Can non-local or higher derivative theories provide alternatives to inflation?

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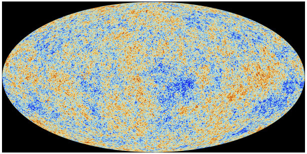
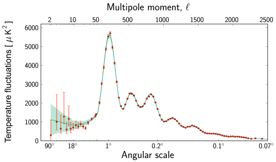
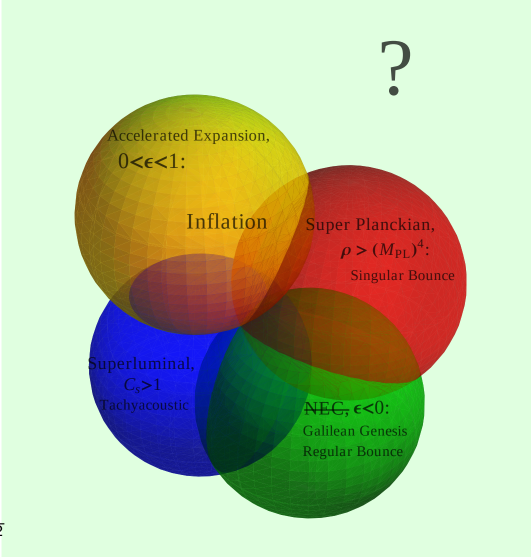
Goal: Are there any other alternatives out there?



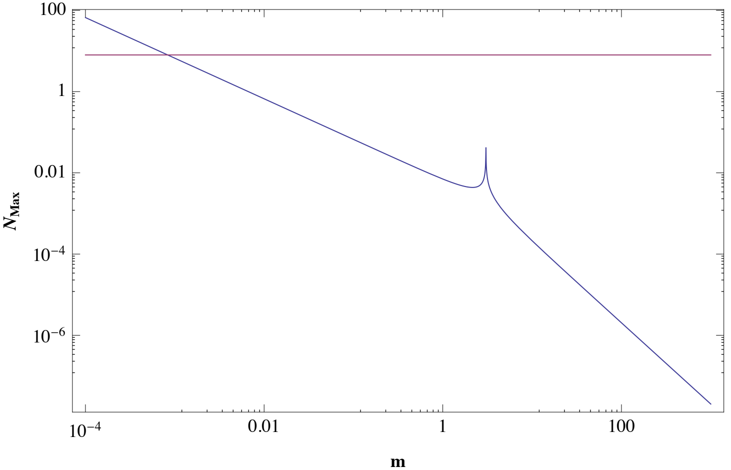


Planck Collaboration

$$\epsilon \equiv -\frac{\dot{H}}{H^2}$$



Summary of the Results [for $m = 1$ see Geshnizjani, Kinney, and Dizgah 2011]



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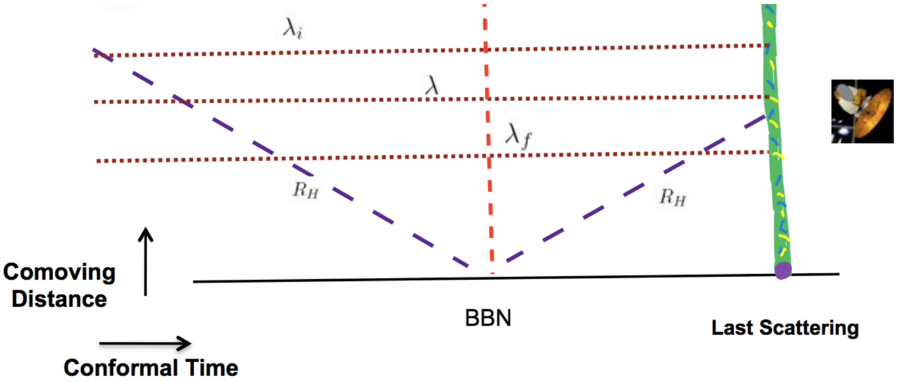
Observational Constraints

- Basic Properties of Primordial perturbations
  - Nearly scale-invariant for 1 Mpc $< \lambda <$ 3000 Mpc
  - Adiabatic
  - Super-Hubble at early times (1 Mpc $\sim 10^4 R_H(z_{BBN})$)
- Other constraints
  - Universe has become Radiation dominated by BBN ($T \sim 1$ MeV, $z \sim 4 \times 10^9$)

Theoretical framework

- Initial conditions: adiabatic vacuum
- Conservation of energy in four dimensions (continuity equation)
- Adiabatic perturbations
- Not many coincidences

How does Inflation work?



$R_H = \frac{1}{aH}$ is the comoving Hubble radius

This mechanism can be generalized to non-inflationary scenarios [Khoury and Piazza (2009)]

$$S_2 = \frac{M_{pl}^2}{2}\int d^3x\ d\tau\ z(\tau)^2\left[\left(\frac{\partial\zeta}{\partial\tau}\right)^2 - c_s(\tau)^2(\nabla\zeta)^2\right].$$

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Time reparametrization and change of variables: $dy = c_s d\tau$, $q \equiv z\sqrt{c_s}$ and $v \equiv M_{pl} \; q \; \zeta$

$$v_k'' + \left(k^2 - \frac{q''}{q} \right) v_k = 0.$$

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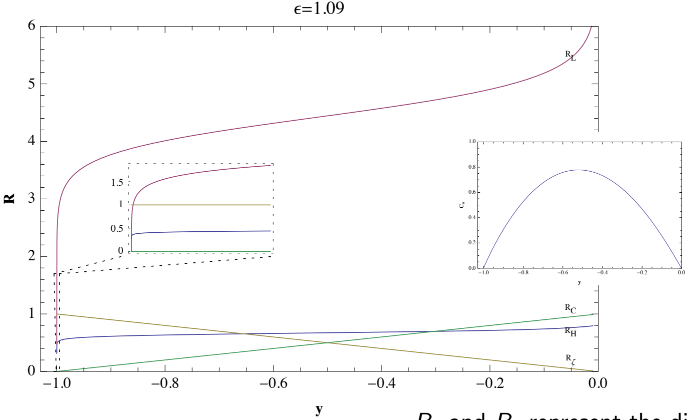
Freezing scale corresponds to :

$$\boxed{R_\zeta \simeq \sqrt{\frac{q}{q''}} \sim -y}$$

which is shrinking in time. In general $R_H \neq R_\zeta$

Different cosmological scales in scenarios with varying speed of sound!



1. Non-accelerating universe
2. scale invariant spectrum
3. subluminal propagation at all times

R_L and R_c represent the distances light and sound each travel since the Planck time.

R_ζ and R_H represent freezout and Hubble radii.

Obtaining scale-invariant spectrum for actions with non-standard spatial derivatives

Action in Fourier space:

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Consistent with $m = 1$, the standard case!

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- Substituting these in continuity equation for $\epsilon > 1$:

$$c_{\max} \ln \frac{\rho_i}{\rho_f} > 10^4 (e^{mN} - 1) \sqrt{|9 - m^2|}$$

Imposing:

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Observational evidence:

$$\lambda_i/\lambda_f \gtrsim 3 \times 10^3 \quad \text{or} \quad N > 8$$

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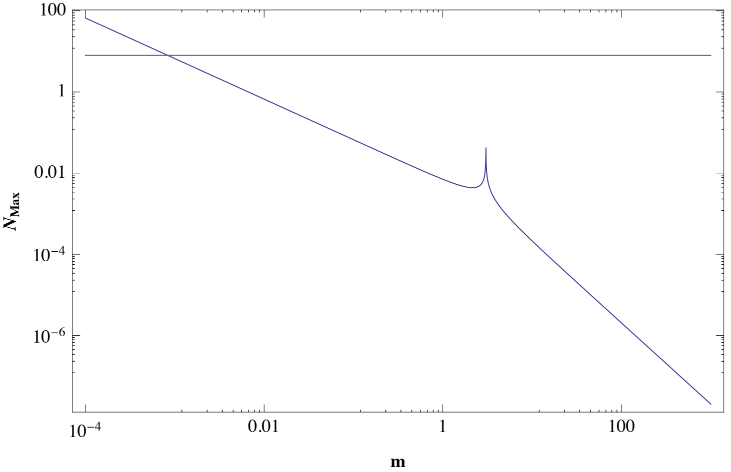

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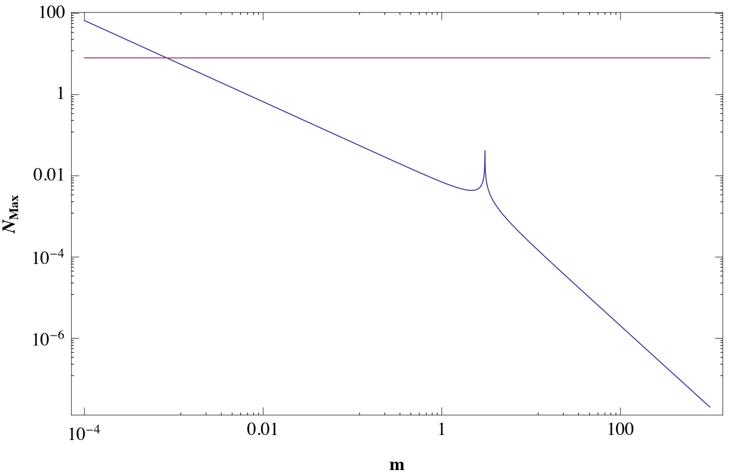

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The End