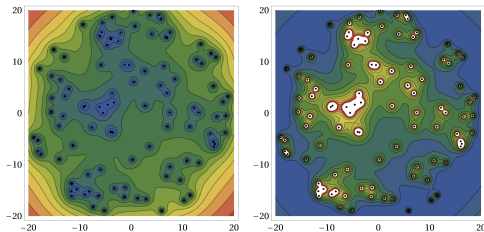


Multicentered Black Holes as Glassy Systems

Edgar Shaghoulian
Stanford University

Out of Equilibrium Statistical Physics and String Theory 2012
University of Michigan



work with Anninos, Anous, Denef, and Konstantinidis

- Glasses
- Multicentered Black Holes
- Supergoop
- Conclusions and Challenges

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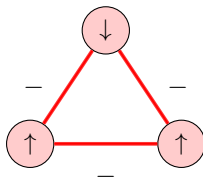
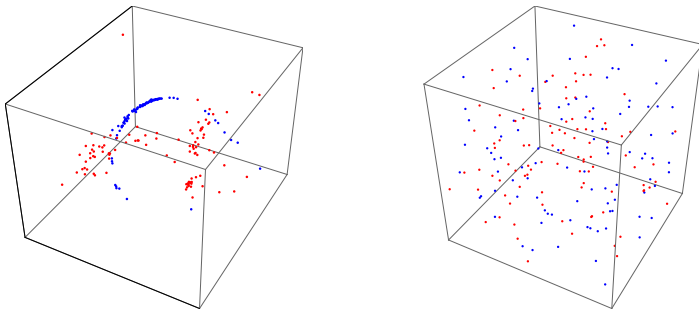


Figure: Frustration begets metastability.

Some words on multicentered black holes



Ground state generated with 100 magnetic and 100 electric centers using gradient descent.

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Not special to supergravity, expected to live beyond supersymmetric limit.

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Other work towards glassy physics in a holographic context includes [\[Adams, Yaida; Kachru, Karch, Yaida; Jottar, Leigh, Minic, Zayas; Saremi\]](#).

SUPERGOOP [Anninos, Anous, Denef, Konstantinidis, ES]



Substringy Supergoop

Low energy behavior of N wrapped branes with charges $\Gamma_p = (p_p, q_p)$ at positions \mathbf{x}_p , i.e. go to the Coulomb branch of the $(0+1)$ -D $\mathcal{N} = 4$ SUSYc quiver quantum mechanics describing this system (we have dimensionally reduced the original $\mathcal{N} = 1$ 4D gauge theory) by integrating out massive chiral multiplets representing stretched string d.o.f.:

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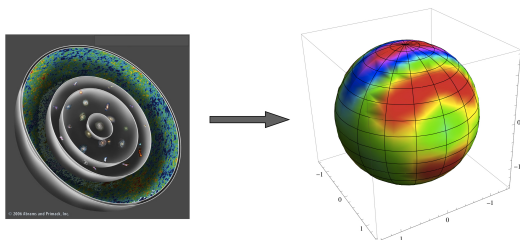
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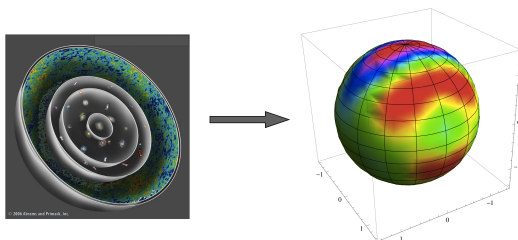
Nonrenormalization theorem responsible for universal appearance of this Hamiltonian: monopoles and dyons in $\mathcal{N} = 2$ SYM, well separated wrapped D-branes representing elementary particles interacting through scalar, vector, and gravitational interactions.

Many centers



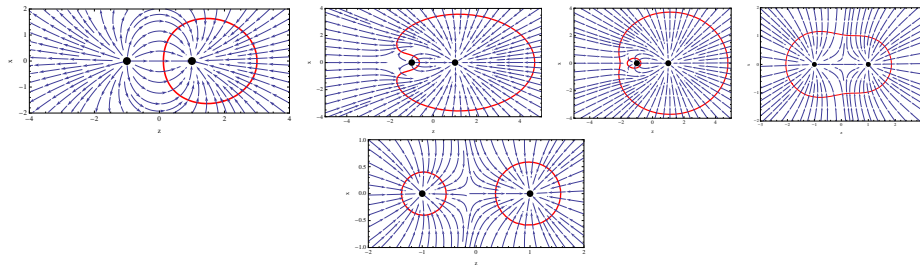
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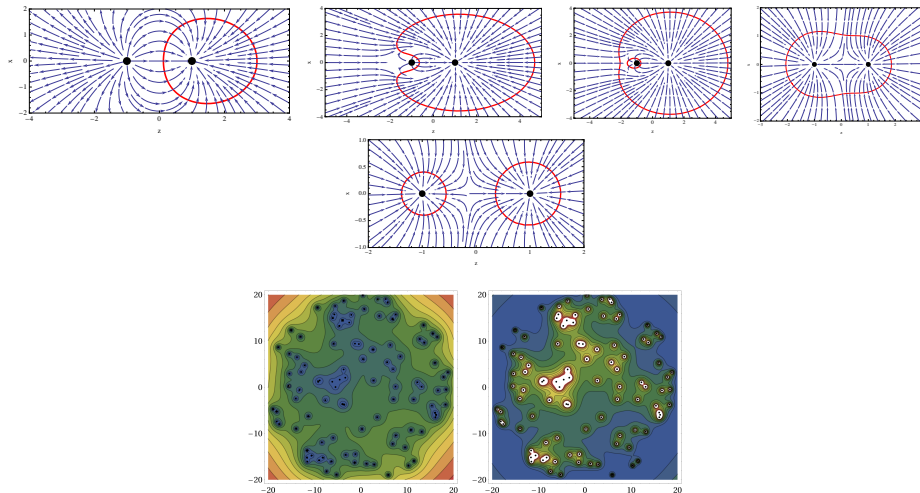


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- Cue video

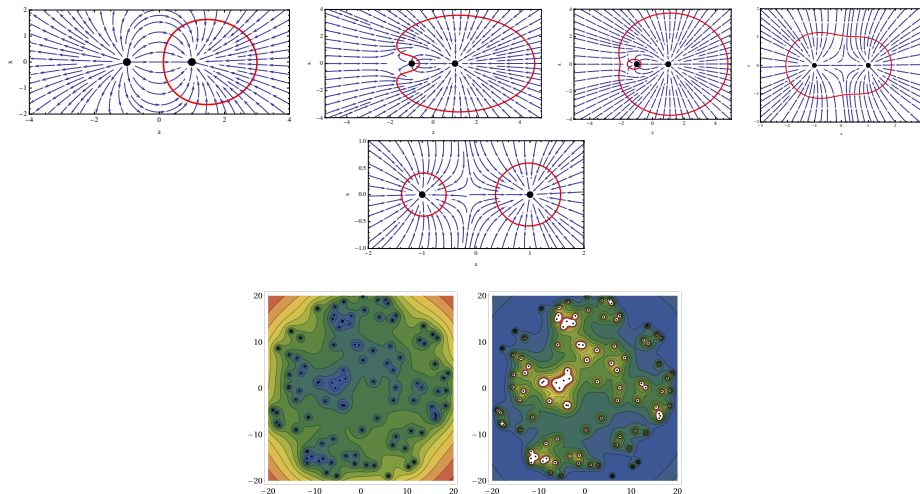
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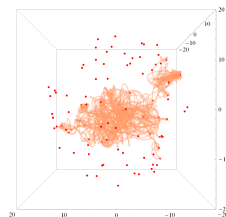
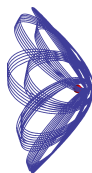
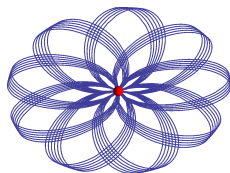


Quantum moduli space computed by finding lowest Landau degeneracy: formula for case of nonuniform B-field everywhere perpendicular to classical moduli space given by flux through surface.

Supergoop Dynamics

Dynamics of a probe particle in a two-centered fixed background is classically integrable! (Reminiscent of Euler-Jacobi 3-body problem; [noticed earlier in \[Nersessian, Ohanyan; Bellucci, Ohanyan; Krivonos, Nersessian, Ohanyan\]](#)). There exists a “hidden” conserved quantity that becomes apparent in prolate spheroidal coordinates.

Dynamics of probe in $N \geq 3$ fixed centers is (naturally) chaotic.



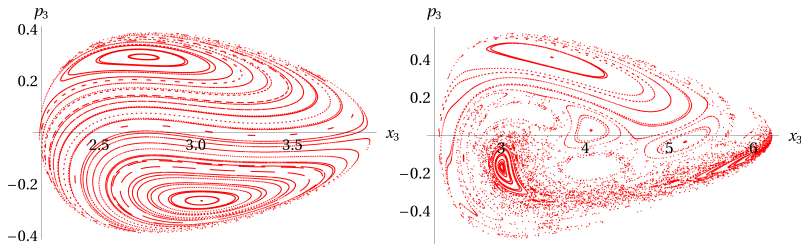
Collinear Supergoop - stringy double pendulum

Simplest setup: supergoop on a line (consistent truncation). Consider two light particles and one fixed center = 4 d.o.f. - 1 conserved quantity = 3. This is analogous to double pendulum, thus we can study Poincaré sections.

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Global chaos sets in before escape energy is reached. Closed loop case seems qualitatively different, at least sometimes.

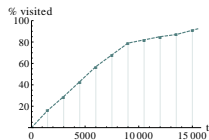
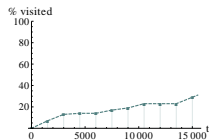
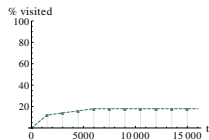
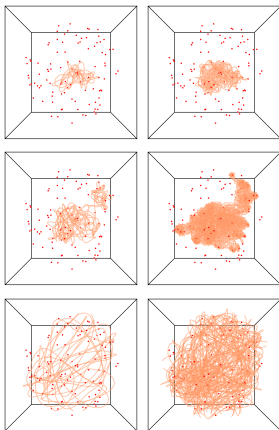


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Summary

- We are attempting to address whether glassiness is a natural language in the context of multicentered BHs, either holographically or directly in the bulk.
- Probe in background of two fixed centers is integrable.
- Classical and quantum degeneracies of probe in two fixed centers have been mapped out.
- Rough picture of transition to chaos.
- Trapping of classical trajectories.
- Many interesting questions remain!