RECONCILIATION OF HIGH ENERGY SCALE MODELS OF INFLATION WITH PLANCK

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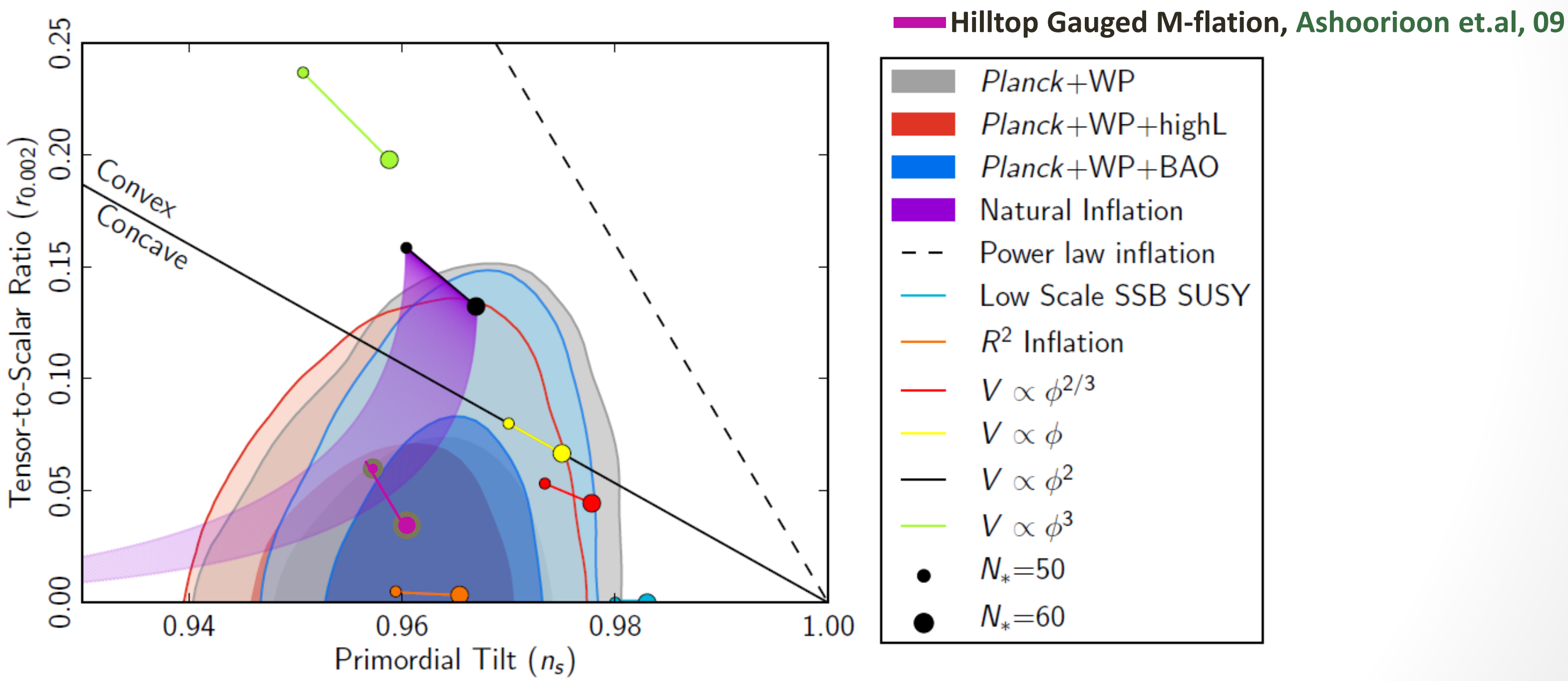
Introduction

The increasingly precise CMB measurements by Planck mission in combination with other cosmological date have ushered us into a precision early Universe cosmology era:

$$n_s \equiv \frac{d\log P_S}{d\log k} + 1 = 0.9603 \pm 0.0073$$

$$r \lesssim 0.11;$$

Planck 2013



One explanation for lack of observation of tensor modes is that gravity is a classical field!

Ashoorioon, B. Dev & A. Mazumdar (2013)

Concave vs. Convex?

- Steinhardt, Loeb (2013) argue that convex models like region (a) is more likely to cause inflation

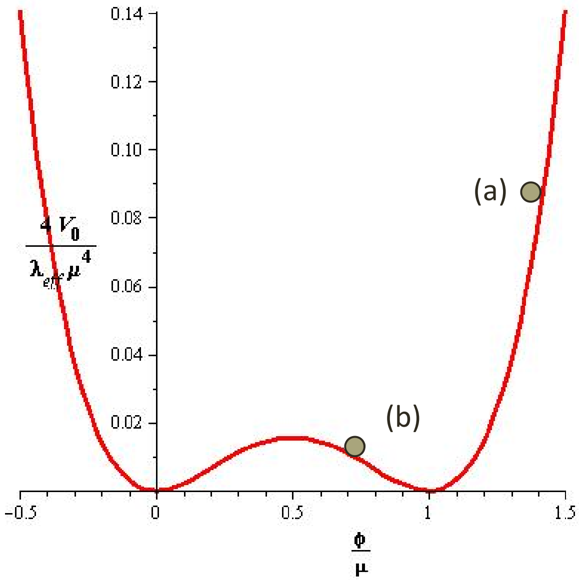
  - In convex models, region (a) once the universe was of Planckian size, M_{pl}^{-1}

$$\rho_G \equiv \frac{1}{2}(\nabla_i \phi)^2 \simeq M_{pl}^4 \qquad \text{decays like } a^{-2}$$

$$\rho_K \equiv \frac{1}{2}\dot{\phi}^2 \simeq M_{pl}^4 \qquad \text{decays like } a^{-6}$$

$$V(\phi) \simeq M_{pl}^4 \qquad \text{constant}$$

$V(\phi)$ comes to dominate the universe and slow-roll inflation starts.



- In concave models, region (b), once the universe was of Planckian size, M_{pl}^{-1},

$V(\phi) \ll \rho_G \simeq \rho_K$ They claim that Inflation is not likely!

Introduction

- CMB fluctuations are traced back to the random quantum fluctuations during inflation period.
- Quantum fluctuation are stretched to cosmological size due to the quasi-exponential expansion of inflation.
- Predictions of inflationary models for the CMB temperature fluctuations depend on the initial condition for fluctuations too.
- It is usually assumed the modes start from the Bunch-Davies (BD) vacuum

$$u_k \to \frac{e^{-ik\tau}}{\sqrt{2k}}, \qquad k\tau \to -\infty$$

- Predictions of inflationary models for the CMB temperature anisotropy depend also on the initial state of the quantum fluctuations.
  - Pre-inflation — Sarangi, Shiu & Van der Schaar (2005); Kinney, Powell (2006)
  - Trans-Planckian effects — Brandenberger et. al. (2000,2001), Ashoorioon et. al. (2004, 2005)

fluctuations start in an excited state, in a non-BD initial state.

Inflationary Setup and Fluctuations

- Let us consider the simplest single scalar field inflationary model:

$$\mathcal{L} = -\frac{M_{pl}^2}{2}\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

- While true in more general cases, we will focus on the quadratic chaotic model,

$$V = \frac{1}{2}m^2\phi^2$$

- The metric scalar and tensor perturbations are parameterized as

$$ds^2 = \mathrm{a}(\tau)^2\left[-(1+2\Phi)d\tau^2 + \left((1-2\Psi)\delta_{ij} + h_{ij}\right)dy^i dy^j\right] \qquad h^{\mathrm{i}}{}_i = \partial^i h_{ij} = 0$$

$$\phi(\tau) = \phi_{hom.} + \delta\phi \qquad \delta\phi \ll \phi_{hom.}$$

- Perturbed Einstein also implies $\Psi = \Phi$ in absence of anisotropic perturbations.
- The equation for gauge-invariant scalar perturbations

$$u_k = -z\left(-\frac{a'}{a}\frac{\delta\phi}{\phi'} + \Psi\right); \qquad \mathcal{H} = \frac{a'}{a} \qquad z = \frac{a\phi'}{\mathcal{H}},$$

$$u_k'' + \left(k^2 - \frac{z''}{z}\right)u_k = 0$$

- In a quasi-deSitter background

$$a(\tau) \simeq -\frac{1}{H\tau}$$

$$\epsilon \equiv 1 - \frac{\mathcal{H}'}{\mathcal{H}^2} \ll 1$$

$$\eta \equiv \epsilon - \frac{\epsilon'}{2\mathcal{H}\epsilon} \ll 1$$

the most generic solution to (4) in the leading order in slow-roll parameters

$$u_k(\tau) = \sqrt{\frac{\pi(-\tau)}{2}}\left[\alpha_k^S H_{\frac{3}{2}}^{(1)}(-k\tau) + \beta_k^S H_{\frac{3}{2}}^{(2)}(-k\tau)\right]$$

where the Bogoliubov coefficients satisfy the Wronskian condition

$$|\alpha_k^S|^2 - |\beta_k^S|^2 = 1$$

- The energy carried away from the inflationary background by the frozen-out perturbations in an e-fold is

$$\delta\rho_0 \sim H^4$$

on the other hand, the change in the background energy density in an e-fold is

$$\Delta\rho_0 \sim \epsilon\rho_0 \sim \epsilon H^2 M_{pl}^2$$

$$\delta\rho_0 \ll \Delta\rho_0 \longrightarrow \frac{H^2}{M_{pl}^2\epsilon} \ll 1$$

which is true recalling COBE normalization!

Bounds on $|\beta_k|$from backreaction

- Any excited state contains massless quanta whose positive pressure can derail Inflation

$$\delta\rho_{non-BD} \sim \delta p_{non-BD} \sim \frac{1}{a(\tau)^4}\int_H^\infty \frac{d^3k}{(2\pi)^3} k\left|\beta_k^S\right|^2$$

$$\delta p'_{non-BD} \sim \delta\rho'_{non-BD} \sim \mathcal{H}\delta\rho_{non-BD}$$

- From the background E.O.M,

$$\dot{H} = -\epsilon H^2 = \frac{(p+\rho)}{2M_{pl}^2} \qquad \ddot{H} = 2\epsilon\eta H^3 = -\frac{(\dot{p} - 3H(p+\rho))}{2M_{pl}^2}$$

one can see that we can avoid derailing the slow-roll inflation if

$$\delta\rho_{non-BD} \lesssim \epsilon\rho_0$$

$$\delta p'_{non-BD} \lesssim \eta\epsilon\mathcal{H}\rho_0$$

The second equation, which is the strongest one, can be written as

$$\int_H^\infty \frac{d^3k}{(2\pi)^3} k\left|\beta_k^S\right|^2 \lesssim \epsilon\eta H^2 M_{pl}^2$$

Bounds on $|\beta_k|$from backreaction

- As a specific example, let us consider the crude model in which

$$\beta_k \simeq \beta_0^S \exp\left(-\frac{k^2}{a(\tau)^2 M^2}\right)$$

D. Boyanovsky, H. J. de Vega, N. G. Sanchez (2006)

or any smooth function in which $|\beta_k|^2$ falls off faster than $k^{-(4+\delta)}$.

This choice corresponds to the situation that new physics kicks in when

$$\frac{k}{a(\tau_0)} = M$$

- Using the above model:

$$\delta\rho_{non-BD} \sim \left|\beta_0^S\right|^2 M^4 \qquad \frac{\delta p'_{non-BD}}{\mathcal{H}} \sim \left|\beta_0^S\right|^2 M^4$$

one obtains the following bound on β_0

$$\beta_0^S \lesssim \frac{\sqrt{\epsilon\eta}\, H M_{pl}}{M^2} \simeq \frac{\epsilon H M_{pl}}{M^2}$$

- As we will see β_0 is not necessarily small.

Effect of an Excited State in the Power Spectra: Scalar Spectrum

- Scalar power spectrum

$$P_S = \frac{k^3}{2\pi^2}\left|\frac{u_k}{z}\right|_{\frac{k}{\mathcal{H}}\to 0}$$

$$P_S = P^S_{B.D.}\gamma_S \qquad P^S_{B.D.} = \frac{1}{8\pi^2}\left(\frac{H}{M_{pl}}\right)^2 \qquad \gamma_S = \left|\alpha^S_k - \beta^S_k\right|^2_{k=\mathcal{H}}$$

$$n_S - 1 \equiv \frac{d\,\ln P_S}{d\ln k}$$

- Choice of new physics in general can affect the tilt, but with our choice of new physics

$$\beta_k \simeq \beta_0 \exp\left(-\frac{k^2}{a(\tau)^2 M^2}\right)$$

there will be no modification to the spectral tilt.

Effect of an Excited State in the Power Spectra: Tensor Spectrum

$$P_{ij}(\boldsymbol{x},\tau) = a(\tau)h_{ij}(\boldsymbol{x},\tau) \qquad p_k''(\tau) + \left(k^2 - \frac{a''}{a}\right)p_k(\tau) = 0$$

During inflation the universe is in a quasi-de-Sitter space-time $a(\tau) = -\frac{1}{H\tau}$

$$p_k(\tau) = \sqrt{\frac{\pi(-\tau)}{2}}\left[\alpha_k^T H_{\frac{3}{2}}^{(1)}(-k\tau) + \beta_k^T H_{\frac{3}{2}}^{(2)}(-k\tau)\right]$$

Wronskian condition: $\left|\alpha_k^T\right|^2 - |\beta_k^T|^2 = 1$

$$P_T = P_{BD}^T \gamma_T \qquad P_{BD}^T = \frac{2}{\pi^2}\left(\frac{H}{M_{pl}}\right)^2 \qquad \gamma_T = |\alpha_T - \beta_T|^2_{k=\mathcal{H}}$$

$$r \equiv \frac{P_T}{P_S} = 16\gamma\epsilon \qquad \gamma \equiv \frac{\gamma_T}{\gamma_S} = \left.\frac{\left|\alpha_k^T - \beta_k^T\right|^2}{\left|\alpha_k^S - \beta_k^S\right|^2}\right|_{k=\mathcal{H}}$$

- The tensor initial state parameters α_T and β_T and those of the scalars are taken to be independent.

$$\int_H^\infty \frac{d^3k}{(2\pi)^3} k \left|\beta_k^T\right|^2 \lesssim \epsilon\eta H^2 M_{pl}^2 \quad \Longrightarrow \quad \beta_0^T \lesssim \frac{\sqrt{\epsilon\eta}\, H M_{pl}}{M^2} \simeq \frac{\epsilon H M_{pl}}{M^2}$$

Violation of the Consistency Relation

- Series of consistency relations relate the tensor and scalar power spectra in single field inflation.

Starobinsky (1985), Stewart & Lyth (1993), Abney, Copeland, Liddle, Kolb (1995)

- The lowest order one takes the form

$$r \equiv \frac{P_S}{P_T} = -16\epsilon = 8n_T$$

- In general, with arbitrary excited initial states for tensor and scalar fluctuations

$$r \equiv \frac{P_S}{P_T} = -16\gamma\epsilon = 8\gamma n_T, \qquad \gamma = \frac{\left|\alpha_k^T - \beta_k^T\right|^2}{\left|\alpha_k^S - \beta_k^S\right|^2}$$

- In Hui & Kinney (2001), different initial conditions between tensor and scalar fluctuations was identified as a signature of short distance physics.

- However large effects on r were dismissed assuming that $\left|\beta_k^T\right|, \left|\beta_k^S\right| \ll 1$

Parameterization of the Parameter Space

- From the Wronskian conditions

$$\begin{cases} \alpha_k^S = \cosh\chi_S\, e^{i\varphi_S} & \alpha_k^T = \cosh\chi_T\, e^{i\varphi_T} \\ \beta_k^S = \cosh\chi_S\, e^{-i\varphi_S} & \beta_k^T = \cosh\chi_T\, e^{-i\varphi_T} \\ \chi_S \simeq \sinh^{-1}\beta_0^S & \chi_T \simeq \sinh^{-1}\beta_0^T \end{cases} \longrightarrow \begin{cases} e_S^{-2\chi_S} \leq \gamma_S \leq e_S^{2\chi_S} \\ e_S^{-2\chi_T} \leq \gamma_T \leq e_S^{2\chi_T} \end{cases}$$

- Using the COBE normalization one obtains:

$$\frac{H}{M_{pl}} \simeq \frac{1}{\sqrt{\gamma_S}} 3.78 \times 10^{-5}$$

that with the help of backreaction condition, $\beta_0^S \leq \frac{\epsilon H M_{pl}}{M^2}$, yields

$$\frac{M^2}{H^2} \lesssim 220 \frac{\sqrt{\gamma_S}}{\sinh\chi_S}$$

- Assuming the scale of new physics for tensor and scalar perturbations to be the same:

$$\frac{M^2}{H^2} \lesssim 220 \frac{\sqrt{\gamma_S}}{\sinh\chi_T}$$

Analysis of the Parameter Space

- For $\chi_T = \chi_S = \chi, \varphi_S = \frac{\pi}{2}, \varphi_T = 0$

$$\frac{M^2}{H^2} \lesssim 440 \frac{1}{1-\sqrt{\gamma}},$$

$$\gamma = e^{-4\chi} \leq 1$$

$\gamma \leq 0.5$ ⟶ $M \lesssim 39\, H;\quad H = 3.78 \times 10^{-5} \gamma^{\frac{1}{4}} M_{pl}$

By decreasing γ one can decrease H from its Bunch-Davies value.

Quasi-BD region, $\chi_S \ll 1$ and general φ_S:

- M can be arbitrary large
- H is very close to its Bunch-Davies value

Typical or large values of χ_S, $\chi_S \geq 1$:

- $\sqrt{\gamma_S} \simeq e^{\chi_S} \sin(\varphi_S)$
- $\sinh \chi_S \simeq \frac{e^{\chi_S}}{2}$
- generic values of φ_S

⟹

- $M \lesssim 21H$ — *with large deviations from BD vacuum, the scale of new physics cannot be arbitrarily larger than H!*
- $H \leq H_{BD}$ — *H can be arbitrarily smaller than its B.D. value #* $\simeq 3.78 \times 10^{-5}$

General Analysis of the Parameter Space

- To make sure that the effective field theory is applicable

$$M \gtrsim H \quad \Longrightarrow \quad \varphi_S \gtrsim 10^{-3}$$

- Desirable value of $M \simeq 20\,H$ is obtained if $\varphi_S \simeq \frac{\pi}{2}$, which will assume now on.

- Backreaction constraint on tensor perturbations

$$\beta_0^T \leq \frac{\sqrt{\epsilon\eta}\,H M_{pl}}{M^2} \simeq \frac{\epsilon H M_{pl}}{M^2} \Longrightarrow \frac{M^2}{H^2} \lesssim 220\,\frac{\sqrt{\gamma_S}}{\sinh\chi_T} \xrightarrow{M \,\simeq\, 20\,H} 2\sinh\chi_T \lesssim \sqrt{\gamma_S} \simeq e^{\chi_S}\sin\varphi_S$$

  - χ_T can be either in the quasi-BD range or typical and large range.

- Depending on the value of χ_T and φ_T one of the following three regions could occur

$$\gamma \simeq \begin{cases} \dfrac{e^{-2\chi_S}}{\sin^2\varphi_S}, & \chi_T \ll 1 \\[2ex] \dfrac{e^{2(\chi_T-\chi_S)}\sin^2\varphi_T}{\sin^2\varphi_S}, & \chi_T \gtrsim 1, \text{ typical } \varphi_T \\[2ex] \dfrac{e^{-2(\chi_T+\chi_S)}}{\sin^2\varphi_S} & \chi_T \gtrsim 1, \tan\varphi_T \lesssim e^{-2\chi_T} \end{cases}$$

as long as the bounds from backreaction are respected $\Longrightarrow$ $\gamma \lesssim 1$

General Analysis of the Parameter Space and the effect on the Lyth Bound

- *suppression of backreaction of scalar and tensor non-BD excitation results in suppression of tensor-to-scalar ratio compared to its BD value,*

- Backreaction considerations sets an upper bound on H

$$H \lesssim 3.78 \times 10^{-5}$$

- For $m^2\phi^2$ model, $\gamma < 0.5$ can bring the model into the 1σ region of the Planck results in the $n_S - r$ plane.

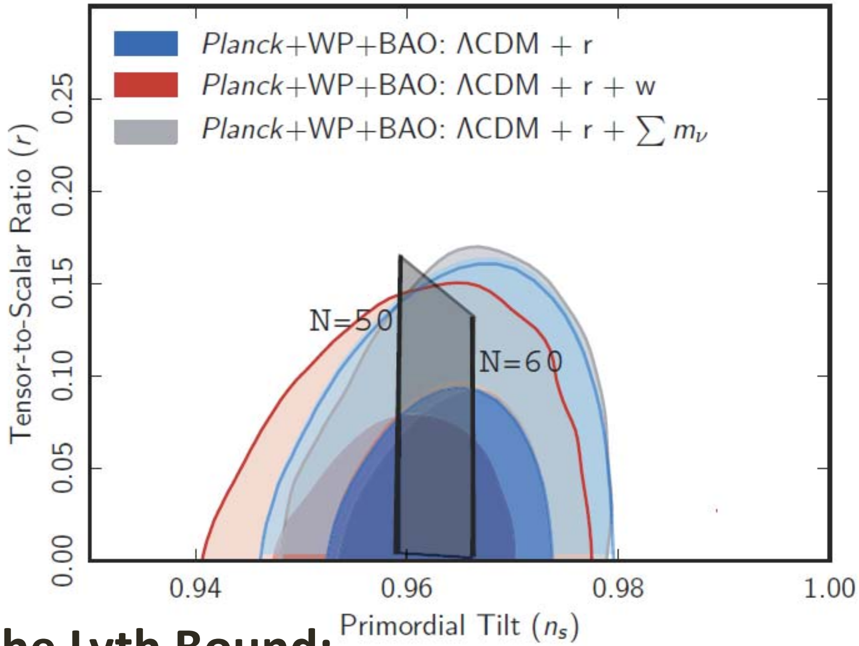
  - For $M \simeq 20\ H$ and $\varphi_S = \frac{\pi}{2}$, $\gamma = e^{2(\chi_T - \chi_S)} \sin^2 \varphi_T$
    - if $\chi_T = \chi_S$, φ_T for which $\sin^2 \varphi_T \lesssim \frac{1}{2}$ can do the job!
    - if $\chi_T < \chi_S$, the ratio can be lowered further.

- Change in the scalar spectral index due to k-dependence in φ_S:

$$\delta(n_S - 1)_{non-BD} \simeq \frac{\partial \ln \gamma_S}{\partial \ln k} \simeq 2 \cot \varphi_S \frac{\partial \varphi_S}{\partial \ln k} \simeq 0 \quad \text{for } \varphi_S \simeq \frac{\pi}{2}$$

- Shift in n_S due to the k-dependence of φ_S is small

General Analysis of the Parameter Space



❖ **Effect on the Lyth Bound:**

- Lyth (1997) bound relates the field excursion in the field space $\Delta\phi$ to r

$$r \lesssim 2.5 \times 10^{-3} \left(\frac{\Delta\phi}{M_{pl}}\right)^2$$

excited states modify it as

$$r \lesssim 2.5 \times 10^{-3} \left(\frac{\Delta\phi}{M_{pl}}\right)^2 \gamma \quad \xrightarrow{\gamma \lesssim 1} \quad r \lesssim r_{BD}$$

Compatibility with the Non-gaussianity Constraints from Planck

The bispectrum could be calculated using the in-in formalism

$$\mathcal{B} \equiv \left\langle \zeta_{\vec{k}_1}\zeta_{\vec{k}_2}\zeta_{\vec{k}_3} \right\rangle = \frac{\pi^3 H^4}{\epsilon M_{pl}^2 k_1^3 k_2^3 k_3^3}\mathcal{A}$$

$$\mathcal{A} \equiv \left[\frac{1-\cos(k_t\tau_0)}{k_t}C_1 + i\,\frac{\sin(k_t\tau_0)}{k_t}C_2 + C_3\sum_{i=1}^{3}\frac{1-\cos(\tilde{k}_i\tau_0)}{\tilde{k}_i} + i\,C_4\sum_{i=1}^{3}\frac{\sin(\tilde{k}_i\tau_0)}{\tilde{k}_i}\right]$$

$$\left(\textstyle\sum_{i<j} k_i^2 k_j^2\right)$$

$$k_t \equiv k_1 + k_2 + k_3$$
$$\tilde{k}_i \equiv k_t - 2k_i$$

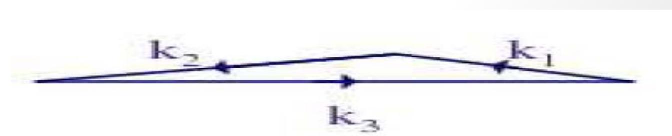
1. Enhancement of the nongaussianity for the flattened configuration, $k_1 + k_2 = k_3$

X. Chen et. al. (2006), Tolley et. al. (2007), Ashoorioon & G. Shiu (2011)

for typical or large χ_S, $\chi_S \gtrsim 1$

$$C_4 \simeq \sin 2\varphi_S \sin^2\varphi_S e^{4\chi_S} \xrightarrow{\varphi_S \simeq \frac{\pi}{2}} C_4 \simeq 0 \longrightarrow$$

no enhancement for the flattened configuration, even before the projection on the 2d CMB surface



Compatibility with the Non-gaussianity Constraints from Planck

1. Enhancement of the nongaussianity for the local configuration, $k_1 \simeq k_2 \gg k_3$

Parker & Agullo (2010)

$$\mathcal{A} \equiv \left[\frac{1-\cos(k_t\tau_0)}{k_t} C_1 + i\,\frac{\sin(k_t\tau_0)}{k_t} C_2 + C_3 \sum_{i=1}^{3} \frac{1-\cos(\tilde{k}_i\tau_0)}{\tilde{k}_i} + i\, C_4 \sum_{i=1}^{3} \frac{\sin(\tilde{k}_i\tau_0)}{\tilde{k}_i}\right] \left(\sum_{i<j} k_i^2 k_j^2\right)$$

$$k_t \equiv k_1 + k_2 + k_3$$
$$\tilde{k}_i \equiv k_t - 2k_i$$

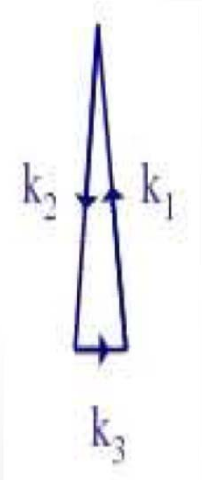
for typical or large χ_S, $\chi_S \gtrsim 1$

$$C_3 \simeq -\frac{e^{4\chi}}{2}\left(2\cos^4(\varphi_S) - 1 - \cos^2(\varphi_S)\right) \xrightarrow{\varphi_S \simeq \frac{\pi}{2}} C_3 \simeq \frac{e^{4\chi}}{2}$$

Since $\frac{H^4}{\epsilon} \propto P_S^2 {\gamma_S}^{-2}\epsilon \propto P_S^2 \epsilon e^{-4\chi_S}$ and $f_{NL} \simeq \frac{\mathcal{B}}{P_S^2}$, powers of $e^{4\chi_S}$ cancel.

$$f_{NL} \simeq \frac{C_3}{|\alpha_k^S - \beta_k^S|^4}\,\epsilon\,\frac{k_S}{k_L}$$

k_S: $\ell \simeq 2500$
k_L: $\ell \simeq 10$



Compatibility with the Non-gaussianity Constraints from Planck

- For $M \simeq 20\, H$, $\varphi_S \simeq \frac{\pi}{2}$ and large $\chi_S, \chi_S \gtrsim 1$

$$f_{NL} \simeq 0.43$$

The amount of NG in the local configuration is even within one sigma range allowed by Planck!

$$f_{NL}^{local} = 2.7 \pm 5.8$$

- Please note that only for $\varphi_S \gtrsim \frac{\pi}{10}$, f_{NL} goes beyond the 2σ level of the Planck data

Conclusion

- Lowering the scale of inflation, $\Lambda \equiv \left(M_{pl}^2 H^2\right)^{1/4}$, from $10^{16} GeV$ involves more fine-tuning of the potential.

$$\Lambda \simeq 10^{15}\ GeV \longrightarrow \epsilon \simeq 10^{-6}$$

fine-tuning of the potential!

- Predictions of inflationary models depend on the initial condition fluctuations.
- non-BD initial condition can be the window through which high scale physics can leave its imprint on the CMB.
- Small backreaction condition does not block large deviations from BD initial conditions.
- Amplitude of fluctuations could be altered by such initial conditions.

Conclusion

- Considering the back reaction constraints, change in the initial conditions for the tensor and scalar spectra in general tend to suppress the tensor/scalar ratio, r. This is obtained for moderate and large values of deviations.

- For large and moderate deviations from the BD vacuum, the scale of new physics, M, cannot go arbitrarily beyond H. For $m^2\phi^2$ model, $M \lesssim \text{few tens of } H$ for such deviations.

- Considering the backreaction constraints, the excited stated do not enhance the non-gaussianity.

Thank you