Multi-ion and Anisotropic Magnetohydrodynamics

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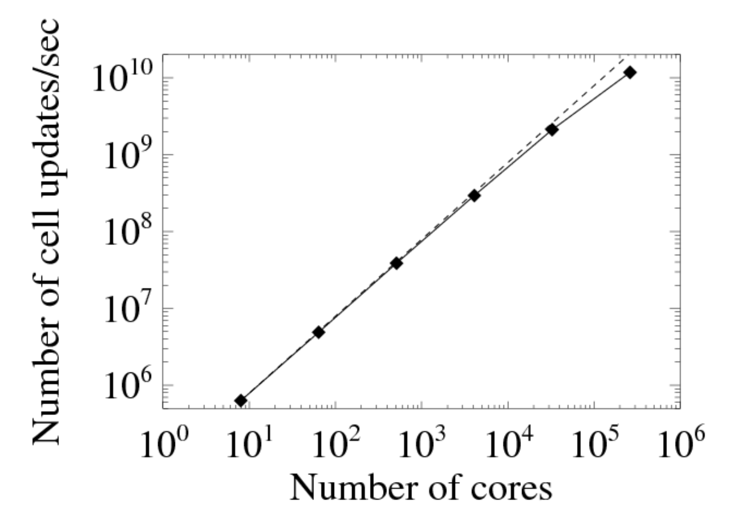
Outline

- BATS-R-US
- MHD equations with multiple ion fluids
- MHD equations with anisotropic pressure
- Instabilities
- Implementation and verification
- Some modeling results
- Summary

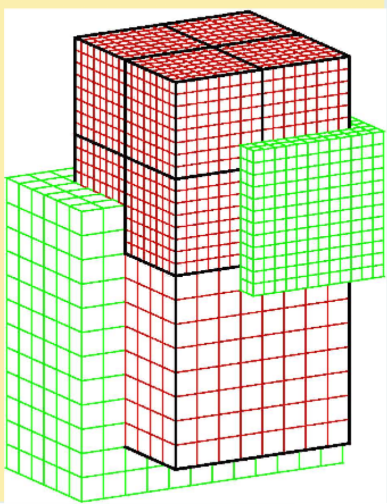
BATS-R-US

Block Adaptive Tree Solar-wind Roe Upwind Scheme

- **Physics**
  - Classical, semi-relativistic and Hall MHD
  - Multi-species, multi-fluid, anisotropic pressure
  - Radiation hydrodynamics multigroup diffusion
  - Multi-material, non-ideal equation of state
  - Solar wind turbulence, Alfven wave heating
- **Numerics**
  - Conservative finite-volume discretization
  - Parallel block-adaptive grid
  - Cartesian and generalized coordinates
  - Splitting the magnetic field into $B_0 + B_1$
  - Divergence B control: 8-wave, CT, projection, parabolic/hyperbolic
  - Shock-capturing TVD schemes: Rusanov, HLLE, AW, Roe, HLLD
  - Explicit, point-implicit, semi-implicit, fully implicit time stepping
- **Applications**
  - Heliosphere, sun, planets, moons, comets, HEDP experiments
- **100,000+ lines of Fortran 90 code with MPI parallelization**



Scaling of BATS-R-US from 8 to 262,144 cores with 40,960 cells/core on the Cray Jaguar system.



Multi-Fluid MHD

- **Multi-fluid MHD has many space physics applications**
  - ionospheric outflow, Earth magnetosphere, Martian ionosphere, outer heliosphere interaction with interstellar medium, etc.
- **BATS-R-US now contains a general multi-fluid solver with arbitrary number of ion and neutral fluids.**
- **Each fluid has separate densities, velocities and temperatures.**
- **One ion fluid + neutrals can be solved as MHD for ions, and HD for neutrals.**
- **Ions and neutrals are coupled by charge exchange and chemical reactions.**
- **Neutrals are coupled by collisions and chemical reactions.**
- **Coupling source terms can be evaluated point-implicitly.**

Multi-Ion MHD Derived

Momentum equations for ion fluids s with charge q_s and electrons with charge $-e$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = +n_s q_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial \rho_e \mathbf{u}_e}{\partial t} + \nabla \cdot (\rho_e \mathbf{u}_e \mathbf{u}_e + I p_e) = -n_e e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + S_{\rho_e \mathbf{u}_e}$$

Express electric field from electron momentum equation neglecting small terms:

$$\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{e n_e} \nabla p_e + \eta \mathbf{J}$$

Obtain electron density from charge neutrality and electron velocity from current:

$$n_e = \frac{1}{e} \sum_s n_s q_s$$

$$\mathbf{u}_e = -\frac{\mathbf{J}}{e n_e} + \mathbf{u}_+$$ where the charge averaged ion velocity is $$\mathbf{u}_+ = \frac{\sum_s n_s q_s \mathbf{u}_s}{e n_e}$$

The electron pressure p_e is either a fixed fraction of total ion pressure, or we solve

$$\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}_e) = -(\gamma - 1) p_e \nabla \cdot \mathbf{u}_e + S_{p_e}$$

Multi-Ion MHD

For each ion fluid *s* we obtain (neglecting resistive terms):

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = S_{\rho_s}$$

Cannot be written in conservative form

Gyration of ions around each other. Can be stiff.

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = \frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B} + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s) = -(\gamma - 1) p_s \nabla \cdot \mathbf{u}_s + S_{p_s}$$

We can also solve for *hydro* energy density $e_s = \rho_s \mathbf{u}_s^2/2 + p_s/(\gamma - 1)$

$$\frac{\partial e_s}{\partial t} + \nabla \cdot [(e_s + p_s)\mathbf{u}_s] = \mathbf{u}_s \cdot \left[\frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B}\right] + S_{e_s}$$

Finally the induction equation with or without the Hall term becomes

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_e \times \mathbf{B}) = 0 \quad \text{or} \quad \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_+ \times \mathbf{B}) = 0$$

Two-Stream Instability

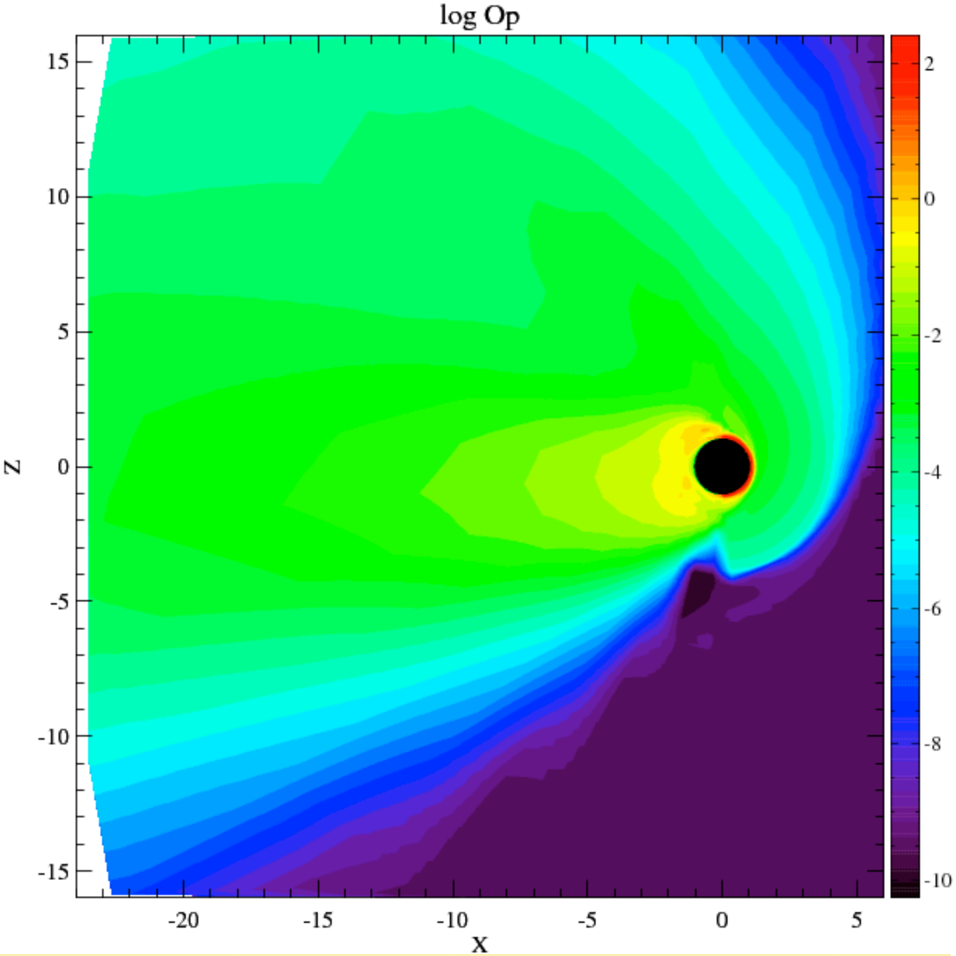
- **Perpendicular ion velocities are coupled through the magnetic field**
- **Parallel ion velocities are not coupled by the multi-ion MHD equations.**
- Two-stream instability restricts the velocity differences parallel to **B**
  - We cannot resolve the two-stream instability
  - Use a simple ad-hoc friction source term in the momentum equations:

$$S_{\rho \mathbf{u}_s}^{friction} = \frac{1}{\tau_c} \sum_{q \neq s} \min(\rho_s, \rho_q)(\mathbf{u}_q - \mathbf{u}_s) \left(\frac{|\mathbf{u}_s - \mathbf{u}_q|}{u_c} \right)^{\alpha_c}$$

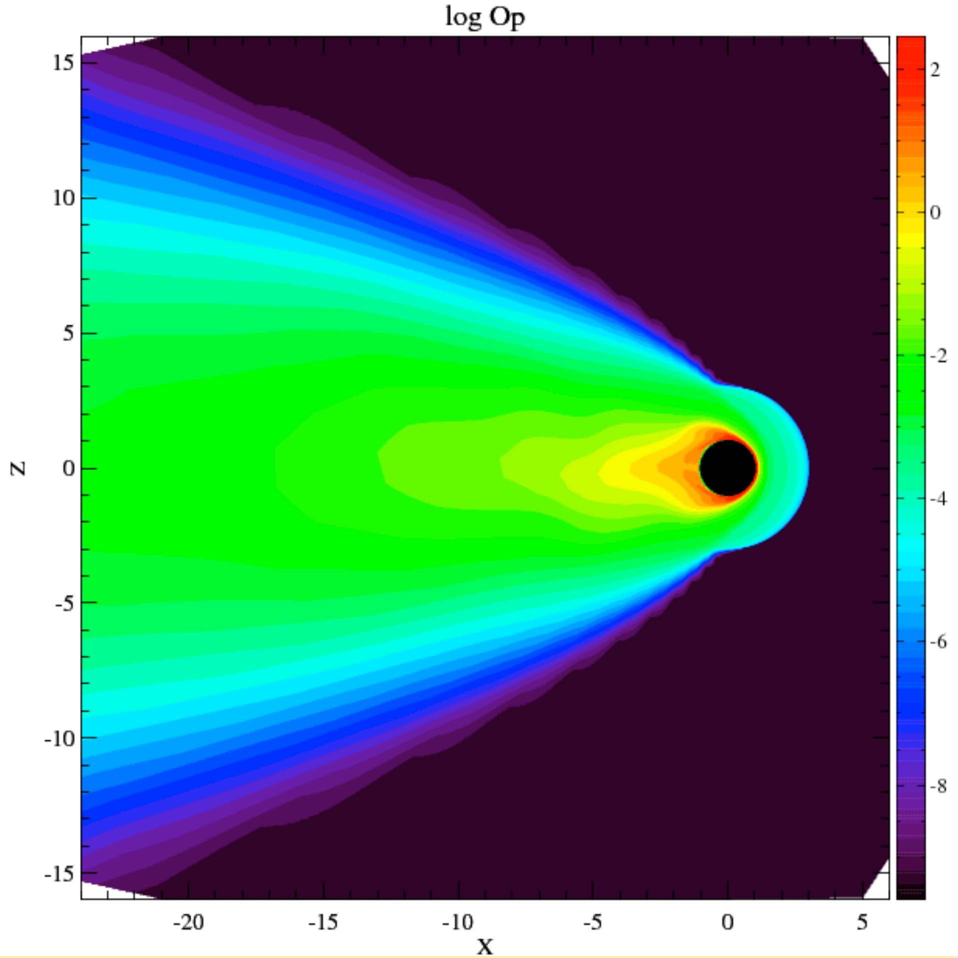
  - Using the minimum of the two densities makes the friction uniformly effective in regions of low and high densities.
  - τ_C is the time scale, u_C is the cut-off velocity, α_C is the cut-off exponent
  - Adjustable parameters. It would be better to use physics based formulas.

O+ Escape from Mars Ionosphere

Multi-fluid MHD



Multi-species MHD



Anisotropic MHD

- **In (nearly) collisionless plasma pressure can be anisotropic**
  - Different pressures parallel and perpendicular to the magnetic field:

$$P = (p_\perp + p_e)I + (p_\parallel - p_\perp)\mathbf{bb} \qquad \mathbf{b} = \mathbf{B}/B$$

- **Where does it matter in space physics?**
  - Reconnection
  - Magnetosphere
  - Inner magnetosphere
  - Solar wind heating

Hall MHD with isotropic electron and anisotropic ion pressure

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[\rho \mathbf{u}\mathbf{u} + p_\perp \mathbf{I} + (p_\parallel - p_\perp)\mathbf{b}\mathbf{b} + p_e \mathbf{I} + \frac{B^2}{2}\mathbf{I} - \mathbf{B}\mathbf{B} \right] = 0$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \left[-\mathbf{u} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{e n_e} - \frac{\nabla p_e}{e n_e} \right] = 0$$

$$\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}_e) = -(\gamma - 1) p_e \nabla \cdot \mathbf{u}_e \qquad \mathbf{u}_e = \mathbf{u} - \frac{\mathbf{J}}{e n_e}$$

$$\frac{\partial p_\parallel}{\partial t} + \nabla \cdot (p_\parallel \mathbf{u}) = -2 p_\parallel \mathbf{b} \cdot (\nabla \mathbf{u}) \cdot \mathbf{b}$$

$$\frac{\partial p}{\partial t} + \nabla \cdot (p \mathbf{u}) = -(\gamma - 1) p_\perp \nabla \cdot \mathbf{u} + (p_\parallel - p)\mathbf{b} \cdot (\nabla \mathbf{u}) \cdot \mathbf{b} \qquad p_\perp = \frac{3p - p_\parallel}{2}$$

Adiabatic index is $\gamma = 5/3$.

Resistive and relaxation terms are implemented but not shown.

Hall and grad P_e terms are implemented but not used in the rest of this work.

Hall MHD with isotropic electron and anisotropic ion pressure

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \left[\rho \mathbf{u}\mathbf{u} + p_\perp \mathbf{I} + (p_\parallel - p_\perp)\mathbf{b}\mathbf{b} + p_e \mathbf{I} + \frac{B^2}{2}\mathbf{I} - \mathbf{B}\mathbf{B} \right] = 0$$

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$$\frac{\partial p_\parallel}{\partial t} + \nabla \cdot (p_\parallel \mathbf{u}) = -2 p_\parallel \mathbf{b} \cdot (\nabla \mathbf{u}) \cdot \mathbf{b}$$

$$\frac{\partial e}{\partial t} + \nabla \cdot \left[\mathbf{u} \left(e + p_\perp + p_e + \frac{\mathbf{B}^2}{2} \right) + \mathbf{u} \cdot \left((p_\parallel - p_\perp)\mathbf{b}\mathbf{b} - \frac{\mathbf{B}\mathbf{B}}{2} \right) \right] = 0$$

$$e = \frac{\rho \mathbf{u}^2}{2} + \frac{\mathbf{B}^2}{2} + \frac{p + p_e}{\gamma - 1}$$

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Characteristic Speeds

- **Entropy:** $\lambda_{1,2,3} = u_x$
- **Alfven:** $\lambda_{4,5} = u_x \pm b_x \sqrt{\frac{B^2 + p_\perp - p_\parallel}{\rho}}$
- **Magnetosonic:**

$$\lambda_{6,7,8,9} = u_x \pm \frac{1}{\sqrt{2\rho}} \Big\{ B^2 + 2p_\perp + \gamma p_e + (2p_\parallel - p_\perp) b_x^2$$

$$\pm \Big[\left(B^2 + 2p_\perp + \gamma p_e + (2p_\parallel - p_\perp) b_x^2 \right)^2$$

$$+ 4 \Big(\frac{1}{4} {p_\perp}^2 (1 - b_x^2) - 3 p_\parallel p_\perp b_x^2 (2 - b_x^2) + 3 {p_\parallel}^2 b_x^4$$

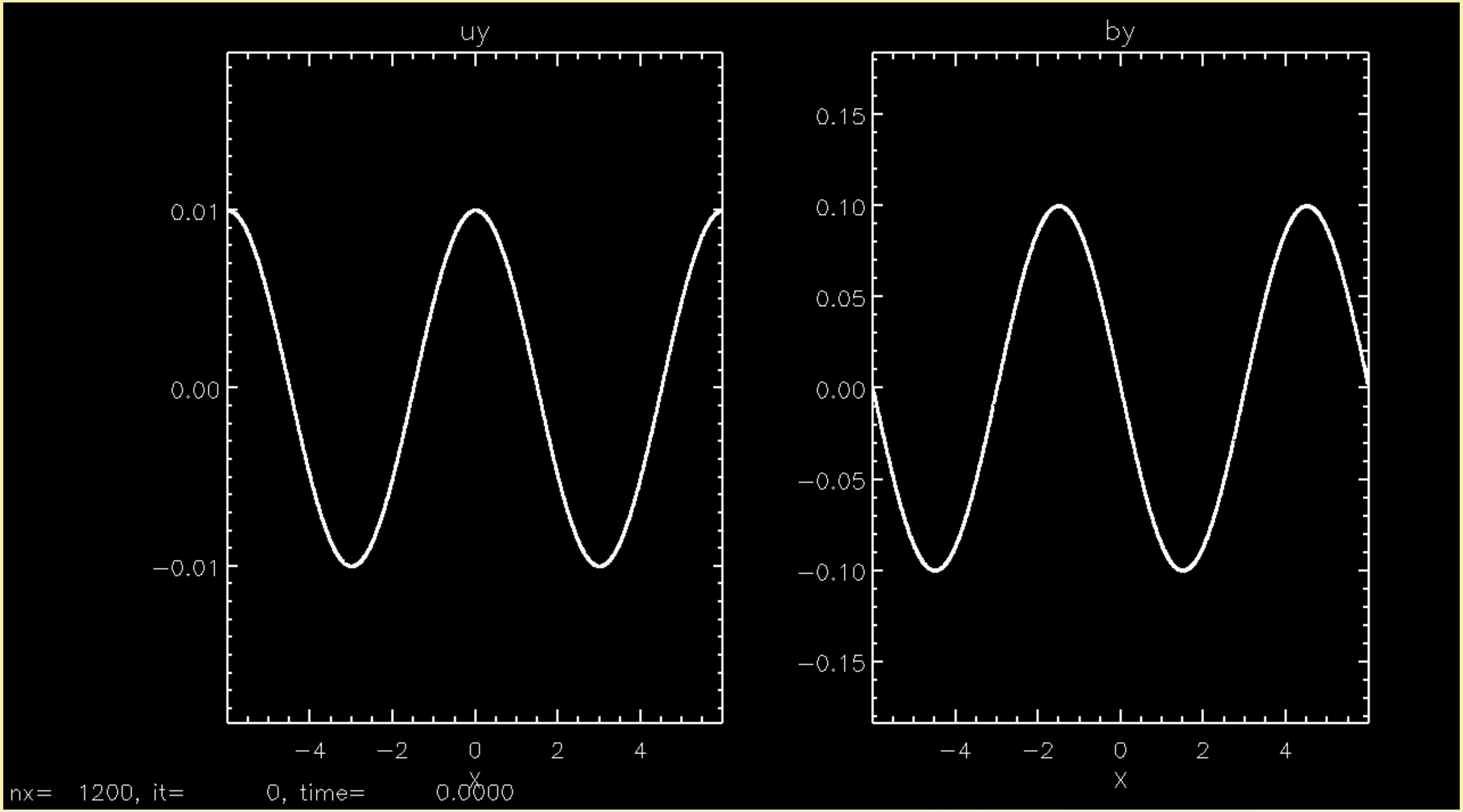
$$+ \frac{5}{3} p_e (4 p_\parallel b_x^2 - p_\perp b_x^2 - 3 p_\parallel) b_x^2 - \left(3 p_\parallel + \frac{5}{3} p_e \right) B_x^2 \Big) \Big]^{1/2} \Big\}^{1/2}$$

- **Agrees with Baranov (1970) with no electron pressure**

Alfvén waves with anisotropic pressure

Circularly polarized Alfvén wave propagates at $v_A = \sqrt{(\mathbf{B}^2 + p_\perp - p_\parallel)/\rho}$

This can become unstable if the parallel pressure is large enough!



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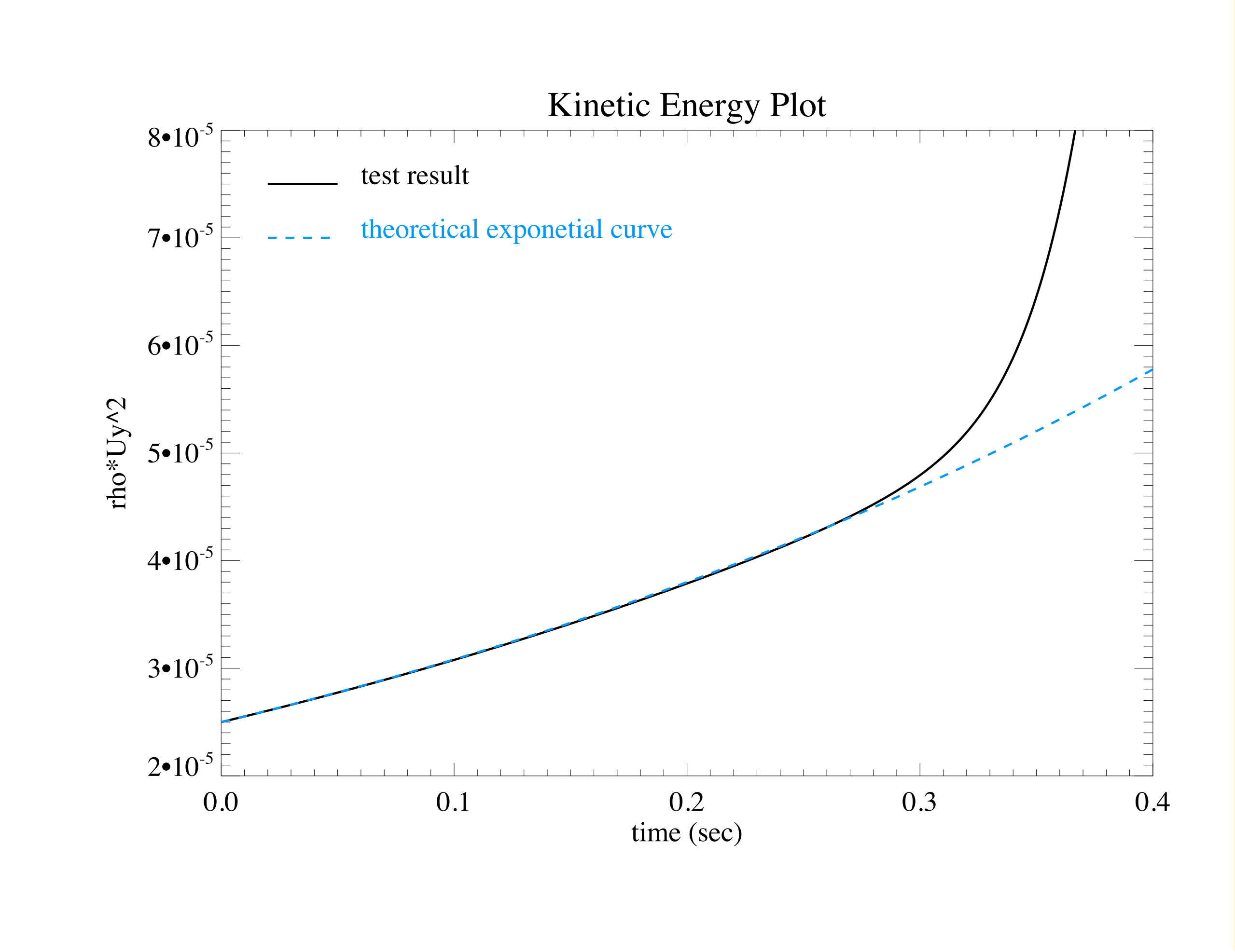


Instabilities Limiting the Anisotropy

- **Instabilities from kinetic theory including finite Larmor radius (FLR) effects:**
  - Fire-hose: $$\frac{p_\parallel}{p_\perp} > 1 + \frac{B^2}{p_\perp}$$ $$\tau_f = \frac{1}{\gamma_{fFLR}(\lambda_f)} = \frac{2}{\Omega_i}\frac{\sqrt{p_\parallel(p_\perp - p_\parallel/4)}}{\Delta p_f}$$
  - Mirror: $$\frac{p_\perp}{p_\parallel} > 1 + \frac{B^2}{2p_\perp}$$ $$\tau_m = \frac{1}{\gamma_m(\lambda_m)} = \frac{3\sqrt{5}}{4\Omega_i}\sqrt{\frac{p_\parallel}{2\Delta p_m}}$$
  - Proton cyclotron: $$\frac{p_\perp}{p_\parallel} > 1 + C_1\left(\frac{\mathbf{B}^2}{2p_\parallel}\right)^{C_2}$$ $$\tau_{ic} = \frac{10^2}{\Omega_i}$$
    - Constants depend on growth rate limit, we use $C_1 = 0.3$ and $C_2 = 0.5$
- **The ion pressure is pushed towards isotropy with relaxation source terms (similar to Birn et al. 1995):**

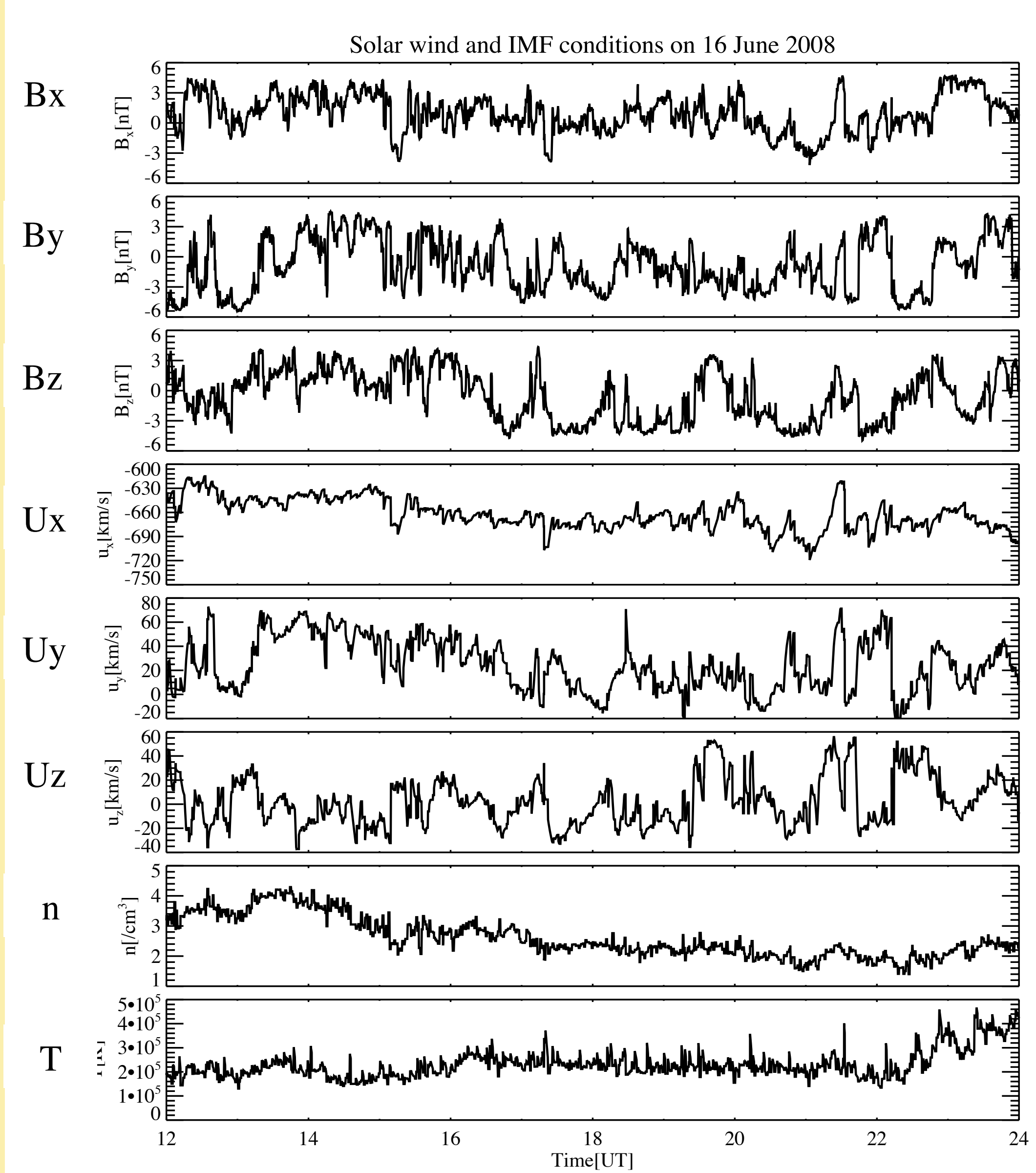
$$\frac{\delta p_\parallel}{\delta t} = \frac{\overline{p}_\parallel - p_\parallel}{\tau}$$

- **We allow for a global relaxation rate as well.**
- **Instabilities also help to provide proper jump conditions.**

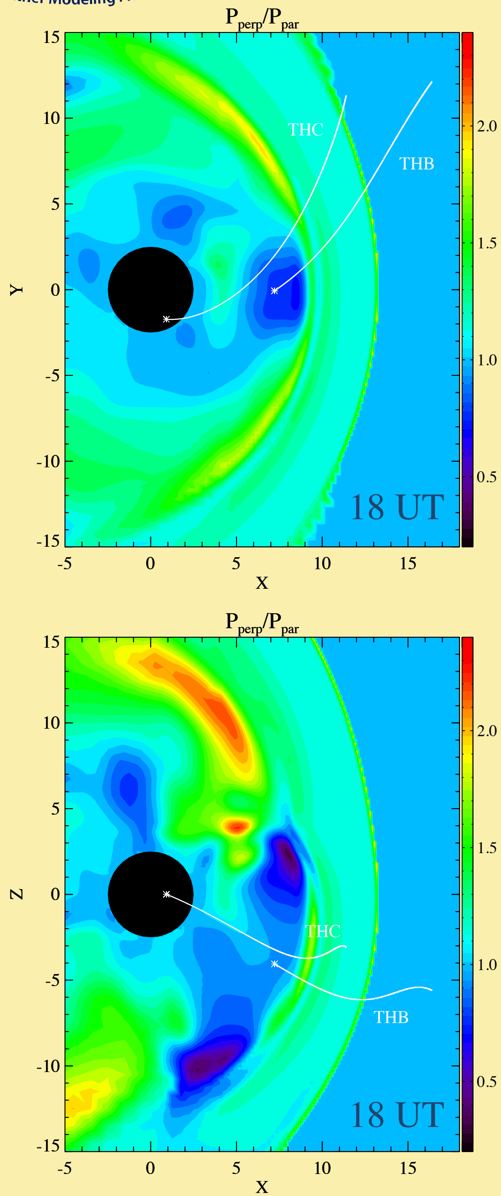
Magnetosphere Simulation, 16 June 2008

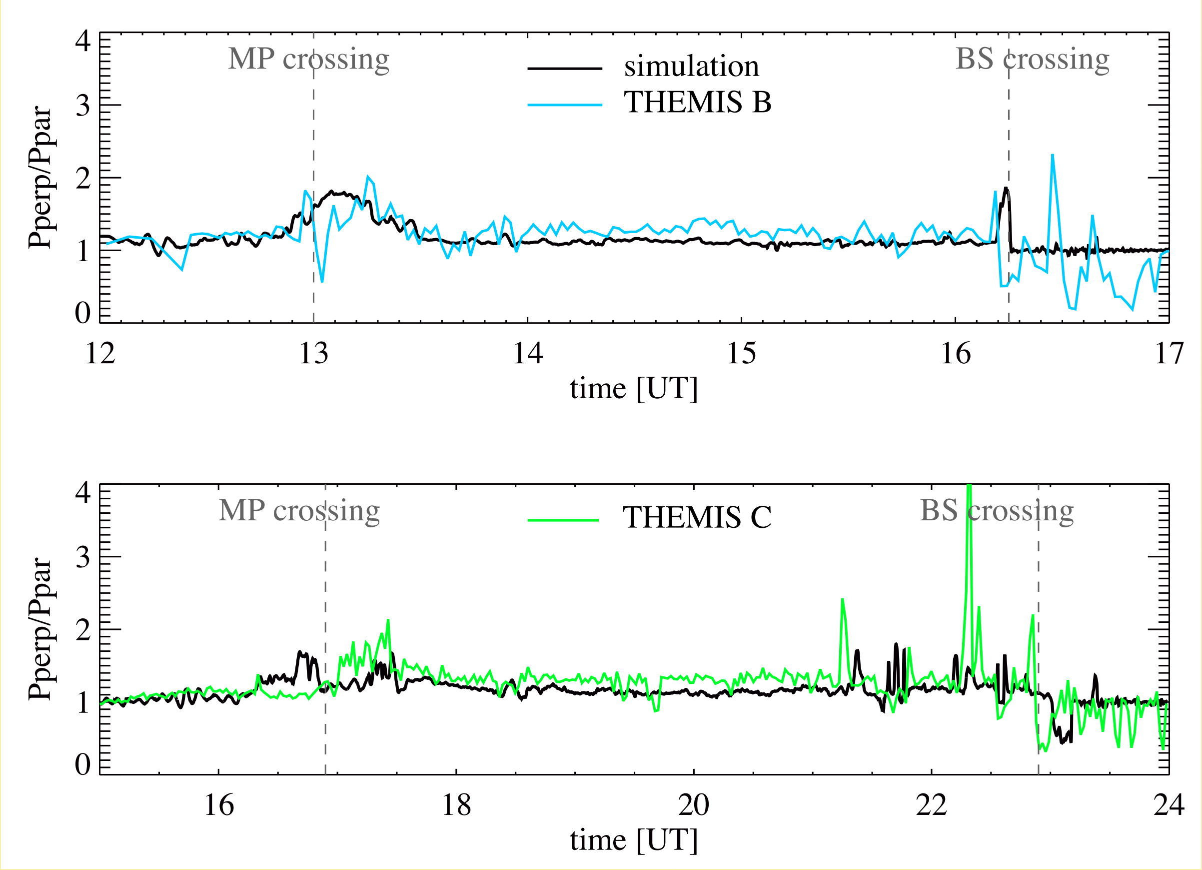
Simulation set-up

- Input solar wind and IMF measured by the ACE and WIND satellites
- Computational domain: X from –224R_e to 32R_e, Y and Z from –128 to 128R_e
- Grid resolution: 1/8R_e where the THEMIS satellites cross the dayside magnetopause

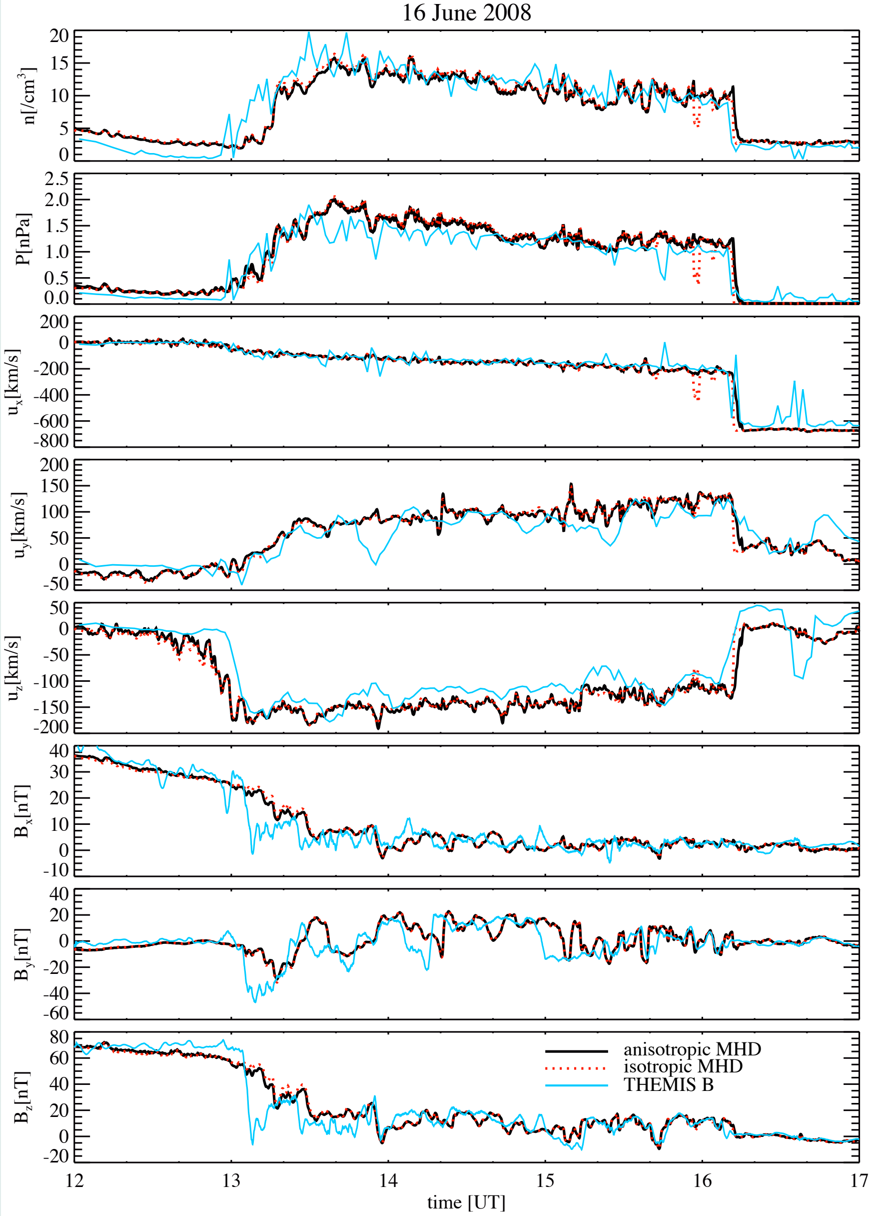


Anisotropy along THEMIS B and C orbits





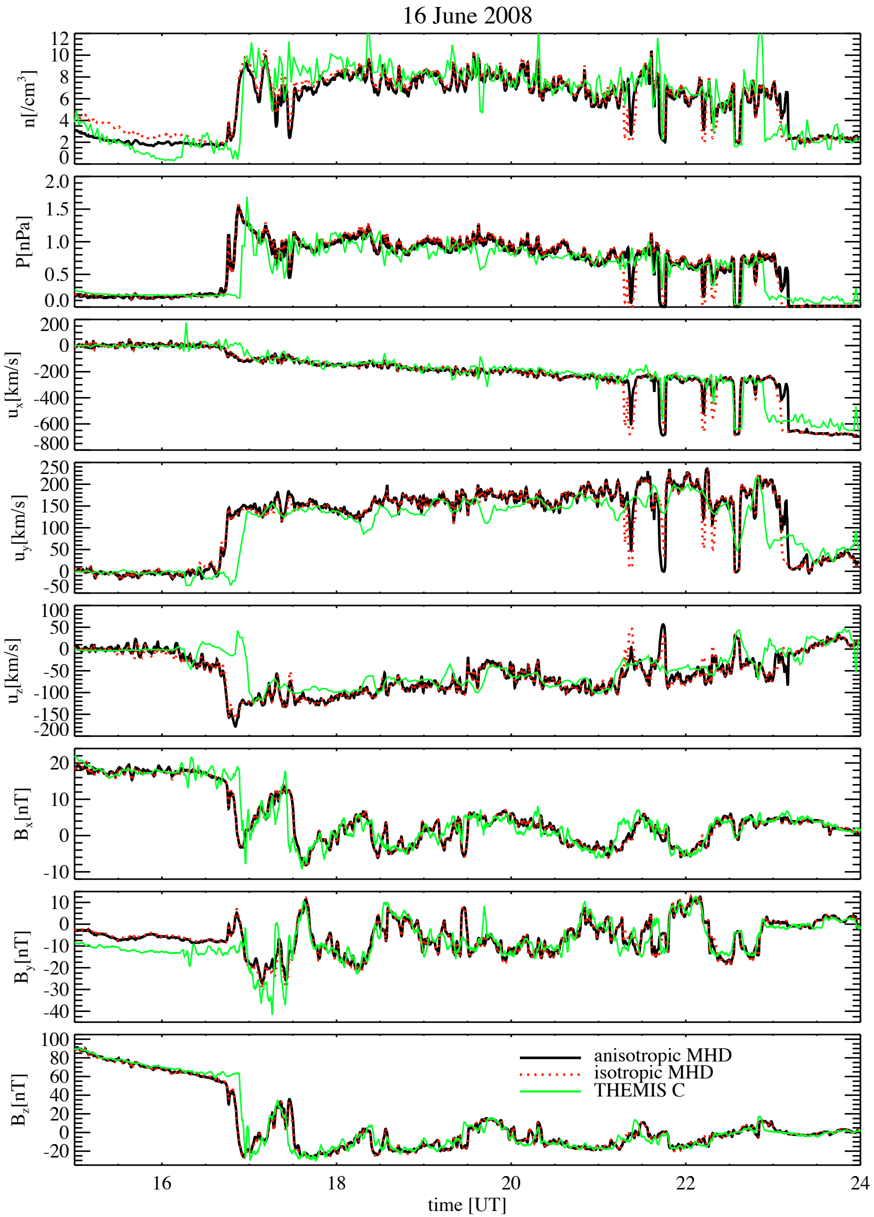
Comparison with THEMIS B measurement



RMS error

	Isotropic MHD	Anisotropic MHD
n [/cc]	2.18	2.29
u_x [km/s]	63.37	52.93
u_y [km/s]	37.67	36.45
u_z [km/s]	45.48	46.12
b_x [nT]	5.81	5.52
b_y [nT]	11.75	11.77
b_z [nT]	11.57	10.86

Comparison with THEMIS C measurement



RMS error

	Isotropic MHD	Anisotropic MHD
n [/cc]	1.66	1.66
u_x [km/s]	80.21	74.22
u_y [km/s]	46.68	45.23
u_z [km/s]	42.52	42.30
b_x [nT]	3.95	3.93
b_y [nT]	6.85	6.70
b_z [nT]	10.13	10.32

Summary

- **We have implemented the multi-ion and anisotropic MHD equations into BATSRUS**
  - Wave speeds are required
  - There is no fully conservative form
  - Point-implicit scheme for stability
  - Sub-grid scale / non-fluid effects need to be taken into account
- **Publications**

  - Adaptive Numerical Algorithms in Space Weather Modeling, G. Toth et al. 2012, Journal of Computational Physics, 231, 870, doi:10.1016/j.jcp.2011.02.006
  - Pressure anisotropy in global magnetospheric simulations: A magnetohydrodynamics model. X. Meng et al. 2012, Journal of Geophysical Research, 117, A08216, doi:10.1029/2012JA017791
  - Classical and Semirelativistic Magnetohydrodynamics with Anisotropic Ion Pressure, X. Meng et al. 2012, Journal of Computational Physics, 231, 3610, doi:10.1016/j.jcp.2011.12.042
  - Multifluid Block-Adaptive-Tree Solar wind Roe-type Upwind Scheme: Magnetospheric composition and dynamics during geomagnetic storms -- Initial results, A. Glocer et al. 2009, Journal of Geophysical Research, 114, A12203, doi:10.1016/j.jastp.2009.01.003