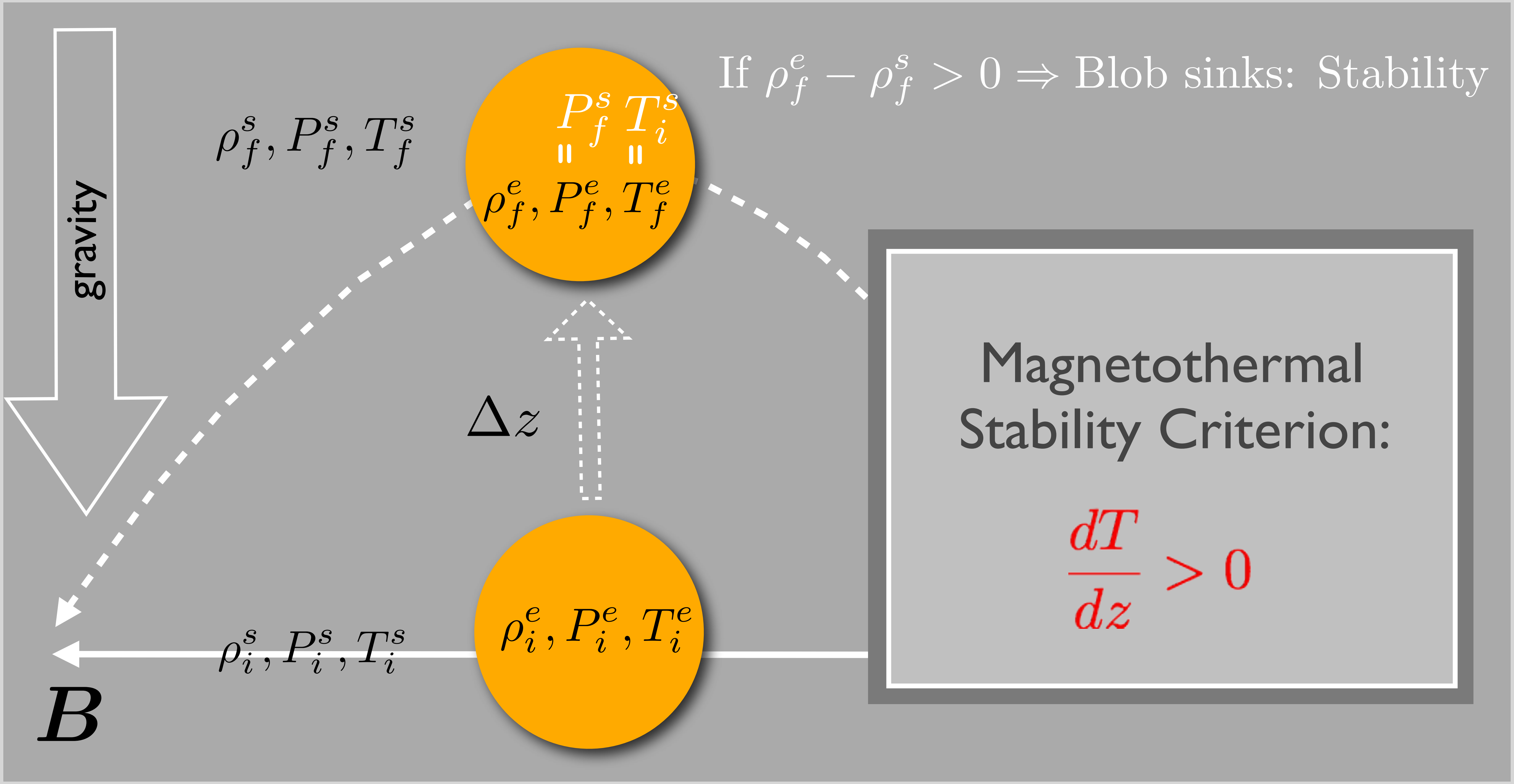
$$\lambda_{\rm mfp} \sim 10\,\rm kpc \ll R \sim Mpc$$

- Fluid description is OK (kinetic-MHD framework)

Anisotropy affects

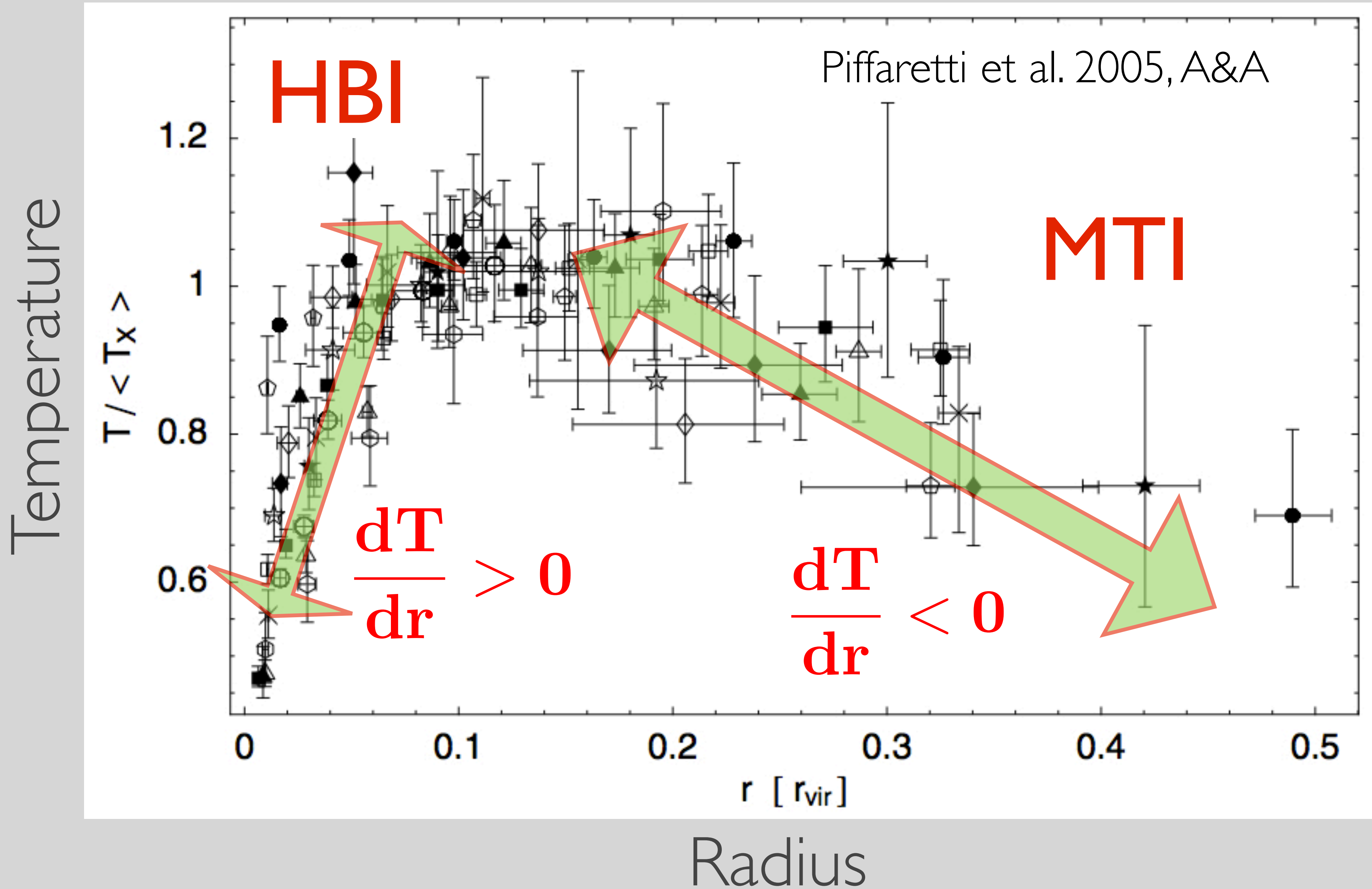
- heat conduction
- momentum transport
- particle diffusion

Heuristic Understanding Of MTI



Similar arguments show HBI stability is $\frac{dT}{dz} < 0$

Temperature Profile in ICM



Balbus 2001
Quataert 2008

ICM is unstable to magneto-thermal (MTI) and heat-flux buoyancy-driven instabilities (HBI)

Composition Gradients in ICM

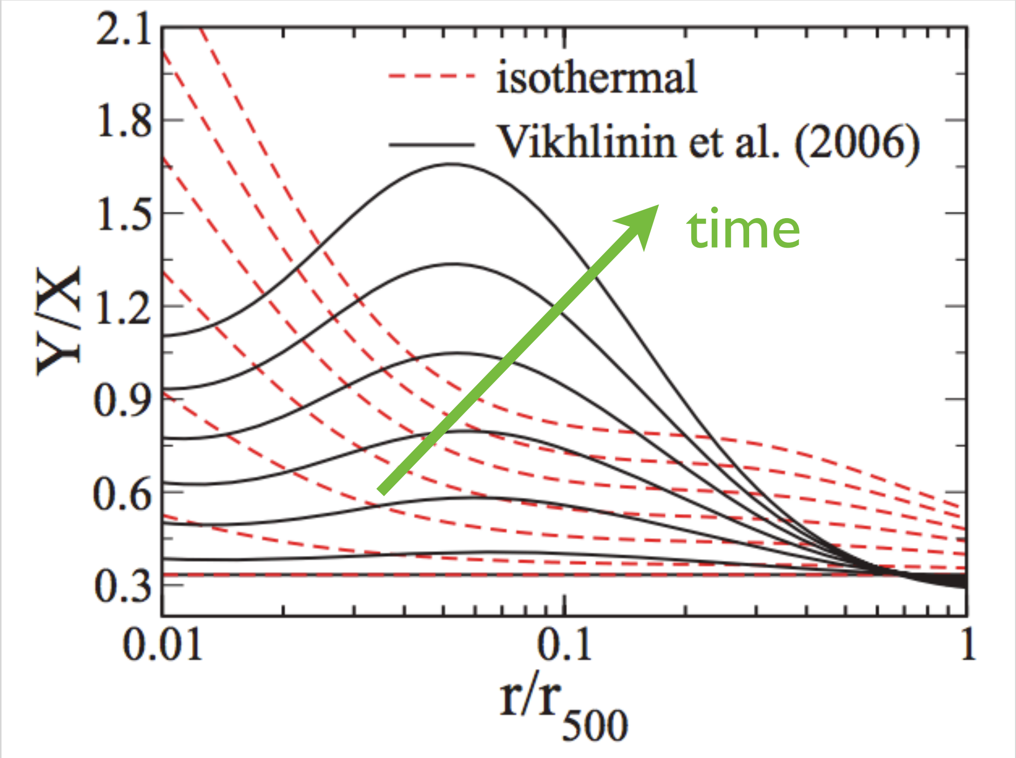
Theoretical Expectations and Observational Issues

- Heavy elements expected to sediment over cluster lifetime
  (Fabian & Pringle 1977, Chuzoy & Nusser 2003, Chuzoy & Loeb 2004, Shtykovskiy & Gilfanov, 2010)

- If He sedimentation is important, it can affect physical and cosmological parameters inferred via X-ray observations
  (Fabian 1994, Qin & Wu 2000, Markevitch 2007, Peng & Nagai 2010, Bulbul et al. 2011)

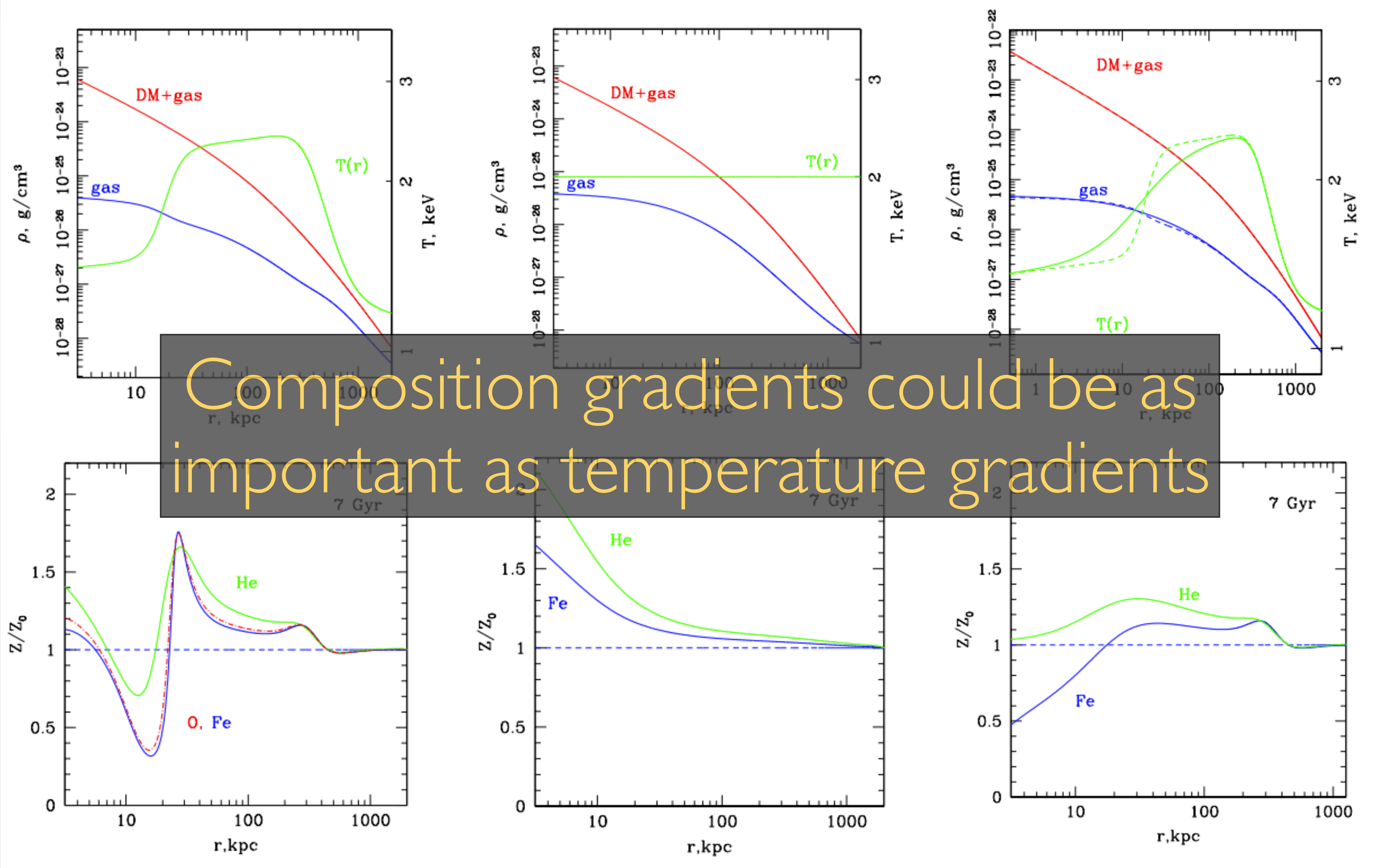
Numerical Modeling

- 1D (spherical symmetry)
- Magnetic fields usually ignored
  (or use simple parametrization to inhibit diffusion)
- Follow evolution for Gyrs (!!!)



Peng & Nagai, 2010

Composition Gradients in ICM



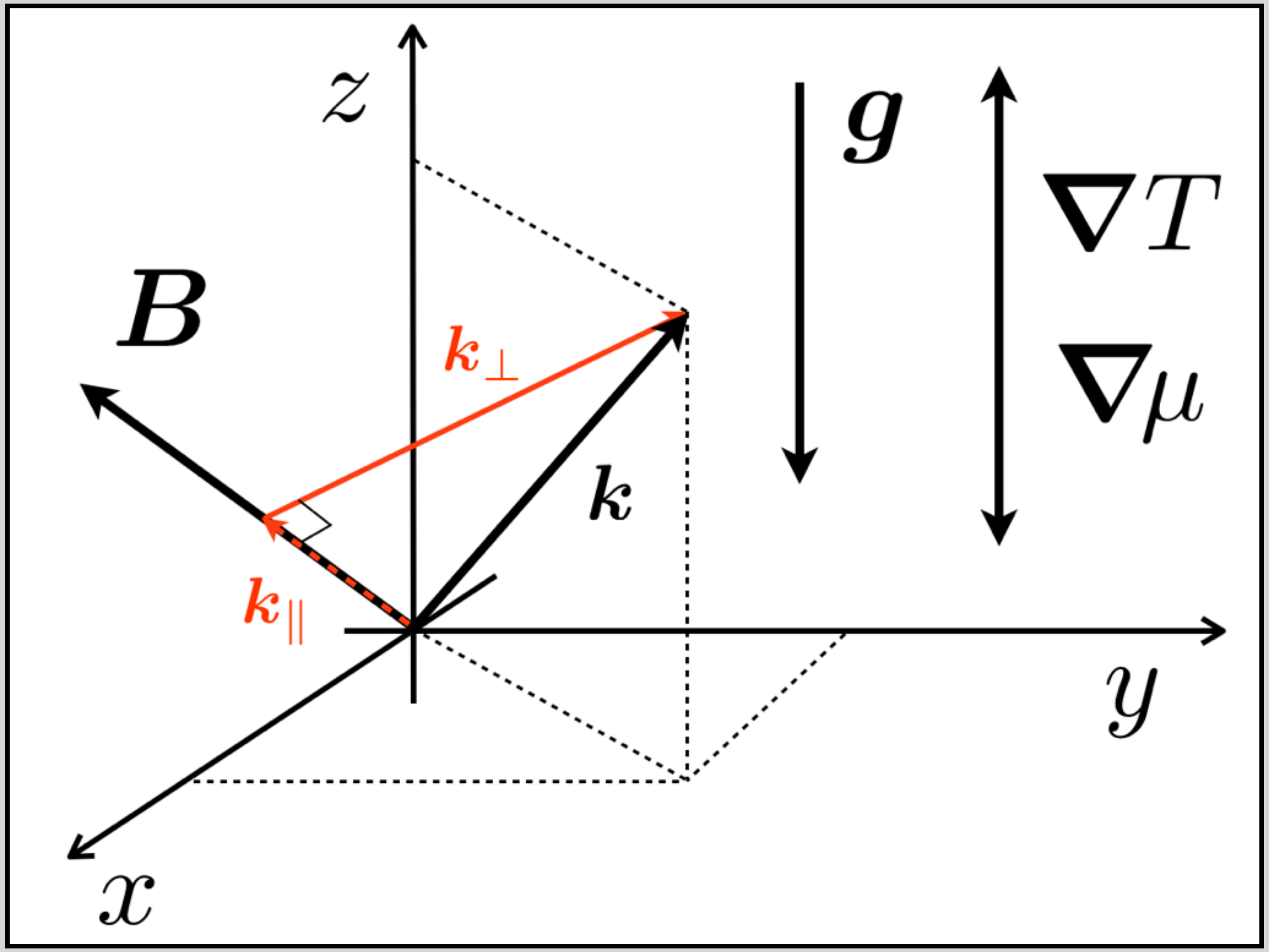
Composition gradients could be as important as temperature gradients

(including thermal diffusion)

Shtykovskiy & Gilfanov, 2010

A "Toy" Model for the ICM

- Dilute, two-component, plane-parallel atmosphere where heat and particle fluxes are anisotropic



$$P = \frac{\rho k_{\mathrm{B}} T}{\mu m_{\mathrm{H}}}$$

$$\frac{1}{\mu} \equiv (1-c)\frac{(1+Z_1)}{\mu_1} + c\frac{(1+Z_2)}{\mu_2}$$

$$\boldsymbol{Q}_{\mathrm{s}} \equiv -\chi \hat{\boldsymbol{b}}(\hat{\boldsymbol{b}}\boldsymbol{\cdot}\boldsymbol{\nabla})T$$

$$\boldsymbol{Q}_{\mathrm{c}} \equiv -D \hat{\boldsymbol{b}}(\hat{\boldsymbol{b}}\boldsymbol{\cdot}\boldsymbol{\nabla})c$$

Anisotropic Transport in ICM

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla}\cdot(\rho \boldsymbol{v}) = 0\,,$$

$$\frac{d\boldsymbol{v}}{dt} = -\frac{1}{\rho}\boldsymbol{\nabla}\cdot\left(\mathsf{P} + \frac{\boldsymbol{B}^2}{8\pi}\mathsf{I} - \frac{B^2}{4\pi}\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}\right) + \boldsymbol{g}\,,$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{B})\,,$$

$$\frac{P}{\gamma - 1}\frac{d}{dt}(\ln P\rho^{-\gamma}) = (p_\perp - p_\parallel)\frac{d}{dt}\ln\frac{B}{\rho^{\gamma-1}} - \boldsymbol{\nabla}\cdot\boldsymbol{Q}_{\rm s}\,,$$

$$\frac{dc}{dt} = -\boldsymbol{\nabla}\cdot\boldsymbol{Q}_{\rm c}\,.$$

$$\mathsf{P} \equiv p_\perp\mathsf{I} + (p_\parallel - p_\perp)\hat{\boldsymbol{b}}\hat{\boldsymbol{b}}$$

Linear Mode Analysis

$$0 = \boldsymbol{k}\cdot\delta\boldsymbol{v}$$

$$\sigma\delta\boldsymbol{v} = -g\frac{\delta\rho}{\rho}\hat{\boldsymbol{z}} - i\boldsymbol{k}v_{\rm th}^2\left(\frac{\delta p_\perp}{P} + \frac{1}{\beta}\frac{\delta B_\|}{B}\right) + ik_\| v_{\rm A}^2\frac{\delta\boldsymbol{B}}{B} - \hat{\boldsymbol{b}}3k_\|^2\nu_\|\delta v_\|$$

$$\sigma\delta\boldsymbol{B} = ik_\| B\delta\boldsymbol{v}$$

$$\sigma\frac{\delta\rho}{\rho} = \frac{N^2}{g}\delta v_z + \frac{\gamma-1}{\gamma}\kappa k_\|^2\frac{\delta T}{T} - i\frac{\gamma-1}{\gamma}\kappa\boldsymbol{k}\cdot\left(\frac{d\ln T}{dz}\delta b_z\hat{\boldsymbol{b}} + b_z\frac{d\ln T}{dz}\frac{\delta\boldsymbol{B}_\perp}{B}\right)$$

$$\sigma\frac{\delta\mu}{\mu} = -\frac{d\ln\mu}{dz}\delta v_z - Dk_\|^2\frac{\delta\mu}{\mu} + iD\boldsymbol{k}\cdot\left(\frac{d\ln\mu}{dz}\delta b_z\hat{\boldsymbol{b}} + b_z\frac{d\ln\mu}{dz}\frac{\delta\boldsymbol{B}_\perp}{B}\right)$$

$$0 = \frac{\delta\rho}{\rho} + \frac{\delta T}{T} - \frac{\delta\mu}{\mu}$$

$$N^2 \equiv \frac{g}{\gamma}\frac{d}{dz}\ln P\rho^{-\gamma} = g\frac{d}{dz}\ln\left(\frac{P^{\frac{1-\gamma}{\gamma}}T}{\mu}\right)$$

Time- & Length-scales Across a Mode

$$\omega_{\rm dyn} \equiv (g/H)^{1/2}$$

$$\tau_{\rm c}^{-1} \equiv (\boldsymbol{k}\cdot\hat{\boldsymbol{b}})^2 \, \kappa \, \frac{(\gamma - 1)}{\gamma}$$

$$\tau_{\rm v}^{-1} \equiv (\boldsymbol{k}\cdot\hat{\boldsymbol{b}})^2 \, 3 \, \nu_{\|}$$

$$\tau_{\rm d}^{-1} \equiv (\boldsymbol{k}\cdot\hat{\boldsymbol{b}})^2 \, D$$

$$\omega_{\rm A} \equiv \boldsymbol{k}\cdot\boldsymbol{v}_{\rm A}$$

plasma physics

$$\tau_{\rm c}^{-1} \simeq 10 k_{\|}^2 \lambda_{\rm mfp} H \, \omega_{\rm dyn}$$

$$\tau_{\rm c}^{-1} \simeq 6 \, \tau_{\rm v}^{-1}$$

Kunz 2011

$$\tau_{\rm v}^{-1} \simeq 9 \, \tau_{\rm d}^{-1}$$

$\omega_{\rm A} \simeq 0$ weak fields!

$$\tau_{\rm c}^{-1} > \tau_{\rm v}^{-1} > \tau_{\rm d}^{-1} \gg \omega_{\rm dyn} \quad \text{if} \quad k_{\|} \gg (\lambda_{\rm mfp} H)^{-1/2}$$ [fast]

$$\omega_{\rm dyn} \gg \tau_{\rm c}^{-1} > \tau_{\rm v}^{-1} > \tau_{\rm d}^{-1} \quad \text{if} \quad k_{\|} \ll (\lambda_{\rm mfp} H)^{-1/2}$$ [slow]

Viscous vs. Diffusion Processes

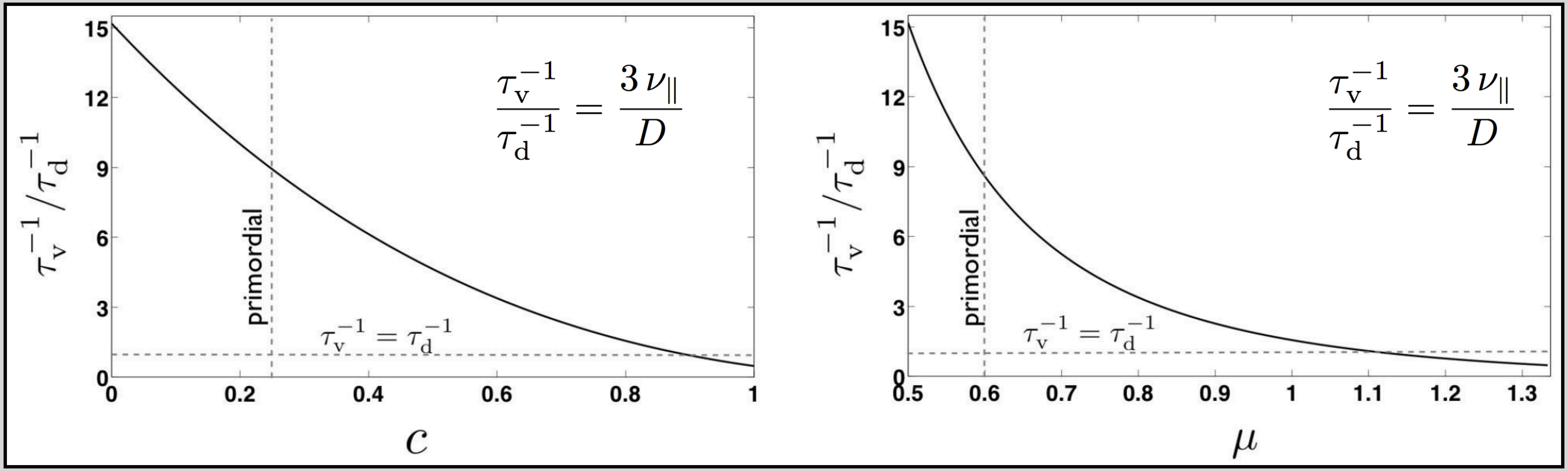
$$p_\parallel - p_\perp = 3\rho\nu_\parallel \left(\hat{\boldsymbol{b}}\hat{\boldsymbol{b}} - \frac{1}{3}\mathsf{I} \right) : \boldsymbol{\nabla v}$$

$$\nu_\parallel = \frac{1}{2}\frac{v_{\rm th}^2}{\nu_{ii}^{\rm eff}}$$

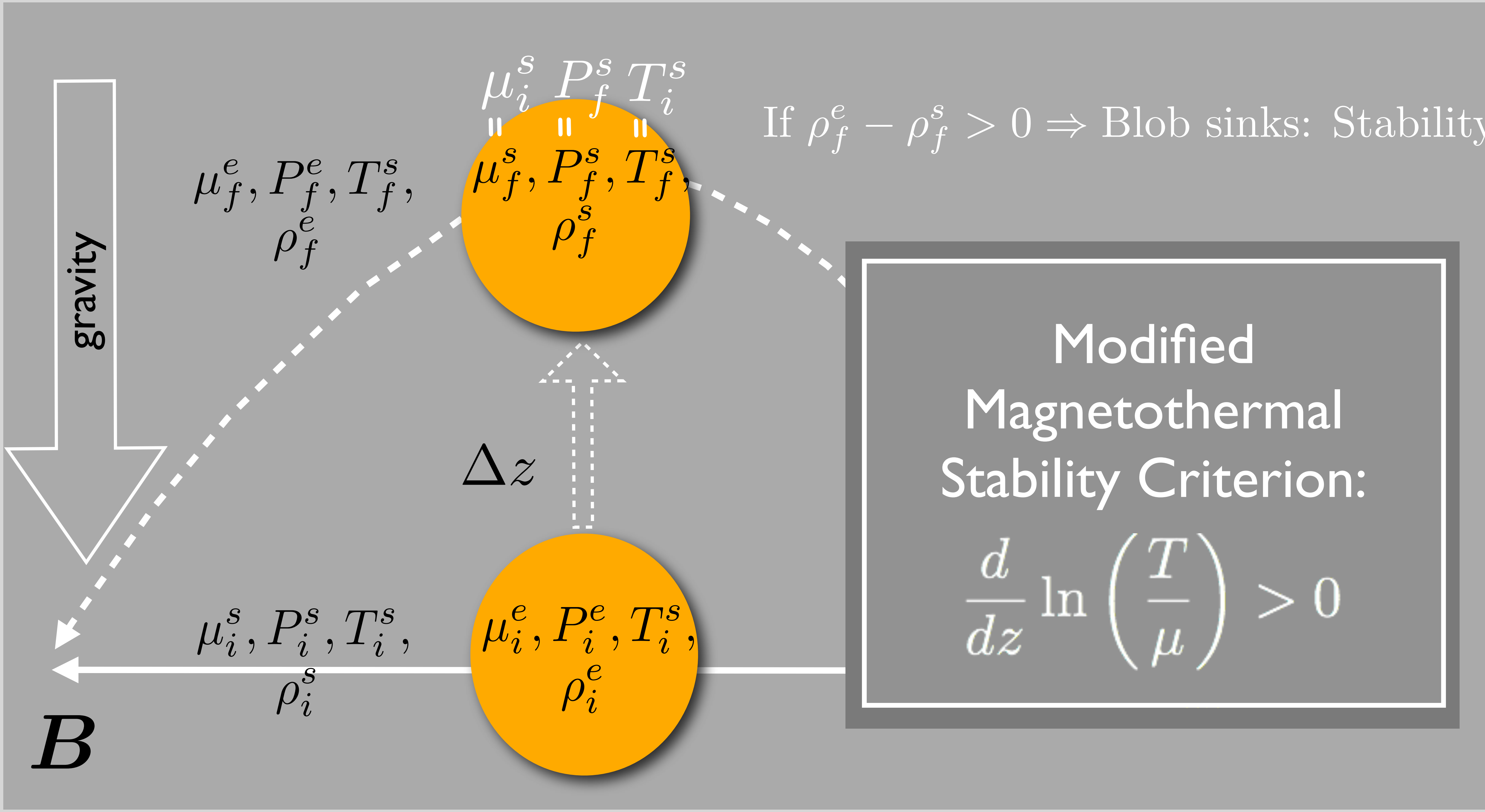
$$\rho\nu_\parallel \simeq \frac{n_{i_1} k_B T}{\nu_{i_1 i_1} + \nu_{i_1 i_2}} + \frac{n_{i_2} k_B T}{\nu_{i_2 i_1} + \nu_{i_2 i_2}}$$

$$\nu_{ij} = \frac{4\sqrt{2\pi}}{3} \left[\frac{\sqrt{m_{ij}} q_i^2 q_j^2 n_j}{m_i (k_B T)^{3/2}} \right] \ln \Lambda_{ij}$$

$$D = \frac{3}{4\sqrt{2\pi}} \frac{m_2 (k_B T)^{5/2}}{\sqrt{m_{12}} q_1^2 q_2^2 \rho \ln \Lambda_{12}} \left[\frac{4 - c}{(2 - c)(8 - 5c)} \right]$$

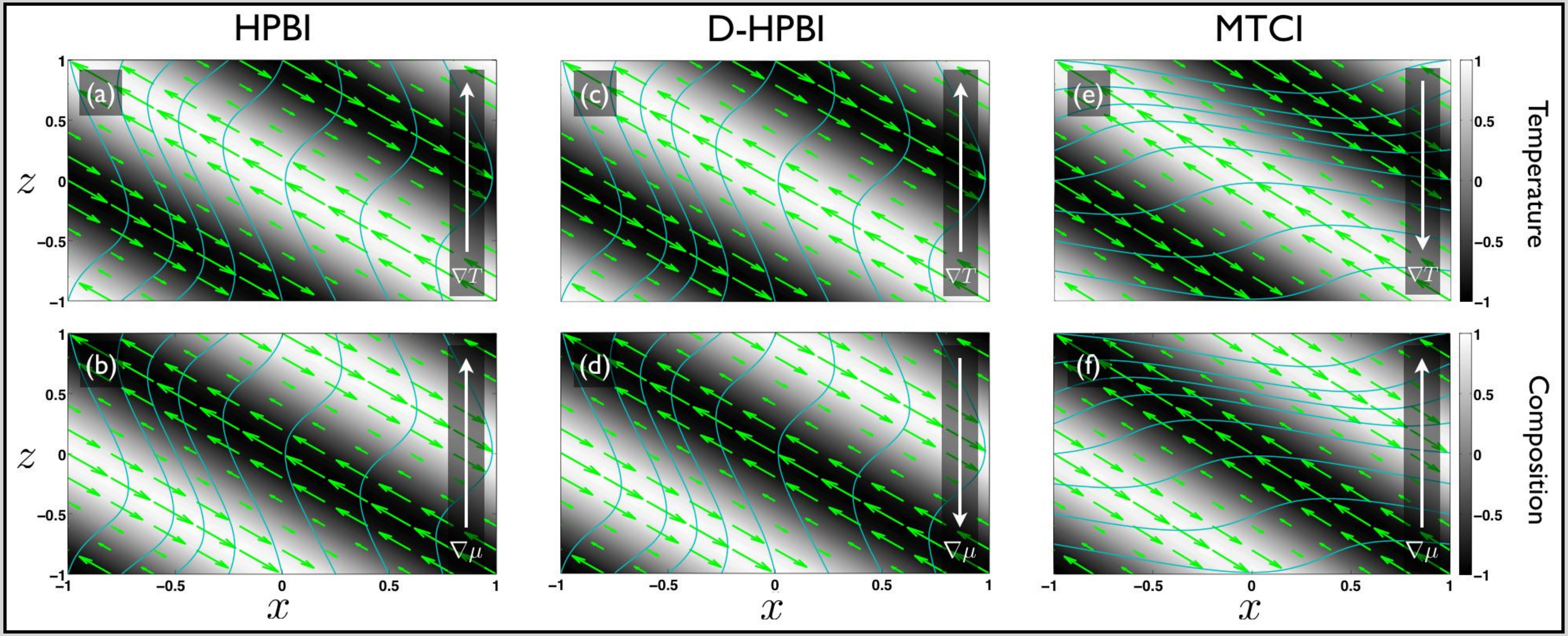


Composition Gradients & MTI



Similar arguments show HBI stability is now $\frac{d}{dz}\ln\left(\frac{T}{\mu}\right) < 0$

Physics Driving Unstable Modes



$$\frac{d\ln T}{dz} > -\frac{d\ln\mu}{dz} \quad \text{HPBI}$$

$$\frac{d\ln T}{dz} > \frac{d\ln\mu}{dz} \quad \text{D-HPBI}$$

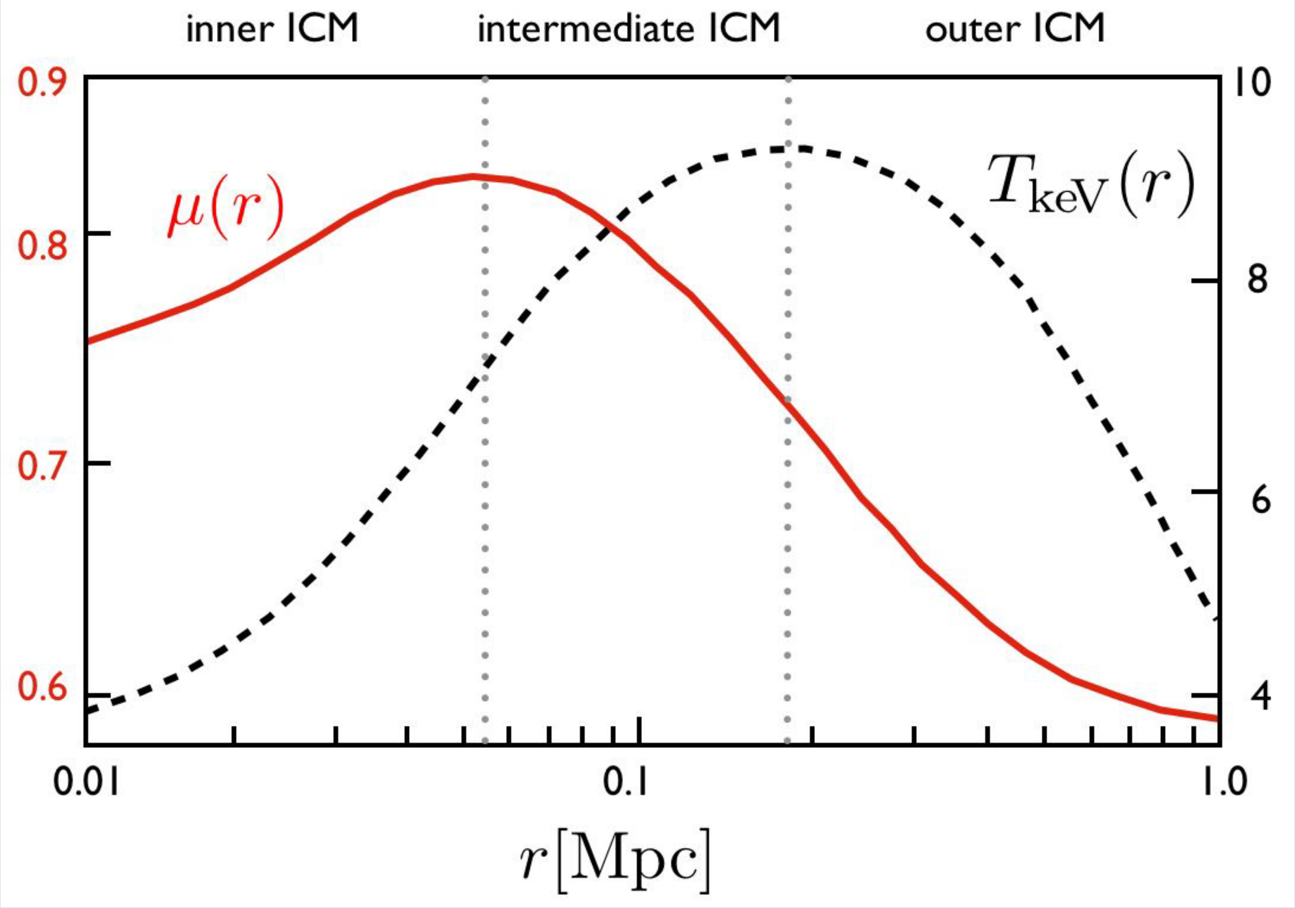
$$\frac{d\ln T}{dz} < \frac{d\ln\mu}{dz} \quad \text{MTCI}$$

$$\sigma^2 \approx g\frac{d\ln(T\mu)}{dz}\frac{k_\perp^2}{k^2}$$

$$\sigma \approx \frac{g}{\tau_{\rm v}^{-1}}\frac{d\ln(T/\mu)}{dz}$$

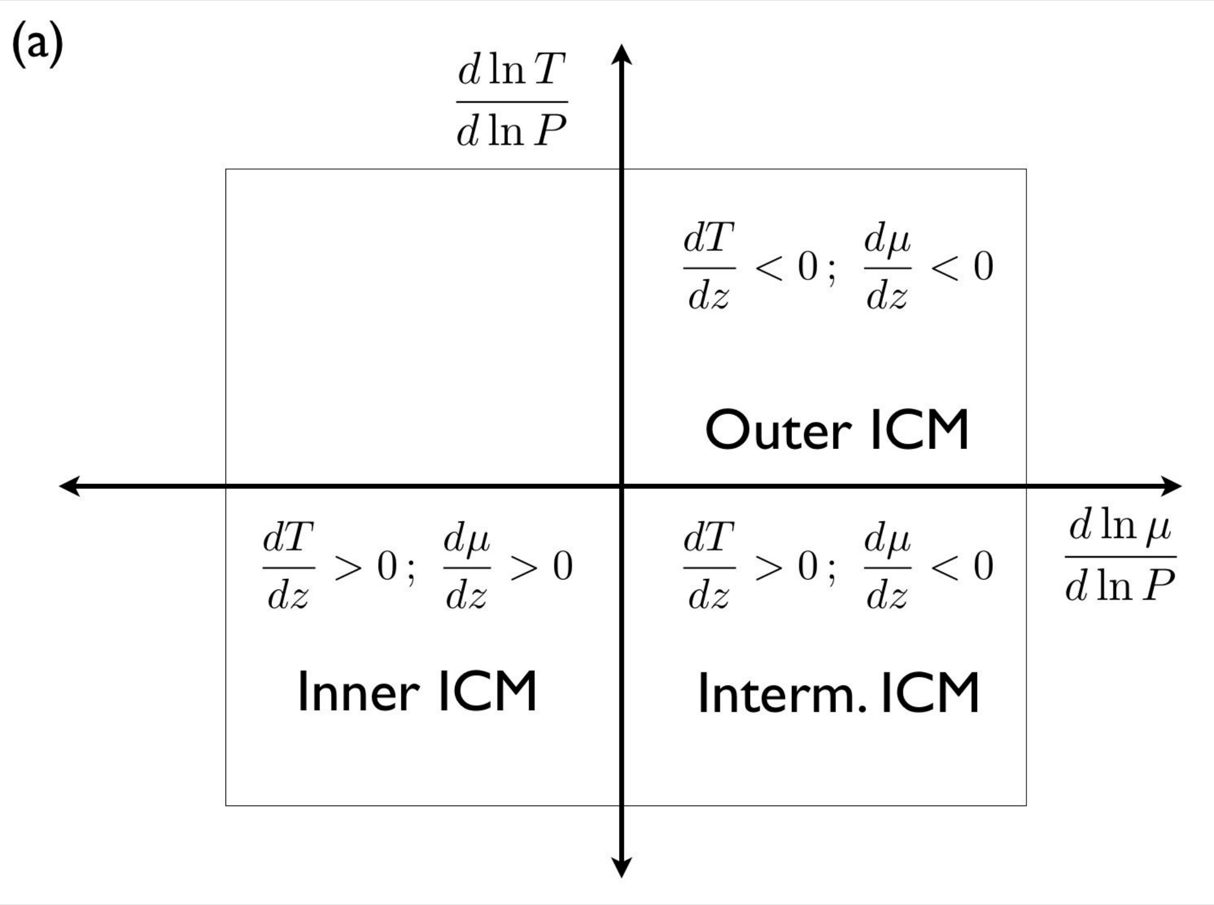
$$\sigma^2 \approx -g\frac{d\ln(T/\mu)}{dz}\frac{k_x^2+k_y^2}{k^2}$$

Astrophysical Context



Peng & Nagai, 2009, ApJ
Bulbul et al. 2011, A&A

Vikhlinin et al. 2006, ApJ

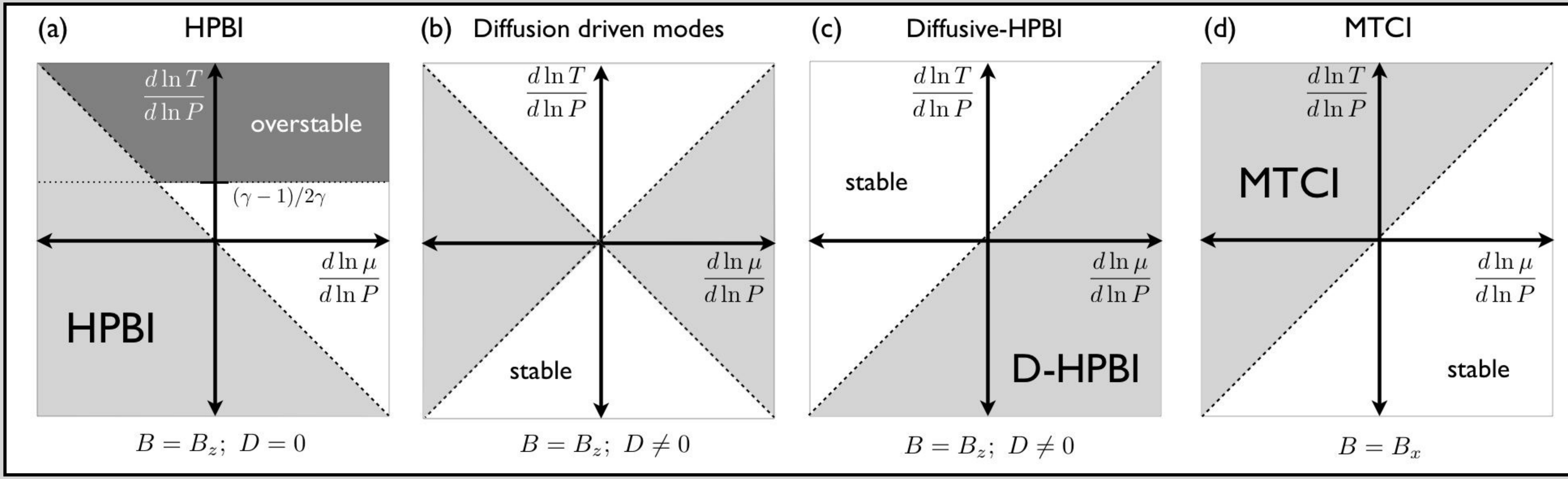


$$\frac{-d}{d\ln P} = \frac{1}{\omega^2_{\rm dyn}}\left(g\frac{d}{dz}\right)$$

ICM stability is subtle, with interesting interplay between thermal and composition structure.

Fast Conduction Limit

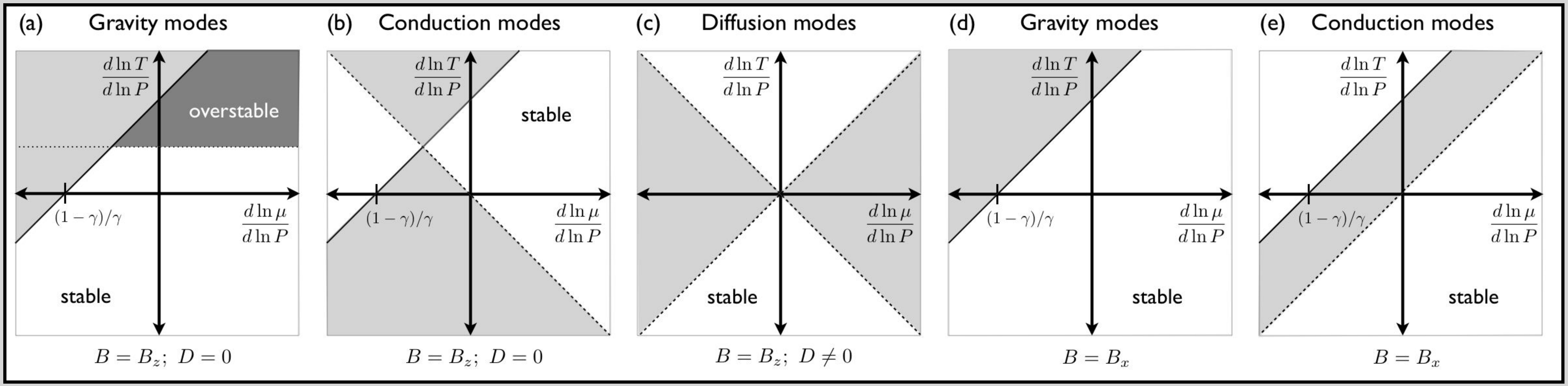
$$\tau_c^{-1} > \tau_v^{-1} > \tau_d^{-1} \gg \omega_{\rm dyn} \quad \text{if} \quad k_{\|} \gg (\lambda_{\rm mfp} H)^{-1/2}$$



- Short || wave-lengths; HBI and MTI modified by mu-gradient
- Overstable gravity-modes, cannot be stabilized by mu-gradient
- Ion-diffusion can drive unstable modes where stable to HPBI

Slow Conduction Limit

$$\omega_{\rm dyn} \gg \tau_{\rm c}^{-1} > \tau_{\rm v}^{-1} > \tau_{\rm d}^{-1} \quad \text{if} \quad k_{||} \ll (\lambda_{\rm mfp} H)^{-1/2}$$



- Long || wave-lengths; HBI and MTI are absent
- Gravity modes can be either stable or over-stable
- Heat conduction and ion-diffusion can drive instabilities

Applicability to ICM

ICM Region	∇T	$\nabla \mu$	β	K_n^{-1}
Outer-ICM	< 0	< 0	10^4	10^1
Interm.-ICM	> 0	< 0	10^3	10^2
Inner-ICM	> 0	> 0	10^2	10^3

$$\beta \equiv \frac{v_{\rm th}^2}{v_{\rm A}^2}$$

$$K_{\rm n} \equiv \frac{\lambda_{\rm mfp}}{H}$$

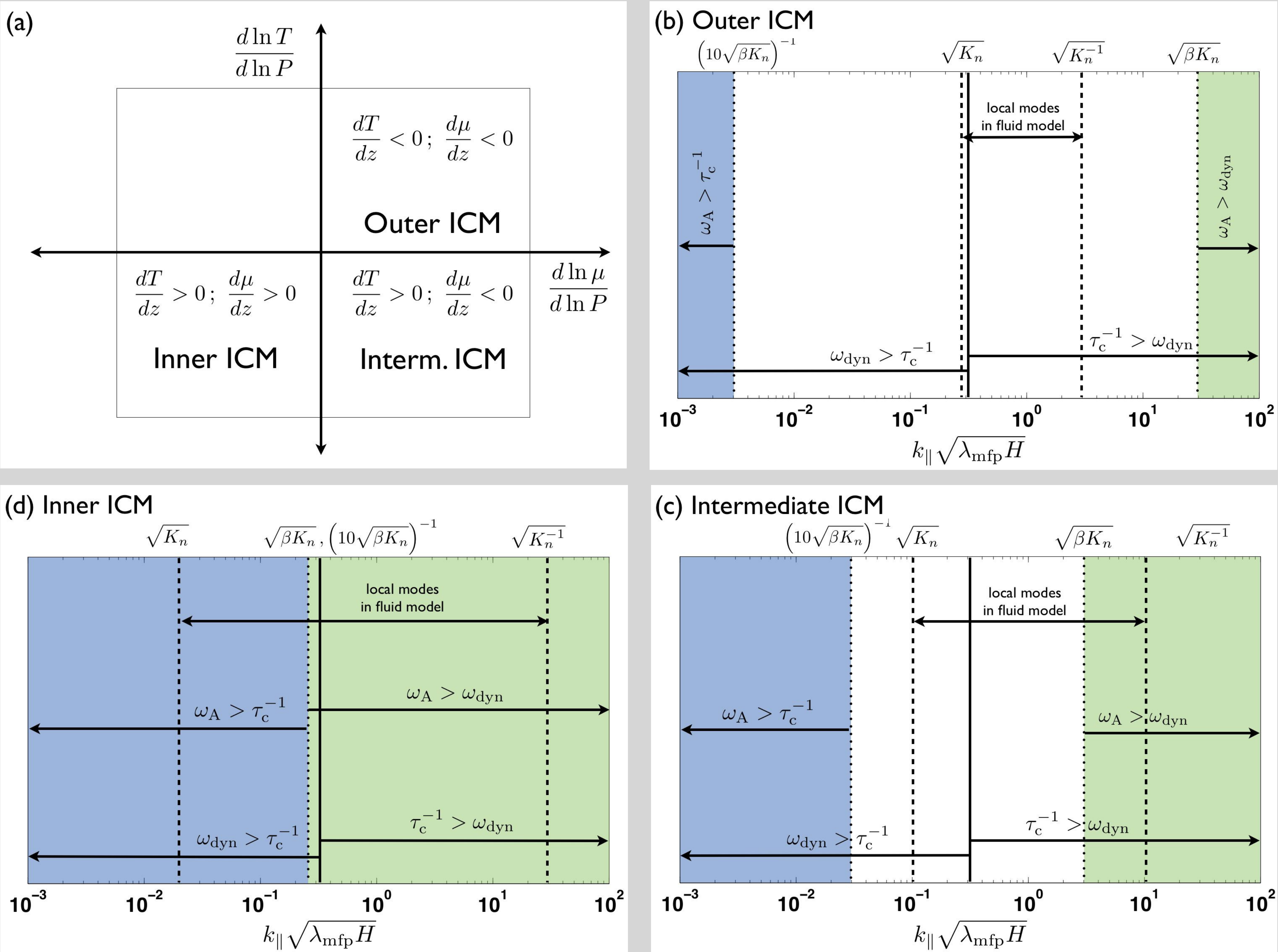
When is the local, fluid model a good approximation?

$$H^{-1} < k_{\|} < \lambda_{\rm mfp}^{-1} \longrightarrow \sqrt{K_{\rm n}} < k_{\|}\sqrt{\lambda_{\rm mfp} H} < \sqrt{K_{\rm n}^{-1}}$$

When is magnetic tension unimportant?

$$k_{\|}(\lambda_{\rm mfp} H)^{1/2} \gg 1\,;\;\; \omega_{\rm A} \ll \omega_{\rm dyn} \ll \tau_{\rm c}^{-1} \longrightarrow 1 \ll k_{\|}\sqrt{\lambda_{\rm mfp} H} \ll \sqrt{\beta K_n} \quad \text{[fast]}$$

$$k_{\|}(\lambda_{\rm mfp} H)^{1/2} \ll 1\,;\;\; \omega_{\rm A} \ll \tau_{\rm c}^{-1} \ll \omega_{\rm dyn} \longrightarrow \frac{1}{10}\frac{1}{\sqrt{\beta K_n}} \ll k_{\|}\sqrt{\lambda_{\rm mfp} H} \ll 1 \quad \text{[slow]}$$



(a)
$\frac{d\ln T}{d\ln P}$
$\frac{dT}{dz} < 0\,;\ \frac{d\mu}{dz} < 0$
Outer ICM
$\frac{dT}{dz} > 0\,;\ \frac{d\mu}{dz} > 0$
$\frac{dT}{dz} > 0\,;\ \frac{d\mu}{dz} < 0$
$\frac{d\ln\mu}{d\ln P}$
Inner ICM
Interm. ICM
(b) Outer ICM
$\left(10\sqrt{\beta K_n}\right)^{-1}$
$\sqrt{K_n}$
$\sqrt{K_n^{-1}}$
$\sqrt{\beta K_n}$
local modes
in fluid model
$\omega_{\rm A} > \tau_{\rm c}^{-1}$
$\omega_{\rm A} > \omega_{\rm dyn}$
$\tau_{\rm c}^{-1} > \omega_{\rm dyn}$
$\omega_{\rm dyn} > \tau_{\rm c}^{-1}$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
$k_\parallel\sqrt{\lambda_{\rm mfp}H}$
(d) Inner ICM
$\sqrt{K_n}$
$\sqrt{\beta K_n}\,,\left(10\sqrt{\beta K_n}\right)^{-1}$
$\sqrt{K_n^{-1}}$
local modes
in fluid model
$\omega_{\rm A} > \omega_{\rm dyn}$
$\omega_{\rm A} > \tau_{\rm c}^{-1}$
$\tau_{\rm c}^{-1} > \omega_{\rm dyn}$
$\omega_{\rm dyn} > \tau_{\rm c}^{-1}$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
$k_\parallel\sqrt{\lambda_{\rm mfp}H}$
(c) Intermediate ICM
$\left(10\sqrt{\beta K_n}\right)^{-1}$
$\sqrt{K_n}$
$\sqrt{\beta K_n}$
$\sqrt{K_n^{-1}}$
local modes
in fluid model
$\omega_{\rm A} > \tau_{\rm c}^{-1}$
$\omega_{\rm A} > \omega_{\rm dyn}$
$\tau_{\rm c}^{-1} > \omega_{\rm dyn}$
$\omega_{\rm dyn} > \tau_{\rm c}^{-1}$
10^{-3} 10^{-2} 10^{-1} 10^{0} 10^{1} 10^{2}
$k_\parallel\sqrt{\lambda_{\rm mfp}H}$

Summary & Prospects

- Composition gradients can be important in cluster stability
- HBI & MTI criteria modified wrt homogeneous medium
- Coupling of "slow" diffusion processes to "fast" instabilities
- Long way to address element sedimentation self-consistently
- Improved models for magnetic field effects on element diffusion