No-Scale Supergravity and Inflation after Planck

- Why Supersymmetry and Inflation
- Supergravity and the η problem
- No-Scale Supergravity
- Planck and $R+R^2$ inflation
- $R+R^2$ gravity and No-Scale Supergravity
- Stabilization

Old New Inflation

Great idea based on 1-loop corrected SU(5) potential for the adjoint:

$$V(\sigma) = A\sigma^4(\ln\frac{\sigma^2}{v^2} - \frac{1}{2})$$

$$A = \frac{5625}{1024\pi^2}g_5^4$$

Linde; Albrecht, Steinhardt

Problems:

- Vacuum structure
- destabilization through quantum fluctuations
- fine tuning (require curvature to be << M_X)
- density fluctuations - $\delta\rho/\rho \sim 100\, g_5^2$

How SUSY can help

Exact Susy - $V_{1\text{-loop}} = 0$

Broken Susy - $$A = \frac{75}{32\pi^2 v^2} g_5^2 m_s^2$$

where $m_s^2 = M_B^2 - M_F^2$

and $\frac{m_s^2}{v^2} \sim 2g_5 \frac{m_{3/2}}{v}$

Ellis, Nanopoulos, Olive, Tamvakis

fixes fine-tuning, $\delta\rho/\rho$, etc. - but isn't really a model

Hilltop-Inflation and WZ models

$$V = \lambda(\phi^2 - v^2)^2$$

require $\lambda \sim 10^{-12}$ for $\delta\varrho/\varrho$
and $(v/M_P)^2 > 65/2\pi$ for slow roll

Linde; Albrecht, Brandenberger

Easily obtained in a WZ model with $W = \frac{\mu}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$

Croon, Ellis, Mavromatos

in a globally supersymmetric model with $V = |\frac{dW}{d\Phi}|^2$

Supergravity

Start with a Kähler Potential

$$G = K + \ln|W|^2$$

Minimal N=1 defined by K = $\phi^i\phi_i^*$

and scalar potential

$$V = e^G\left[G_i(G^{-1})^i_j G^j - 3\right] + \mathrm{D-terms}$$

or

$$V = e^{\phi^i\phi_i^*}\left[|\frac{\partial W}{\partial\phi^i} + \phi_i^* W|^2 - 3|W|^2\right] + \mathrm{D-terms}$$

for minimal N=1

Typically, m^2 ~ H^2

η-problem!

Supergravity

Constructing Models $W = \mu^2 \sum_n \lambda_n \phi^n$

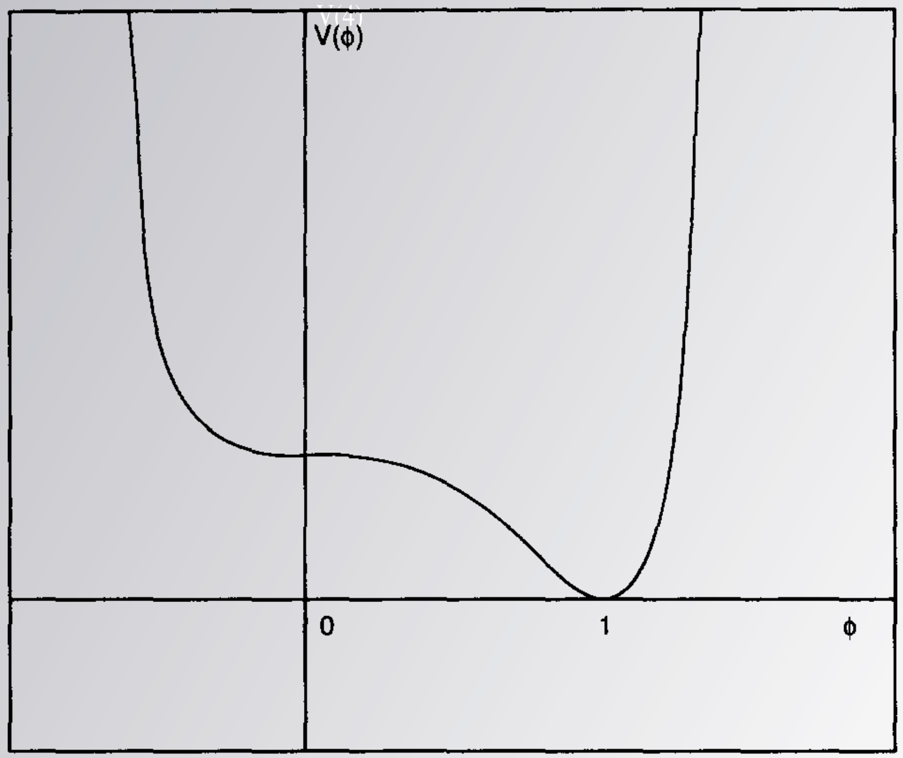
Nanopoulos, Olive, Srednicki, Tamvakis

μ^2 fixed by amplitude of density fluctuations, $\lambda_n \sim \mathcal{O}(1)$

Simplest example, $W = \mu^2 (1 - \phi)^2$

Holman, Ramond, Ross

$$\begin{aligned} V &= \mu^4 e^{|\phi|^2} \Big[1 + |\phi|^2 - (\phi^2 + \phi^{*2}) - 2|\phi|^2(\phi + \phi^*) \\ &\quad +5|\phi^2|^2 + |\phi|^2(\phi^2 + \phi^{*2}) - 2|\phi^2|^2(\phi + \phi^*) + |\phi^3|^2 \Big] \\ &\simeq \mu^4 (1 - 4\phi^3 + \frac{13}{2}\phi^4 + \cdots \end{aligned}$$



Supergravity

Generic Models

$$K = SS^* + (\phi - \phi^*)^2 + \cdots$$

with $W = Sf(\Phi)$

resulting in $V = |f(\phi)|^2$

Kawasaki, Yamaguchi, Yanagida; Kallosh, Linde, Rube; Kallosh, Linde, Olive, Rude

Easily generates any potential
(which is a perfect square)

No-Scale Supergravity

Natural vanishing of cosmological constant (tree level) with the supersymmetry scale not fixed at lowest order. (Also arises in generic 4d reductions of string theory.)

$$K = -3\ln(T + T^* - \phi^i {\phi_i}^*/3)$$

$$V = e^{\frac{2}{3}K} \left| \frac{\partial W}{\partial \phi^i} \right|^2$$

Globally supersymmetric potential once K (canonical) picks up a vev

No-Scale Supergravity

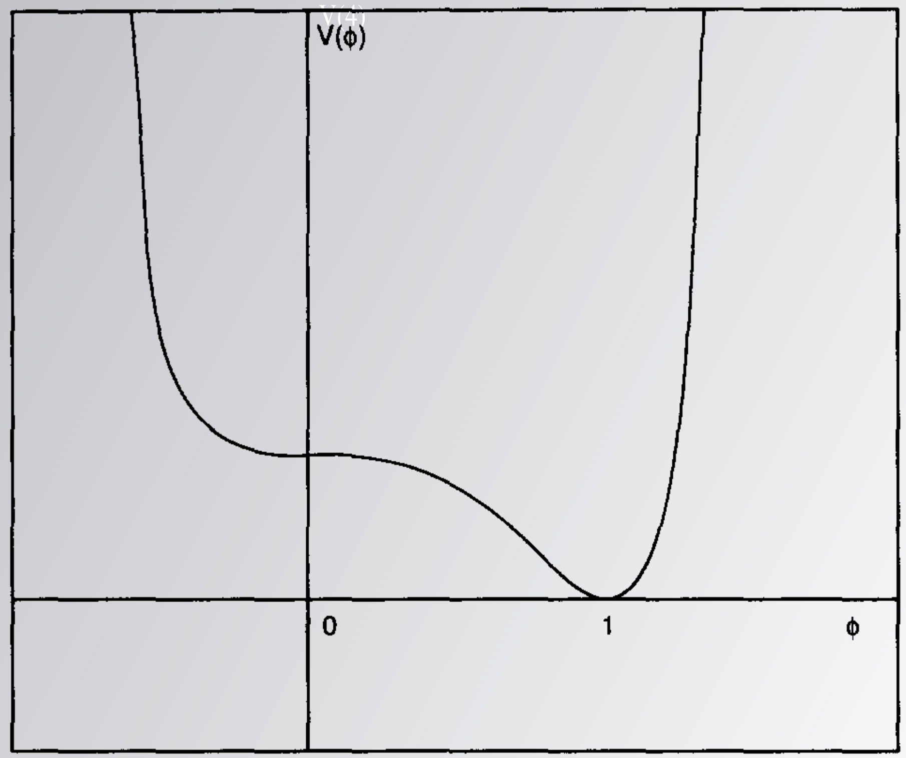
Constructing Models $W = \mu^2 \sum_n \lambda_n \phi^n$

Ellis, Enqvist, Nanopoulos, Olive, Srednicki

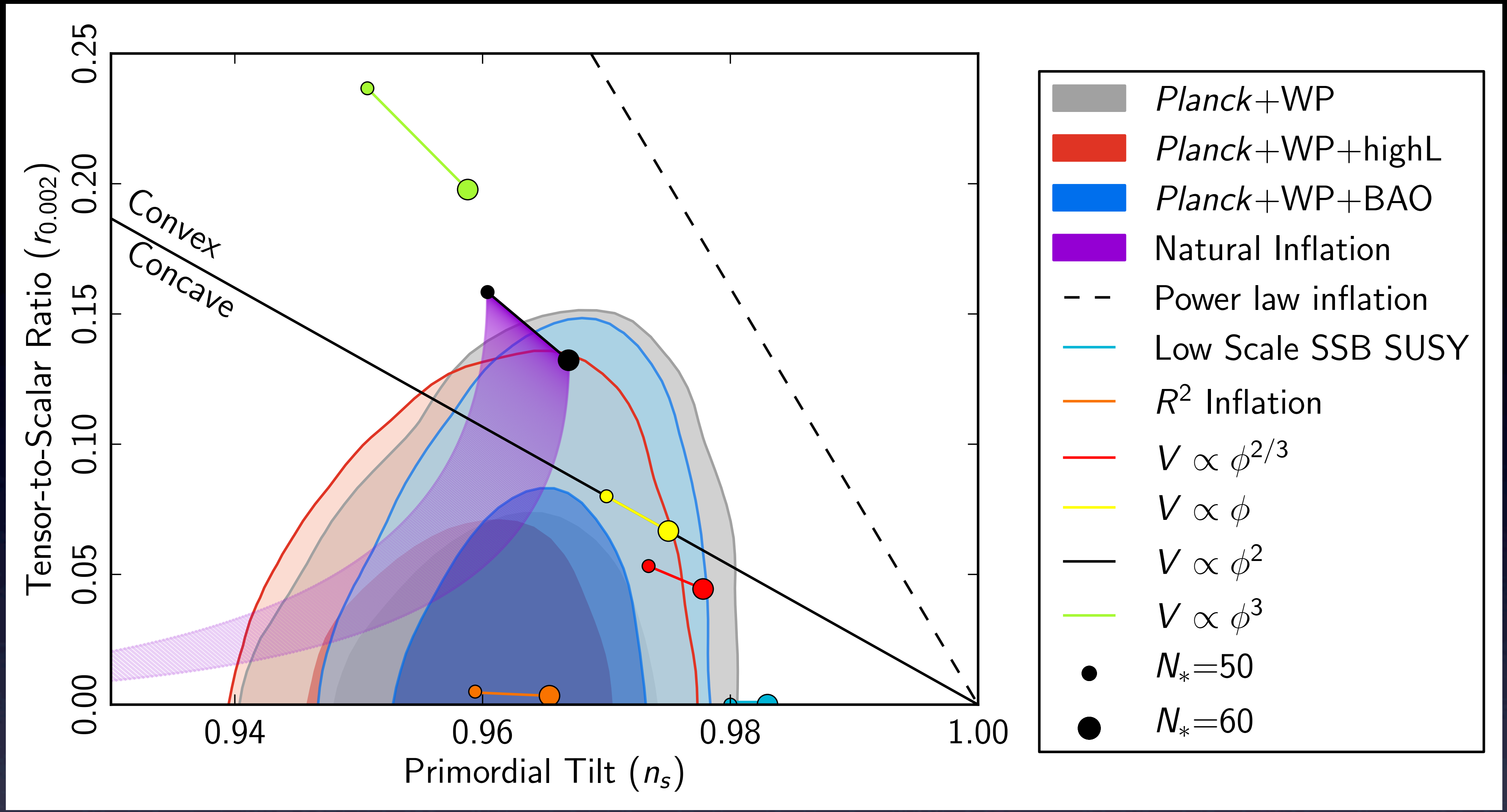
μ^2 fixed by amplitude of density fluctuations, $\lambda_n \sim \mathcal{O}(1)$

Simplest example, $W = \mu^2(\phi - \phi^4/4)$

$$V = \mu^4 |1 - \phi^3|^2$$



Planck Results



$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta = \frac{V''}{V} \qquad n_s = 1 - 6\epsilon + 2\eta \qquad r = 16\epsilon$$

Trouble for simple supergravity and no-scale models:

$$\epsilon \simeq \frac{1}{72N^4} \ll 1$$ OK

$$\eta \simeq -\frac{1}{2N}$$ not OK

$$n_s \sim .933$$

R+R^2 Inflation

$$S = \frac{1}{2}\int d^4x\sqrt{-g}(R + R^2/6M^2)\,,$$

Starobinsky

where M ≪ M$_P$

With $\tilde{g}_{\mu\nu} = (1 + \varphi/3M^2)g_{\mu\nu}$

and $\varphi' = \sqrt{\frac{3}{2}}\ln\left(1 + \frac{\varphi}{3M^2}\right)$

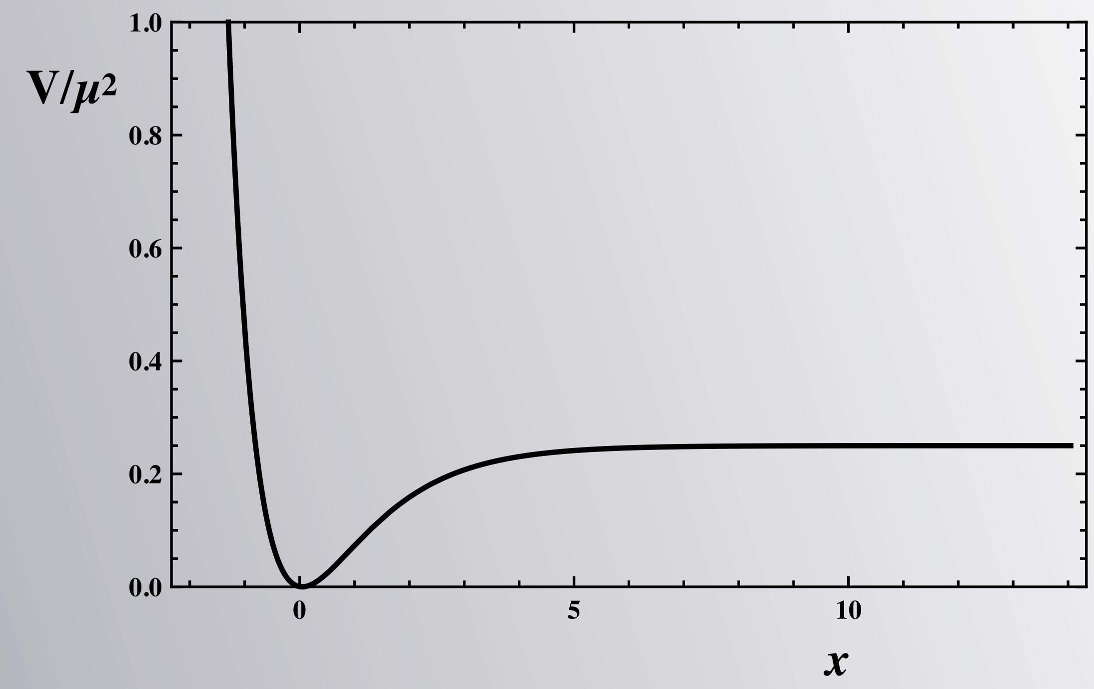
conformally equivalent to:

$$S = \frac{1}{2}\int d^4x\sqrt{-\tilde{g}}\left[\tilde{R} + (\partial_\mu\varphi')^2 - \frac{3}{2}M^2(1 - e^{-\sqrt{2/3}\varphi'})^2\right]$$

$$V = \frac{3}{4}M^2(1 - e^{-\sqrt{2/3}\varphi'})^2$$

R+R^2 Inflation

$$V = \frac{3}{4}M^2(1 - e^{-\sqrt{2/3}\varphi'})^2$$
$$= \mu^2 e^{-\sqrt{2/3}x}\sinh^2(x/\sqrt{6})$$



$x=\varphi/M_P$, $\mu^2 = 3M^2$

Slow Roll parameters:

$$\epsilon = \frac{1}{3}\text{csch}^2(x/\sqrt{6})e^{-\sqrt{2/3}x},$$
$$\eta = \frac{1}{3}\text{csch}^2(x/\sqrt{6})\left(2e^{-\sqrt{2/3}x} - 1\right)$$

μ is again set by the normalization of the quadrupole

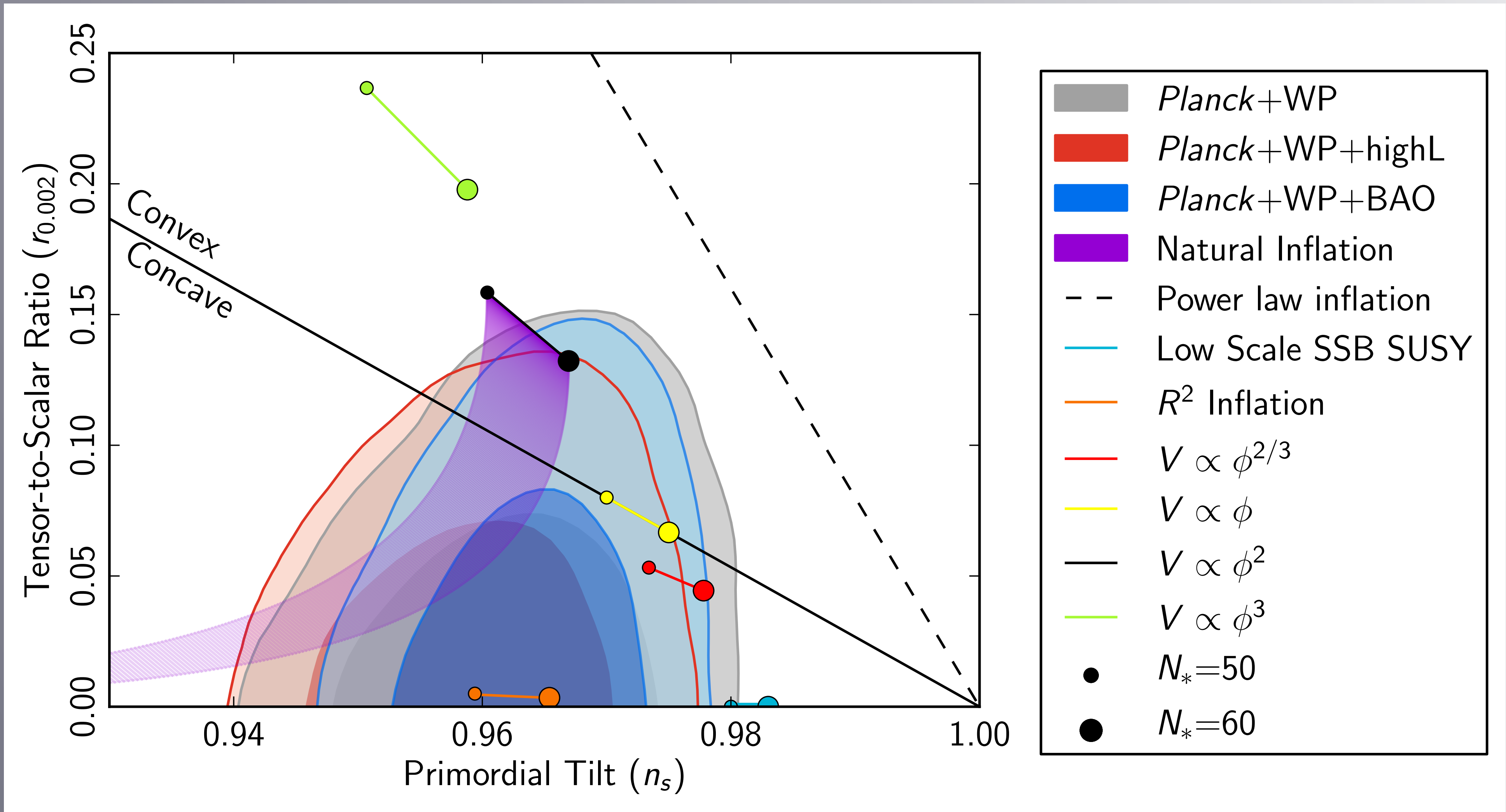
$$A_s = \frac{V}{24\pi^2\epsilon} = \frac{\mu^2}{8\pi^2}\sinh^4(x/\sqrt{6})$$

$\Longrightarrow \mu = 2.2 \times 10^{-5}$ for $N = 55$

$x_i = 5.35$

R+R^2 Inflation

For N=55, n_s = 0.965; r = .0035



No-Scale models revisited

Can we find a model consistent with Planck?

Ellis, Nanopoulos, Olive

Start with WZ model again: $W = \frac{\hat{\mu}}{2}\Phi^2 - \frac{\lambda}{3}\Phi^3$

Assume now that T picks up a vev: 2<Re T> = c

$$\mathcal{L}_{eff} \; = \; \frac{c}{(c - |\phi|^2/3)^2}|\partial_\mu \phi|^2 - \frac{\hat{V}}{(c - |\phi|^2/3)^2}$$

Redefine Inflaton to a canonical field χ

$$\phi = \sqrt{3c}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$$

No-Scale models revisited

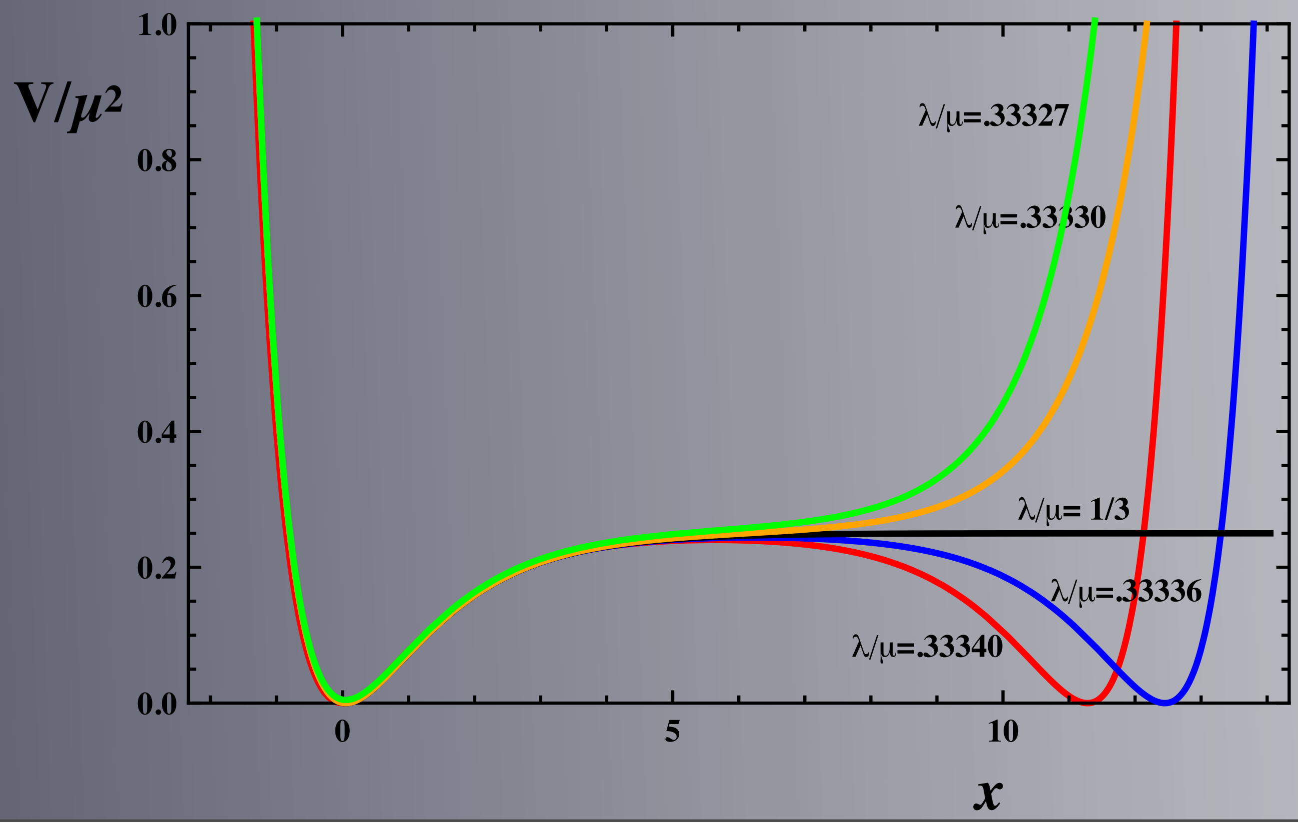
The potential becomes:

$$V = \mu^2 \left| \sinh(\chi/\sqrt{3}) \left(\cosh(\chi/\sqrt{3}) - \frac{3\lambda}{\mu} \sinh(\chi/\sqrt{3}) \right) \right|^2$$

$$\hat{\mu} = \mu\sqrt{(c/3)}$$

For λ=μ/3, this is exactly the R + R^2 potential

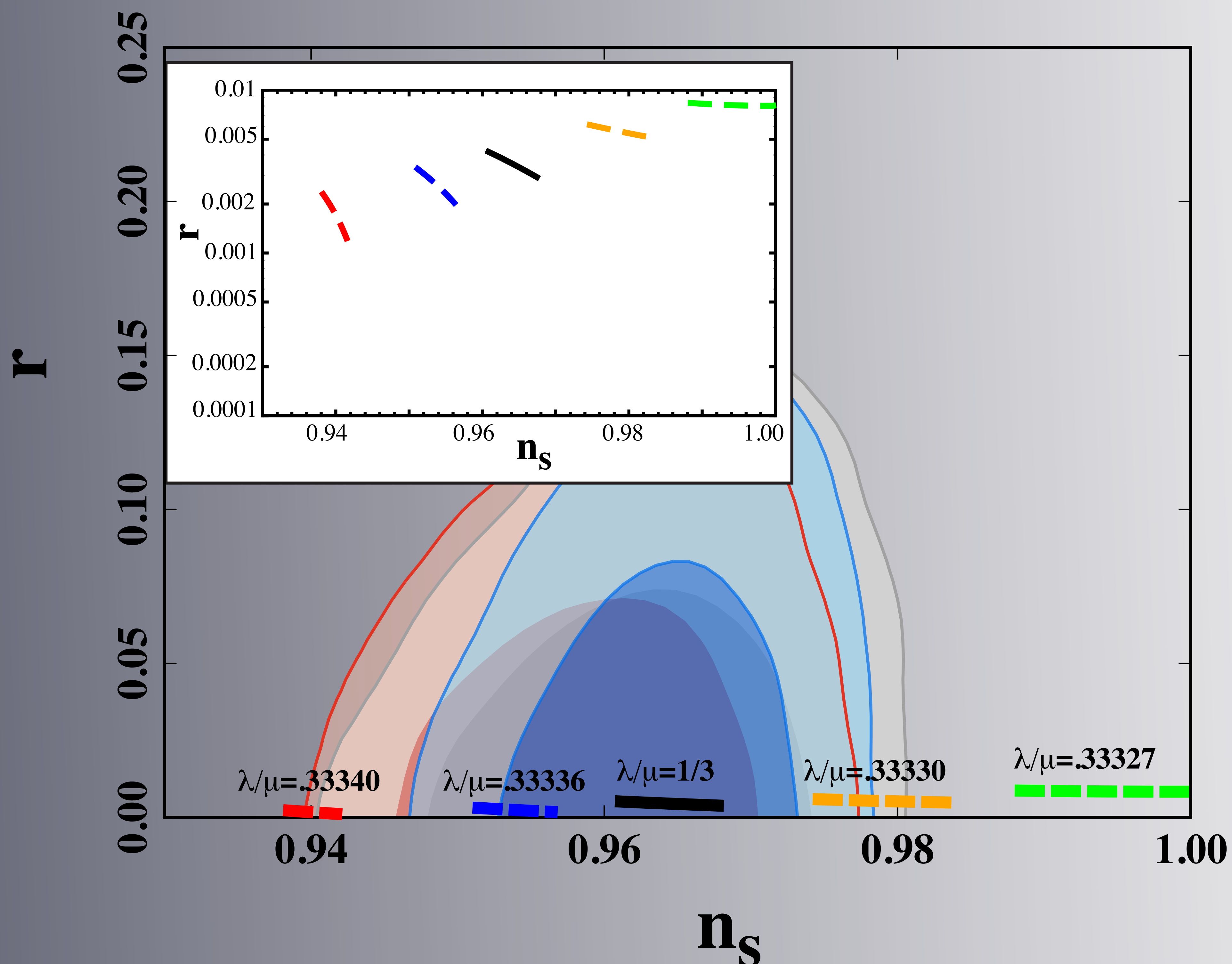
$$V = \mu^2 e^{-\sqrt{2/3}x} \sinh^2(x/\sqrt{6})$$



$$\chi = (x + iy)/\sqrt{2}$$

No-Scale models revisited

How well does this do vis a vis Planck?



Classes of R+R^2 in No-Scale Supergravity

Utilizing the no-scale symmetry, we can write

Ellis, Nanopoulos, Olive

$$K = -3\ln\left(1 - \frac{|y_1|^2 + |y_2|^2}{3}\right)$$

$$y_1 = \left(\frac{2\phi}{1+2T}\right) \; ; \; y_2 = \sqrt{3}\left(\frac{1-2T}{1+2T}\right)$$

or

$$T = \frac{1}{2}\left(\frac{1 - y_2/\sqrt{3}}{1 + y_2/\sqrt{3}}\right) \; ; \; \phi = \left(\frac{y_1}{1 + y_2/\sqrt{3}}\right)$$

with

$$W(T,\phi) \rightarrow \widetilde{W}(y_1, y_2) = \left(1 + y_2/\sqrt{3}\right)^3 W$$

Classes of R+R^2 in No-Scale Supergravity

So is the inflaton T or ϕ?

1) T-fixed (ϕ-inflaton) or y_2-fixed y_1-inflaton

Starobinsky potential found when

$$\hat{V} = M^2|\phi|^2|1-\phi/\sqrt{3}|^2$$

with field redefinition $(y_i, \phi) = \sqrt{3}\tanh\left(\frac{\chi}{\sqrt{3}}\right)$

2) ϕ-fixed (T-inflaton)

Starobinsky potential found when

$\hat{V} = 3M^2|T-1/2|^2$ or $\hat{V} = 12M^2|T|^2|T-1/2|^2$

with field redefinition $T = \frac{1}{2}e^{2\chi/\sqrt{3}}$ or T $\rightarrow$ 1/(4T)

Classes of R+R^2 in No-Scale Supergravity

Example 1:

$$W = M\left[\frac{y_1^2}{2}\left(1+\frac{y_2}{\sqrt{3}}\right)-\frac{y_1^3}{3\sqrt{3}}\right]$$

Ellis, Nanopoulos, Olive

or $$W = M\left[\frac{\phi^2}{2}-\frac{\phi^3}{3\sqrt{3}}\right]$$

or (reversing y_1 and y_2) $$W = \frac{M}{4}(T-1/2)^2(1+10T+2\sqrt{3}\phi)$$

Example 2:

$$W = My_1y_2(1+y_2/\sqrt{3})$$

or $$W = \sqrt{3}M\phi(T-1/2)$$

Cecotti; Linde, Kallosh

or (reversing y_1 and y_2) $$W = M\left[\sqrt{3}(T^2-1/4)\phi+(T-1/2)\phi^2\right]$$

⋮

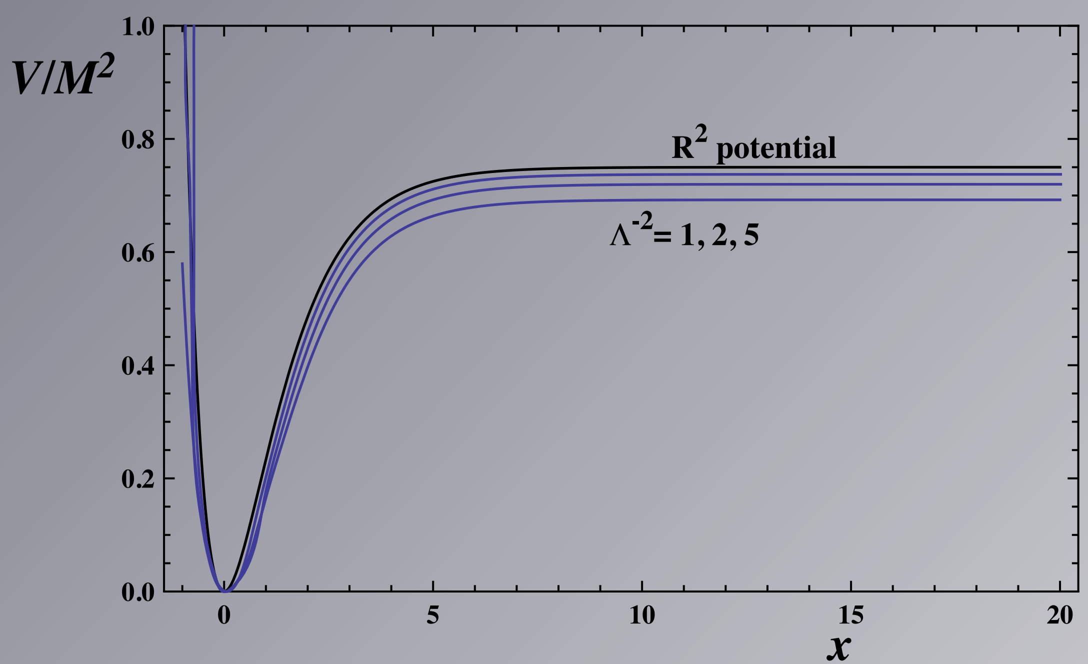
Stabilization

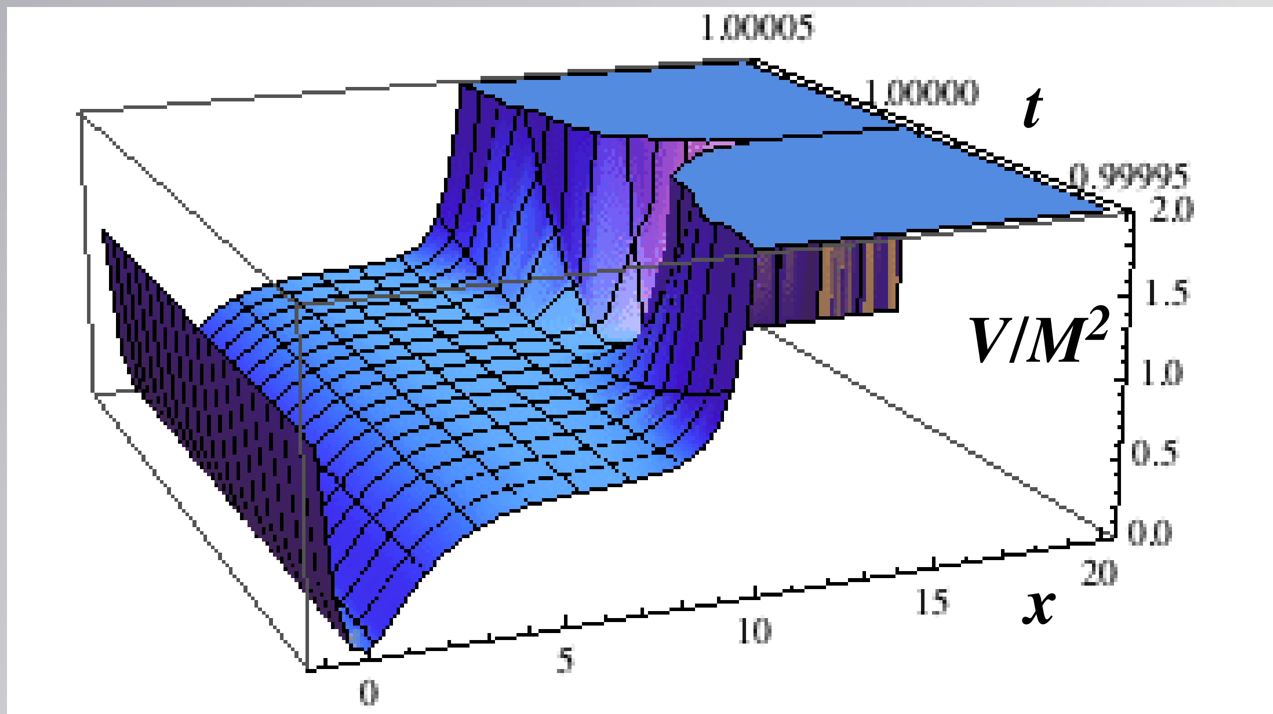
Have so far *assumed* a vev for one of the two fields

$$K = -3\ln\left(1 - \frac{|y_1|^2 + |y_2|^2}{3} + \frac{|y_2|^4}{\Lambda^2}\right), \quad \Lambda < M_P$$

or

$$K = -3\ln\left(T + T^* - \frac{|\phi|^2}{3} + \frac{(T+T^*-1)^4 + d(T-T^*)^4}{\Lambda^2}\right)$$





Summary

- Broad connection between $R+R^2$ models and no-scale supergravity.
- The Starobinsky model of inflation can be realized with either modulus T or ‘matter’ field ϕ with a simple WZ superpotential.
- Exact $R+R^2$ found for specific choices of couplings leading to $n_s = 0.965$ but $r \sim .003$ is rather generic.
- To do: supersymmetry breaking; connection to the standard model, reheating....