Effective Closure Schemes for ICM Dynamics

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Schekochihin *et al.*, *ApJ* **629**, 139 (2005)
Schekochihin & Cowley, *Phys. Plasmas* **13**, 056501 (2006)
Schekochihin *et al.*, *PRL* **100**, 081301 (2008)
Schekochihin *et al.*, *MNRAS* **405**, 291 (2010)
Rosin *et al.*, *MNRAS* **413**, 7 (2011)
Kunz *et al.*, *MNRAS* **410**, 2446 (2011)
Rincon *et al.*, in preparation (2012)

$$\mu = \frac{mv_\perp^2}{2B}$$

[Kulsrud (1997)]

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Equations

To describe the dynamics of the ICM, we write evolution equations for moments of the particle distribution and magnetic fields:

$$\frac{dn}{dt} = -n\nabla \cdot \mathbf{u} \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$$

$$m_i n_i \frac{d\mathbf{u}}{dt} = -\nabla \cdot \left(\mathsf{P} + \mathsf{I}\frac{B^2}{8\pi} - \frac{\mathbf{BB}}{4\pi} \right)$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E} = \nabla \times (\mathbf{u} \times \mathbf{B}) + [\mathrm{FLR}]$$

[AAS *et al.*, *MNRAS* **405**, 291 (2010)]

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$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} + [\mathrm{FLR}]$$

[AAS *et al.*, *MNRAS* **405**, 291 (2010)]

Pressure Tensor

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$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} + [\mathrm{FLR}]$$

$$\mathsf{P} = \sum_s \mathsf{P}_s = \sum_s m_s \int d^3\mathbf{w}\,\mathbf{ww} f_s(t,\mathbf{r},\mathbf{w}) \qquad \mathbf{w} = \mathbf{v} - \mathbf{u}$$

$$= p_\perp(\mathsf{I} - \mathbf{bb}) + p_{||}\mathbf{bb} + [\mathrm{FLR}]$$

[AAS *et al.*, *MNRAS* **405**, 291 (2010)]

CGL

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$$m_i n_i \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\mathbf{bb}\left(p_\perp - p_\parallel + \frac{B^2}{4\pi}\right) + [\mathrm{FLR}]\right]$$

$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u} - \mathbf{B}\nabla\cdot\mathbf{u} + [\mathrm{FLR}]$$

$$p_\perp \frac{d}{dt}\ln\frac{p_\perp}{nB} = -\nu(p_\perp - p_\parallel) + [\text{heat fluxes}]$$

conservation of $\mu = \dfrac{mv_\perp^2}{2B}$

$$p_\parallel \frac{d}{dt}\ln\frac{p_\parallel B^2}{n^2} = -2\nu(p_\parallel - p_\perp) + [\text{heat fluxes}]$$

Typically $p_\perp - p_\parallel \ll p$ (tbc)
Also ignore $d\ln n/dt$ for simplicity (wrong, tech. speaking)

[AAS *et al.*, *MNRAS* **405**, 291 (2010)]

Heating and Pressure Anistropy

To describe the dynamics of the ICM, we write evolution equations for moments of the particle distribution and magnetic fields:

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$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u}$$

$$\frac{3}{2}n\frac{dT}{dt} = \underbrace{-\nabla\cdot\mathbf{q}}_{\text{heat flux}} + \underbrace{(p_\perp - p_\|)\frac{d\ln B}{dt}}_{\text{viscous heating}} - \underbrace{n_i n_e \Lambda}_{\text{cooling}} \qquad p = nT$$

$$\frac{d(p_\perp - p_\|)}{dt} = 3p\frac{d\ln B}{dt} - 3\nu(p_\perp - p_\|) + [\text{heat fluxes}]$$

Typically $p_\perp - p_\| \ll p$ (tbc)
Also ignore $d\ln n/dt$ for simplicity (wrong, tech. speaking)

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heat flux — viscous heating — cooling

$$\Delta = \frac{p_\perp - p_\parallel}{p} \approx \frac{1}{\nu} \left(\frac{d \ln B}{dt} + \dots \right)$$

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heat flux   viscous heating

Braginskii viscosity

$$\Delta = \frac{p_\perp - p_\parallel}{p} \approx \frac{1}{\nu}\left(\frac{d\ln B}{dt} + \dots\right) = \frac{\mathbf{bb}:\nabla\mathbf{u}}{\nu}$$

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Instabilities

$$m_i n_i \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\mathbf{bb}\left(p\Delta + \frac{B^2}{4\pi}\right) + [\mathrm{FLR}]\right]$$

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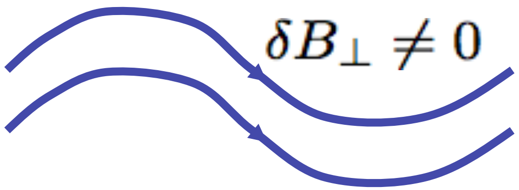
These equations are unstable to fast, short-scale perturbations:

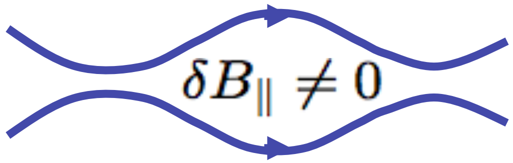
$\Delta < -\dfrac{2}{\beta}$ FIREHOSE



destabilised Alfvén wave

$\Delta \gtrsim \dfrac{1}{\beta}$ MIRROR

(kinetic! but a wimpy fluid version exists)



resonant instability

Marginality

For typical **cluster** parameters:

$$\Delta \sim \frac{1}{\beta} \sim 0.01$$

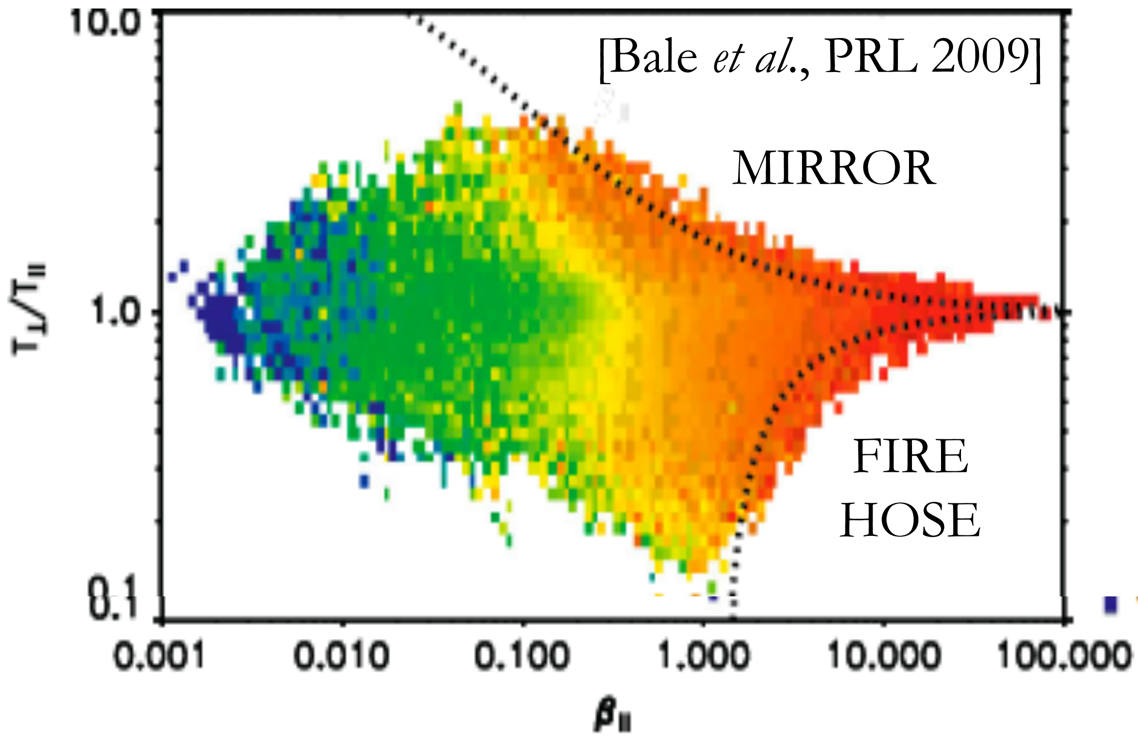
We know large scales (local equlibrium) will be marginal, but how to model this depends on how marginality is achieved

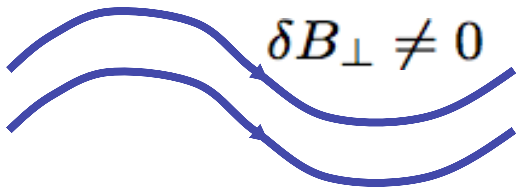
$$\Delta < -\frac{2}{\beta}$$ FIREHOSE

$$\Delta \gtrsim \frac{1}{\beta}$$ MIRROR

(kinetic! but a wimpy fluid version exists)

In the **solar wind**:





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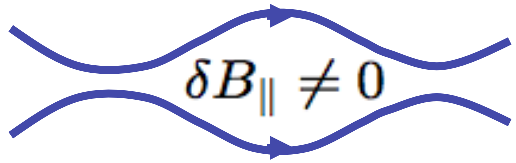

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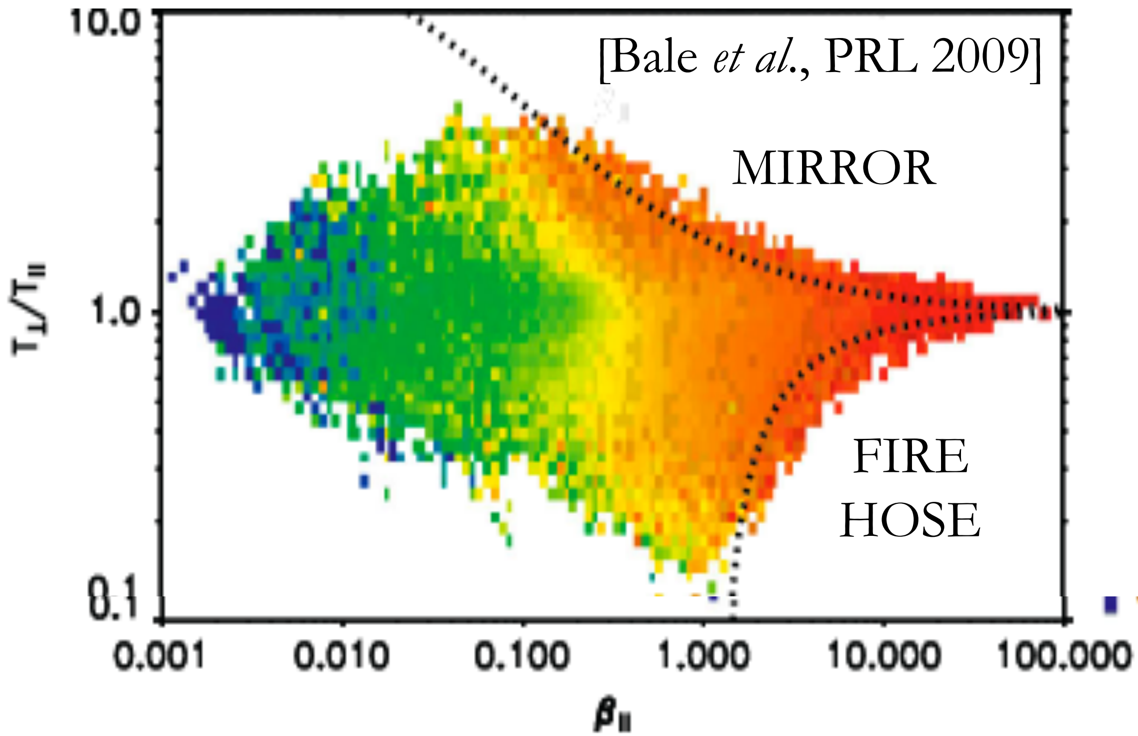

NB: The reason you care about this is that you can't move anything in the ICM without stretching/ (de)compressing the m. field. This has to be consistent with the conservation of $\mu = mv_\perp^2/2B$ *for each particle (so even weak* B *matters!) Pressure anisotropies and the resulting instabilities are the ICM's way of satisfying this constraint.*

Closure

$$m_i n_i \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\mathbf{bb}\left(p\Delta + \frac{B^2}{4\pi}\right) + [\mathrm{FLR}]\right]$$

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heat flux — viscous heating

Braginskii viscosity

Closure must enforce:

$$\Delta = \frac{p_\perp - p_\parallel}{p} \approx \frac{1}{\nu}\left(\frac{d\ln B}{dt} + \dots\right) = \frac{\mathbf{bb}:\nabla\mathbf{u}}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

- Enhanced particle scattering isotropises pressure

→ Closure provides more collisions: $\nu \to \nu_{\mathrm{eff}} \gg \nu$ [Sharma et al. 2006; AAS & Cowley 2006]

- Magnetic field structure and evolution modified to offset change

→ Closure limits rate of strain:

$$\frac{d\ln B}{dt} = \mathbf{bb}:\nabla\mathbf{u} \in \frac{\nu}{\beta}[-2, 1]$$

[Kunz et al. 2011]

Sharma et al. Closure

$$m_i n_i \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\mathbf{bb}\left(p\Delta + \frac{B^2}{4\pi}\right) + [\mathrm{FLR}]\right]$$

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Implementation: $\nu_{\rm eff} = \nu_{\rm Coulomb}$ if $\Delta \in [-2, 1]/\beta$

(in CGL eqns) $\nu_{\rm eff} = \boldsymbol{\nu} \gg \nu_{\rm Coulomb}$ if $\Delta \notin [-2, 1]/\beta$

So Δ does not limit rate of change of magnetic field;

(parallel) viscosity $p/\nu_{\rm eff}$ **drops** to naught

($\Rightarrow$smaller scales, larger rates of strain develop, $\mathrm{Re}_{\rm eff} \to \infty$)

Kunz et al. Closure

$$m_i n_i \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\mathbf{bb}\left(p\Delta + \frac{B^2}{4\pi}\right) + [\mathrm{FLR}]\right]$$

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Implementation: if $\mathbf{bb}:\nabla\mathbf{u} \notin [-2,1](\nu/\beta)$ add large effective viscosity in momentum eqn to damp excess rate of strain

(cf. LES models/Syrovatskii)

So rate of change of magnetic field stays $\dfrac{d\ln B}{dt} = \nu\Delta \in \dfrac{\nu}{\beta}[-2,1]$

viscosity of supercritical motions **increases**

(motions stay large-scale, $\mathrm{Re}_{\mathrm{eff}} \to 0$)

Implications for Heating

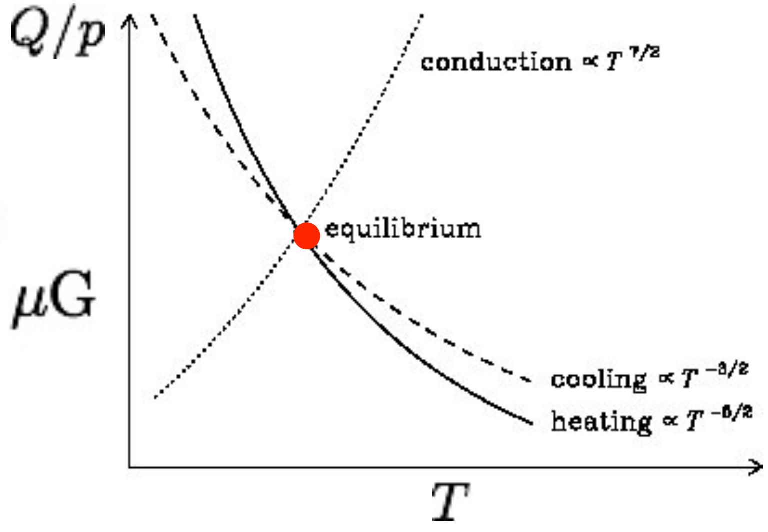
Kunz et al. closure

$$Q^+ = p\Delta \frac{d\ln B}{dt} = p\nu\Delta^2 \sim p\frac{\nu}{\beta^2}$$

$$\sim 10^{-25} \left(\frac{B}{10\,\mu\mathrm{G}}\right)^4 \left(\frac{T}{2\,\mathrm{keV}}\right)^{-5/2} \frac{\mathrm{erg}}{\mathrm{s\,cm}^3}$$ viscous heating

$$Q^- \sim 10^{-25} \left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^2 \left(\frac{T}{2\,\mathrm{keV}}\right)^{1/2} \frac{\mathrm{erg}}{\mathrm{s\,cm}^3}$$ Brehmsstrahlung cooling

- Thermally stable ICM



- If $Q^+ \sim Q^-$,

$$B \sim 10 \left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{T}{2\,\mathrm{keV}}\right)^{3/4} \mu\mathrm{G}$$

- If $m_i n_i u^2/2 \sim B^2/8\pi$,

$$u \sim 10^2 \left(\frac{T}{2\,\mathrm{keV}}\right)^{3/4} \frac{\mathrm{km}}{\mathrm{s}}$$

[Kunz *et al.*, *MNRAS* **410**, 2446 (2011)]

Implications for Heating

Sharma et al. closure

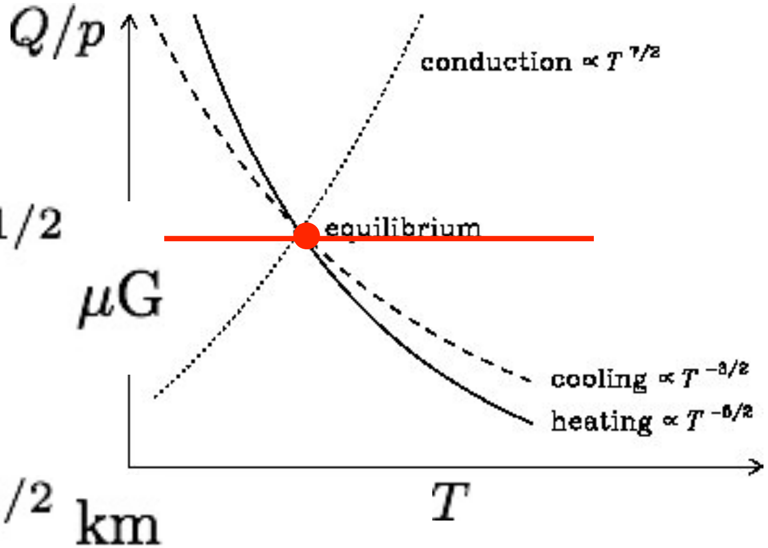
$$Q^+ = p\Delta \frac{d\ln B}{dt} = p\nu_{\rm eff}\Delta^2 \sim p\frac{\Omega_i}{\beta^4} \qquad \nu_{\rm eff} \sim \gamma_{\rm mirror} \sim \Omega_i \Delta^2$$

$$\sim 10^{58}\left(\frac{B}{10\,\mu\mathrm{G}}\right)^9 \left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^{-4} \left(\frac{T}{2\,\mathrm{keV}}\right)^{-4} \frac{\mathrm{erg}}{\mathrm{s\,cm}^3}$$

$$Q^- \sim 10^{-25}\left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^2 \left(\frac{T}{2\,\mathrm{keV}}\right)^{1/2} \frac{\mathrm{erg}}{\mathrm{s\,cm}^3}$$

Brehmsstrahlung cooling

- Thermally **unstable**



- If $Q^+ \sim Q^-$,

$$B \sim 10^{-8}\left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^{2/3} \left(\frac{T}{2\,\mathrm{keV}}\right)^{1/2} \mu\mathrm{G}$$

- If $m_i n_i u^2/2 \sim B^2/8\pi$,

$$u \sim 10^{-2}\left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^{1/6} \left(\frac{T}{2\,\mathrm{keV}}\right)^{1/2} \frac{\mathrm{km}}{\mathrm{s}}$$

Implications for Heating

Sharma et al. closure

$$Q^+ = p\Delta \frac{d\ln B}{dt} = p\nu_{\rm eff}\Delta^2 \sim p\frac{\Omega_i}{\beta^4}$$

$$\boxed{\nu_{\rm eff} \sim \gamma_{\rm mirror} \sim \Omega_i \Delta^2}$$

Of course, this is a naive estimate!
In fact,

$$\gamma_{\rm mirror} \sim \Omega_i \left(\Delta - \frac{1}{\beta}\right)^2$$

so can bring $\nu_{\rm eff}$ down by staying very close to threshold,

$$\Delta - \frac{1}{\beta} \ll \frac{1}{\beta}$$

Can't make progress without microphysical theory of $\nu_{\rm eff}$ …

NB: Since this closure implies a dramatic reduction of viscosity near thresholds, this becomes very difficult to resolve numerically, so typically one just uses a large $\nu_{\rm eff}$, but not one that depends on Δ, β and Ω_i, which changes the dynamics (also heating, dynamo)

Implications for Heating

$$\boxed{Q^+ = p\Delta\frac{d\ln B}{dt} = p\nu\Delta^2 \sim p\frac{\nu}{\beta^2}}$$

One might say: *surely, heating = energy injection,*
so how can it be limited by microphysics?

- System can have "impedance": how much energy it will accept depends on local value of β
- If you take the view that there is no impedance, then

$$Q^{\text{in}} \sim \frac{m_i n_i u^3}{L} \sim \frac{m_i n_i}{L}\left(\frac{B}{4\pi m_i n_i}\right)^3 \sim Q^+(B,T) \sim Q^-(n_e,T)$$

Using Kunz et al. scalings, $T \sim 2\left(\frac{L}{0.2\,\text{kpc}}\frac{n_e}{0.1\,\text{cm}^{-3}}\right)^{4/7}\text{keV}$

(this is not terribly useful as we do not know the injection scale; alternatively, can use this to predict L from observations)

[Kunz *et al.*, *MNRAS* **410**, 2446 (2011)]

Implications for Turbulent Dynamo

$$\frac{d\ln B}{dt} = \mathbf{bb}:\nabla\mathbf{u} = \nu\Delta \in \frac{\nu}{\beta}[-2,1]$$

Suppose there is enough stirring to keep Δ at the threshold

$$\frac{d\ln B}{dt} \sim \frac{\nu}{\beta} = \frac{\nu}{8\pi p}B^2$$

Explosive growth, but (using **Kunz et al. closure**) it takes a long time to get going:

$$t_{\rm growth} \sim \frac{\beta_0}{\nu} \sim \beta_0 \times 10 \left(\frac{n_e}{0.1\,{\rm cm}^{-3}}\right)^{-1} \left(\frac{T}{2\,{\rm keV}}\right)^{3/2} {\rm yrs}$$

$$t_{\rm cool} \sim 10^8 \left(\frac{n_e}{0.1\,{\rm cm}^{-3}}\right)^{-1} \left(\frac{T}{2\,{\rm keV}}\right)^{1/2} {\rm yrs}$$

So this can efficiently restore fields from $B \gtrsim 10^{-8}{\rm G}$
to current values $B \sim 10^{-5}{\rm G}$,
but for growth from a tiny seed, need a different mechanism

[cf. AAS *et al.*, *ApJ* **629**, 139 (2005)]

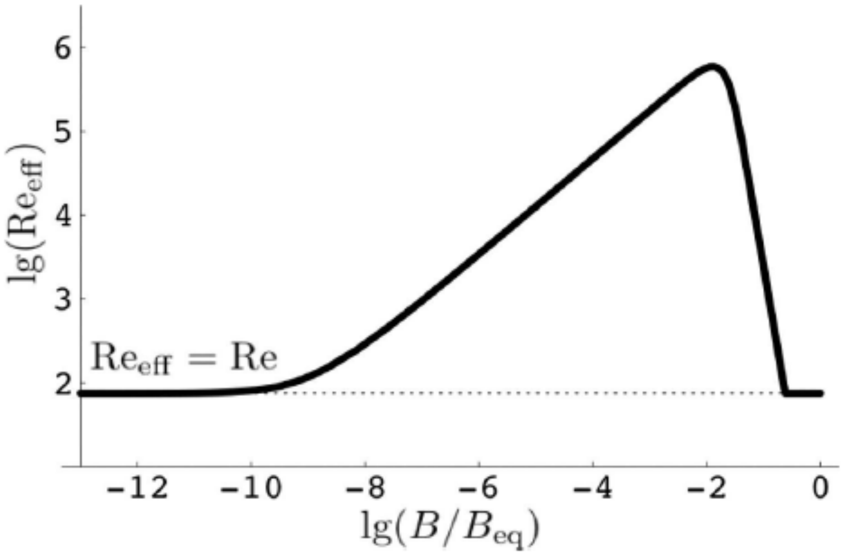
Implications for Turbulent Dynamo

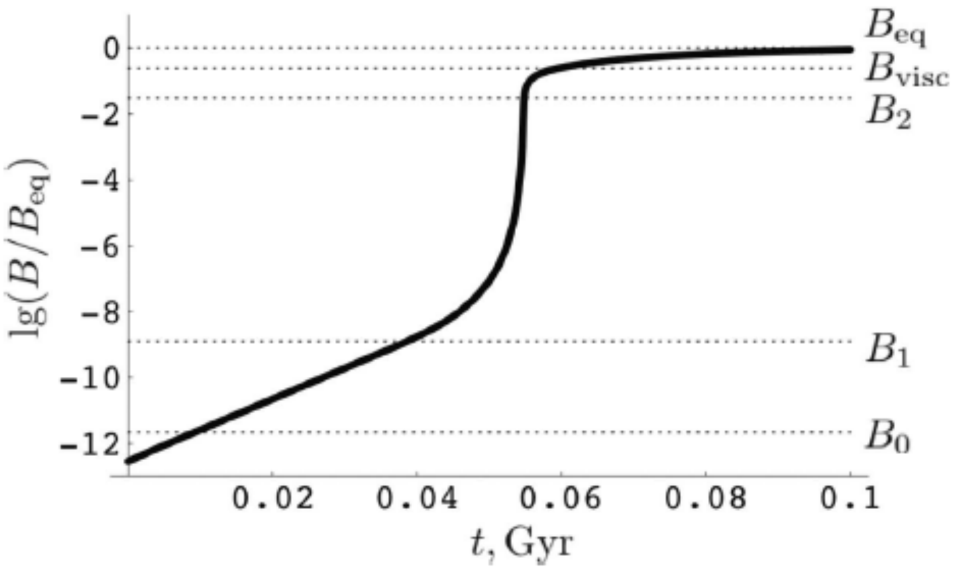
$$\frac{d\ln B}{dt} = \mathbf{bb}:\nabla\mathbf{u} = \nu\Delta \in \frac{\nu}{\beta}[-2,1]$$

Sharma et al. closure: $\nu \to \nu_{\rm eff} = \nu + \Omega_i\left(\Delta - \frac{1}{\beta}\right)^2$ (AAS & Cowley 2006 version)

$$\frac{d\ln B}{dt} \sim \frac{u}{L}\mathrm{Re}_{\rm eff}^{1/2} \sim \frac{u}{L}\left(\frac{uL}{v_{{\rm th}i}^2}\nu_{\rm eff}\right)^{1/2}$$

Explosive growth again and can massage the model to grow from small seeds ($B \sim 10^{-17}$ G) to observed strengths in short times:





[AAS & Cowley, *Phys. Plasmas* **13**, 056501 (2006)]

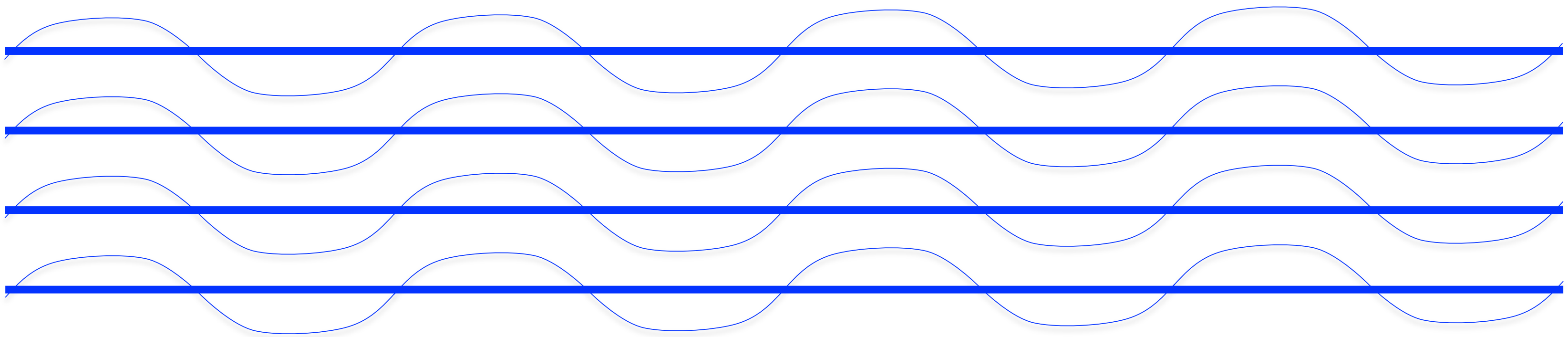
Microphysics: Firehose

$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = \frac{1}{\nu}\,\overline{\frac{d\ln B}{dt}} = \frac{1}{\nu}\left(-\left|\frac{d\ln B_0}{dt}\right| + \frac{1}{2}\frac{d}{dt}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2}\right) \to -\frac{2}{\beta}$$

macroscale field (large but slow)

microscale fluctuations (small but fast)



[AAS *et al.*, *PRL* **100**, 081301 (2008); Rosin *et al.*, *MNRAS* **413**, 7 (2011)]

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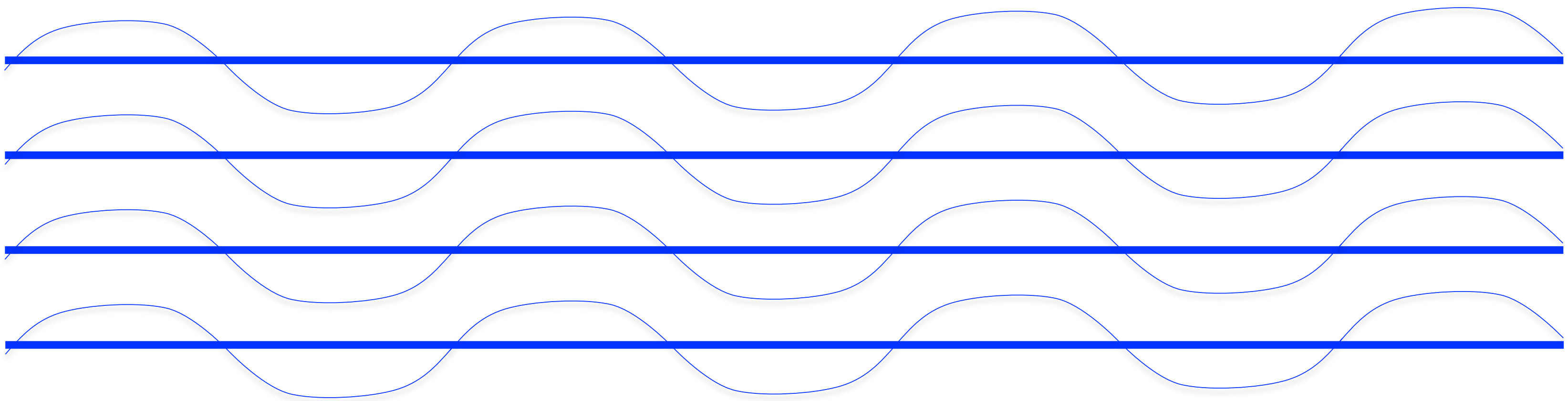

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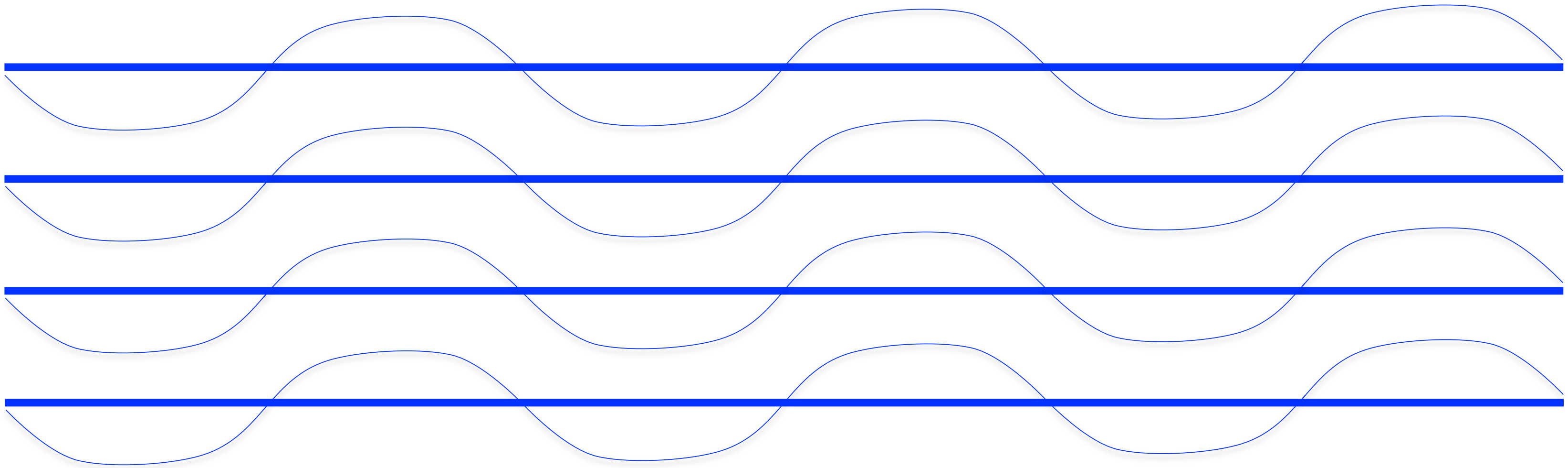
macroscale field (large but slow) — microscale fluctuations (small but fast)



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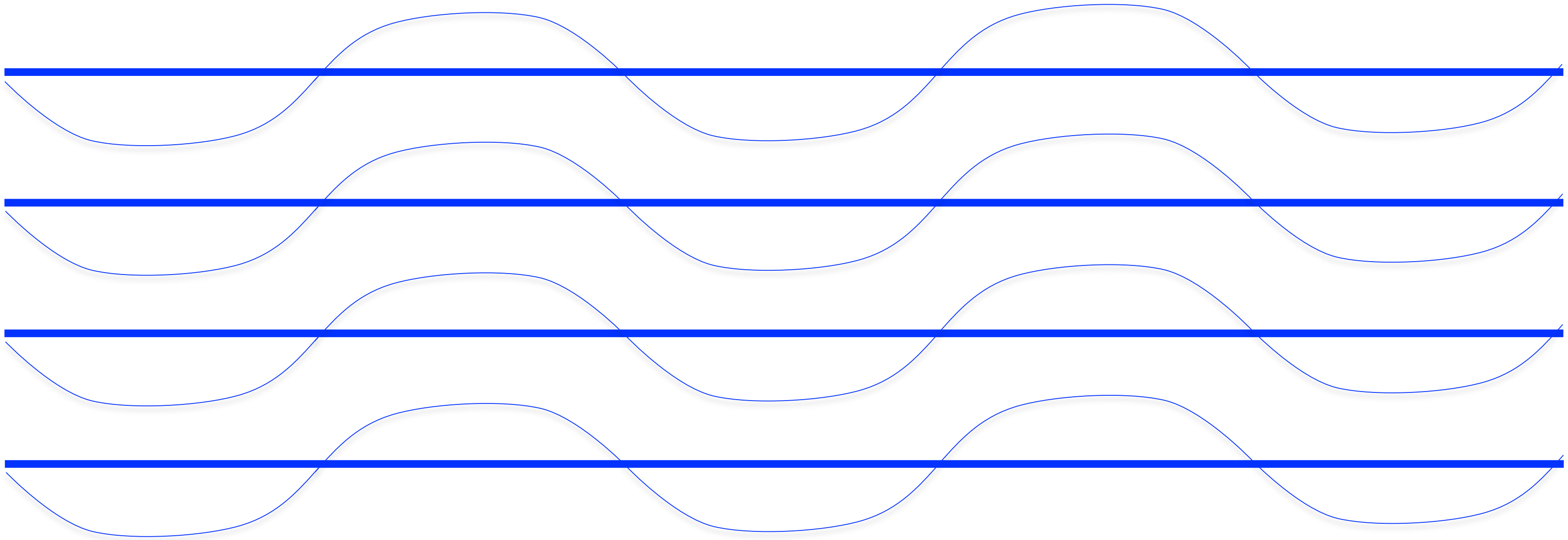
Microphysics: Firehose

$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = \frac{1}{\nu}\overline{\frac{d\ln B}{dt}} = \frac{1}{\nu}\left(-\underbrace{\left|\frac{d\ln B_0}{dt}\right|}_{\substack{\text{macroscale}\\\text{field}\\\text{(large but slow)}}} + \underbrace{\frac{1}{2}\frac{d}{dt}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2}}_{\substack{\text{microscale}\\\text{fluctuations}\\\text{(small but fast)}}}\right) \to -\frac{2}{\beta}$$



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- Amplitude $\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} = \left(\left|\frac{d\ln B_0}{dt}\right| - \frac{2\nu}{\beta}\right)t$
- Scale $k_\| \rho_i = 2\sqrt{2}\left|\Delta + \frac{2}{\beta}\right|^{1/2} \sim \frac{1}{\sqrt{\Omega_i t}} \ll 1$

(scale too large to break conservation of μ, so no scattering)

[AAS *et al., PRL* **100**, 081301 (2008); Rosin *et al., MNRAS* **413**, 7 (2011)]

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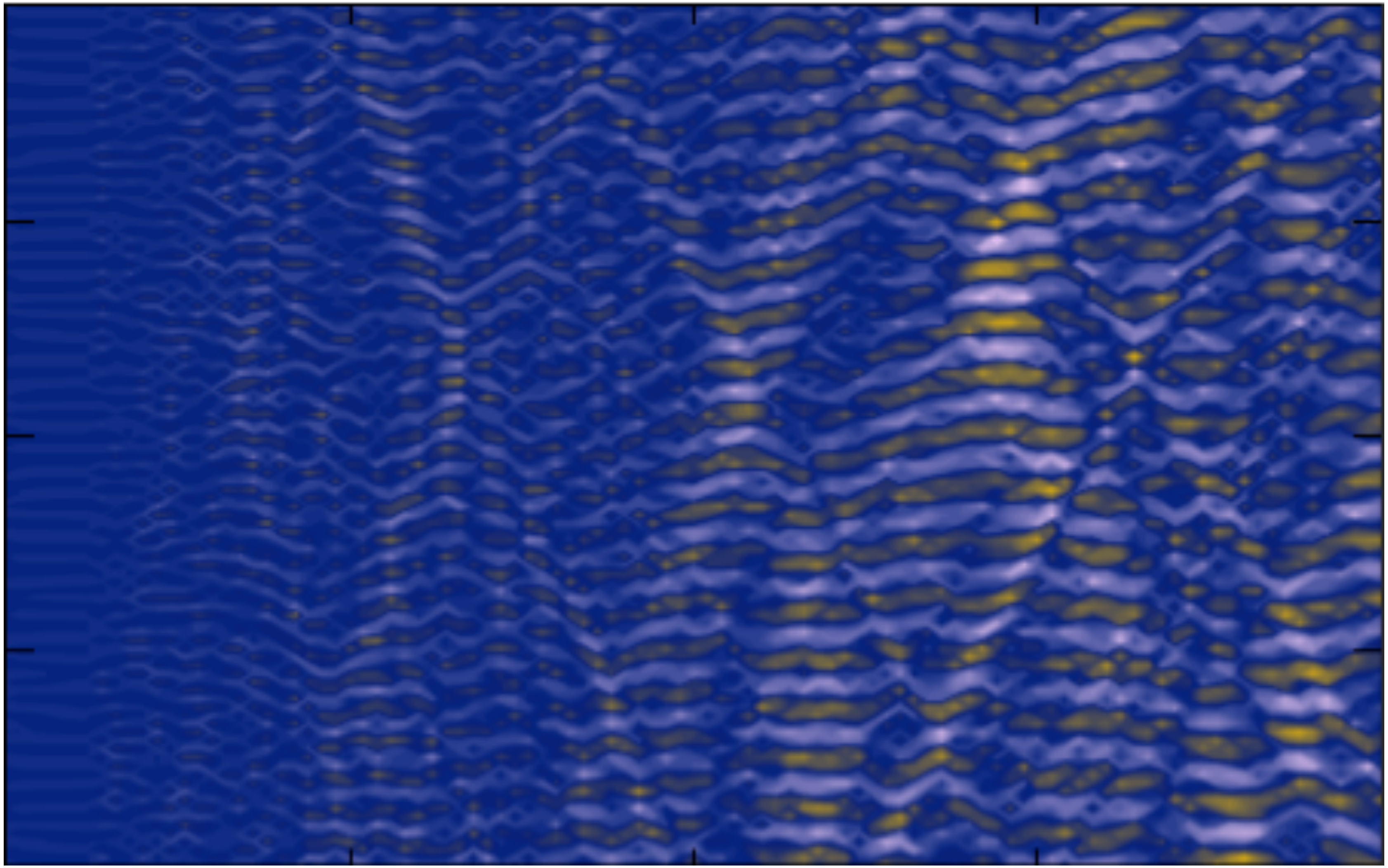

[Rosin *et al.*, *MNRAS* **413**, 7 (2011)]

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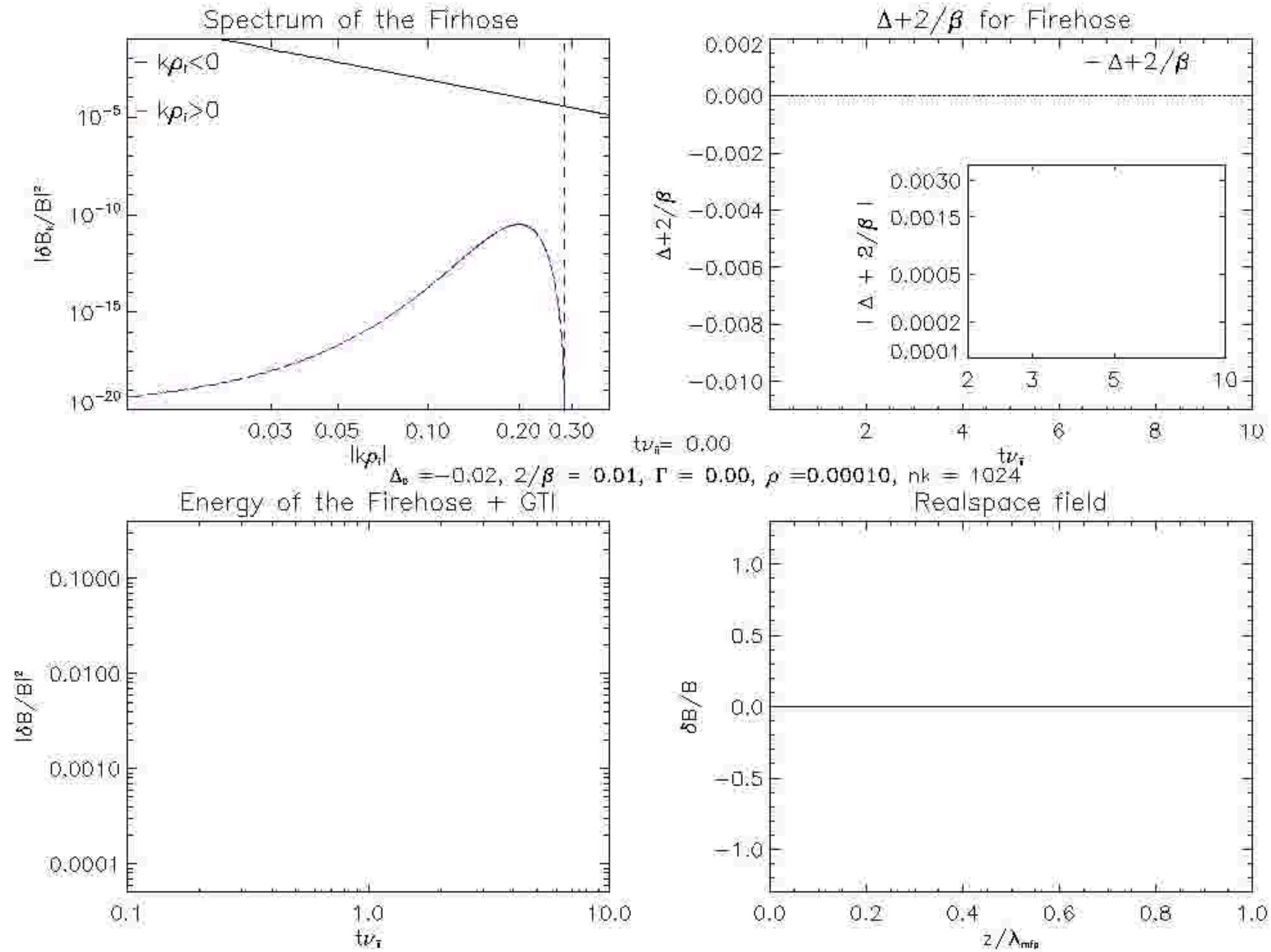

[Rosin *et al.*, *MNRAS* **413**, 7 (2011)]

Microphysics: Mirror

$$\overline{B} = B_0 + \overline{\delta B_\parallel}$$

$$\Delta = \frac{1}{\nu}\overline{\frac{d\ln B}{dt}} = \frac{1}{\nu}\left(\frac{d\ln B_0}{dt} + \frac{d}{dt}\frac{\overline{\delta B_\parallel}}{B_0}\right) \to \frac{1}{\beta}$$

mirror-trapped
particles in holes
(fraction $\sim |\delta B_\parallel / B_0|^{1/2}$)

[AAS *et al.*, *PRL* **100**, 081301 (2008); Rincon *et al.* (2012)]

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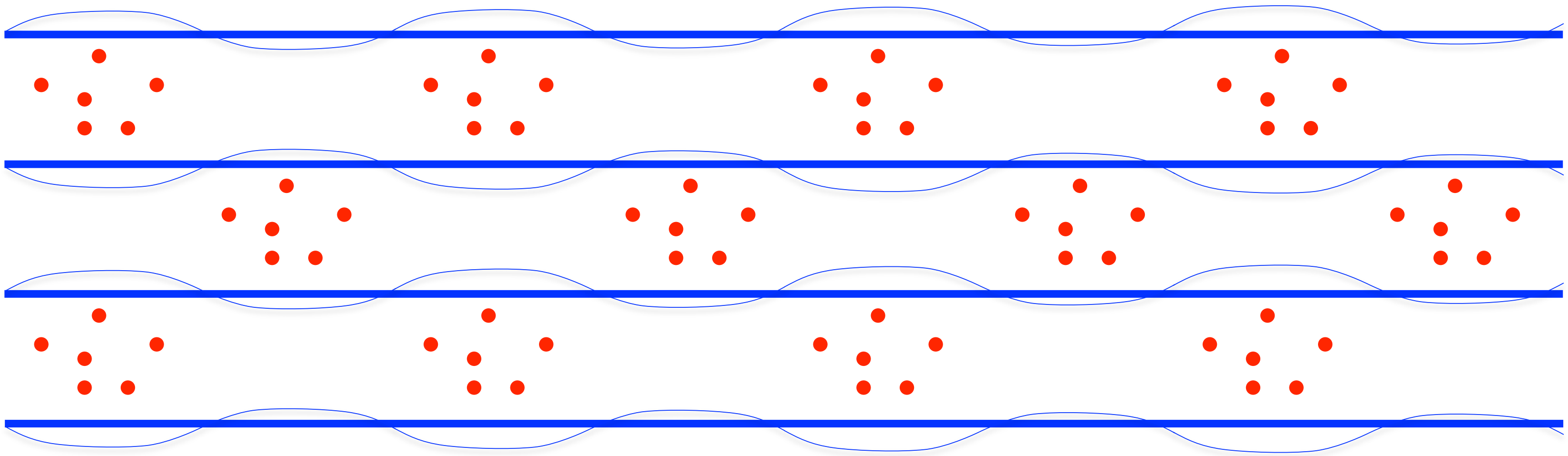

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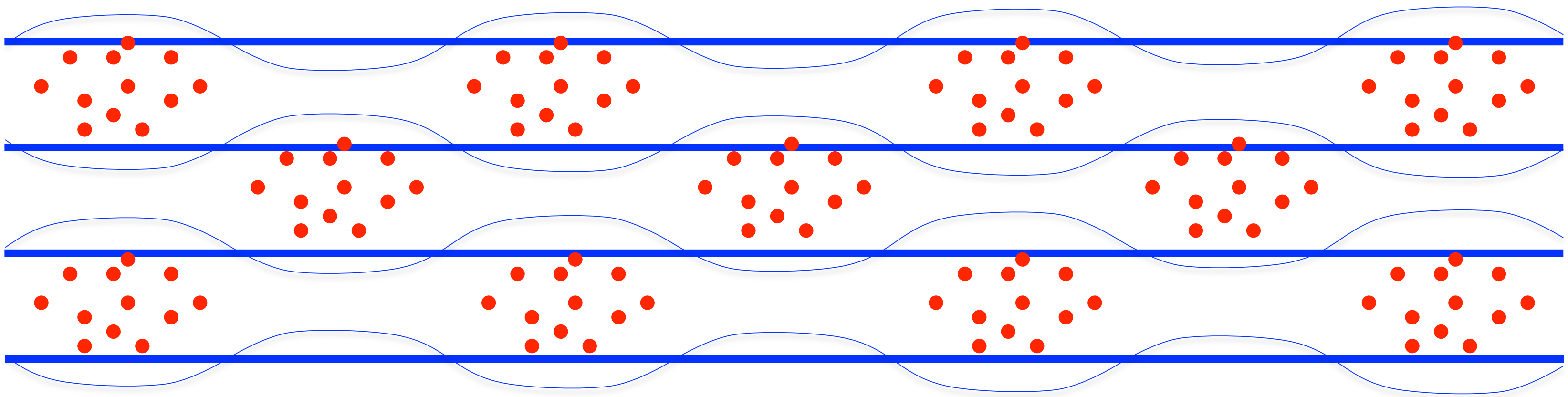

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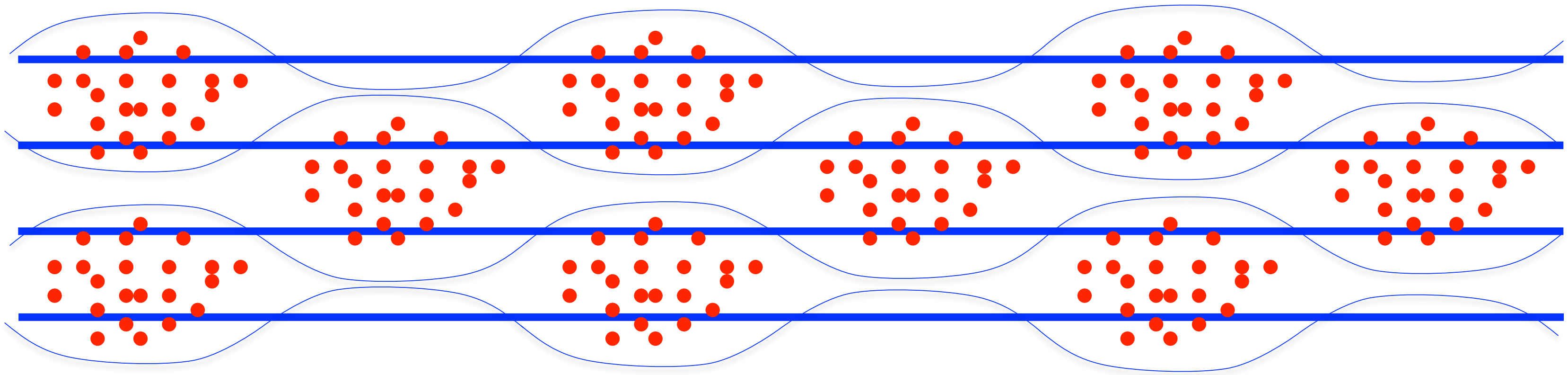

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mirror-trapped
particles in holes
(fraction $\sim |\delta B_\parallel / B_0|^{1/2}$)

- Amplitude $\left|\frac{\delta B_\parallel}{B_0}\right| \sim \left[\left(\frac{d\ln B_0}{dt} - \frac{\nu}{\beta}\right) t\right]^{2/3}$

- Scale $k_\parallel \rho_i \sim \Delta - \frac{1}{\beta} \ll 1$

(scale too large to break conservation of μ , so no scattering)

[AAS *et al.*, *PRL* **100**, 081301 (2008); Rincon *et al.* (2012)]

Conclusions

I have no conclusions yet…

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I have no conclusions yet… OK, here are some:

- Moving magnetised ICM around at large scales requires a closure model for the net effect of pressure-anisotropy-driven microscale instabilities
- The effective closure must keep pressure anisotropy within thresholds set by the local β [AAS *et al.*, *MNRAS* **405**, 291 (2010)]
- This can be achieved EITHER **via locally enhanced collisionality [I]** OR **via controlling local rate of strain [II]** **(enhanced viscosity, LES-style)**
- It matters which you choose:
  - Smaller [I] vs. larger [II] viscosity
  - Different predictions for heating [Kunz *et al.*, *MNRAS* **410**, 2446 (2011)] (only [II] gives thermally stable ICM as far as I can tell)
  - Turbulent dynamo is explosive in both cases, but with different time scales [AAS & Cowley, *Phys. Plasmas* **13**, 056501 (2006)]
- To the best my understanding of the microphysics, there is no anomalous scattering, so **I prefer closure [II]**

[AAS *et al.*, *PRL* **100**, 081301 (2008); Rosin *et al.*, *MNRAS* **413**, 7 (2011)]