# $f(\phi)F^2$ mechanism and solid inflation: broken spatial invariance during inflation

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- Motivations
- Solid inflation
- The $\mathcal{L}_\phi - f(\phi)F^2$ model
- Prolonged anisotropic inflation
- Violation of $\lim_{q\to 0}\langle\zeta_{\vec{q}}\,\zeta_{\vec{k}}\,\zeta_{-\vec{k}}\rangle = -(n_s-1)\,P_\zeta(q)\,P_\zeta(k)$ both in amplitude and shape

- Effective field theory $\equiv$ description of a system through the lowest dim. $\hat{O}$ compatible with the underlying symmetry. Very fruitful in many areas of physics, including inflation

  Cheung et al '08

- Universe is statistically isotropic and homogeneous (barring anomalies). On the contrary, time translational invariance is broken.

$$\phi(x) = \phi^{(0)}(t + \pi(x)) \simeq \phi^{(0)}(t) + \partial_t \phi^{(0)} \cdot \pi(x)$$

EFT for $\pi \supset$ operators respecting spatial invariance. Measurements fix their coefficients

- Given improved data, consider broken spatial invariance. New operators and signatures.

- In this talk, 2 $\neq$ models with common features, not obtained in other models of inflation.

# Elastic / Solid inflation

Grizinov '04

Endlich, Nicolis, Wang '12

Medium driving inflation has FT description of a solid. Divide it in cells.

$\vec{\phi}\,(t, \vec{x})$ is the position at time $t$ of the cell that initially was at $\vec{x}$.

Solid at rest : $\phi^i\,(t, \vec{x}) = x^i$

Triplet of inflatons; to reconcile with homogeneity & isotropy:

$$B^{ij} \equiv g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \quad , \quad X \equiv \text{tr } B \;, \; Y \equiv \frac{\text{tr } B^2}{(\text{tr } B)^2} \;, \; Z \equiv \frac{\text{tr } B^3}{(\text{tr } B)^3} \quad , \quad \mathcal{L} = F\,[X, Y, Z]$$

(3 independent combs. Normalized so that only $X$ sensitive to volume)

$$\Rightarrow \quad \rho = -F \quad , \quad \rho + p = -\frac{2}{3}X\,F_X \quad , \quad F_X \equiv \frac{\partial F}{\partial X}$$

To have inflation, $X\,F_X \ll F$: solid very weakly affected by expansion

Phonons : $\phi^i = x^i + \pi^i(t,\vec{x})$ , $\pi^i = \frac{\partial_i}{\sqrt{-\nabla^2}}\pi_L + \pi_T^i$

Deeply inside the horizon:

$$c_L^2 = 1 + \frac{2}{3}\frac{X^2 F_{XX}}{X F_X} + \frac{8}{9}\frac{F_Y + F_Z}{X F_X} \ , \ c_T^2 = 1 + \frac{2}{3}\frac{F_Y + F_Z}{X F_X}$$

Therefore all $X\,F_X$ , $X^2\,F_{XX}$ , $F_Y + F_Z \ll F$

In spatially flat gauge $\zeta = -H\frac{\delta\rho}{\dot\rho} = \frac{1}{3}\partial\cdot\pi$

$$\delta T_{ij,\mathrm{scalar}} = a^2 M_p^2 \dot{H}\zeta \left[2\,(3-2\epsilon+\eta)\,\delta_{ij} - \left(3+3c_L^2-2\epsilon+\eta\right)\left(3\hat{k}_i\hat{k}_j - \delta_{ij}\right)\right]$$

- $P_\zeta$ slowly growing outside the horizon
- $B_\zeta$ has nontrivial angular dependence in the squeezed limit

Endlich, Nicolis, Wang ’12

# Vector fields and anisotropic inflation

Initial theoretical interest (Wald '83; Kaloper '91; Barrow, Hervic '05), then WMAP anomaly (now understood to be a systematics).

$\mathcal{L} \supset V\left(A^2\right)$ Ford '89

$\mathcal{L} \supset \lambda\left(A^2 - v^2\right)$ Ackerman, Carroll, Wise '07

$\mathcal{L} \supset A^2 R$ Golovnev, Mukhanov, Vanchurin '08

In these models, the longitudinal polarization is a ghost Contaldi, Himmetoglu, MP '08

Preserve U(1), but $-\frac{1}{4}F^2 \rightarrow -\frac{f(t)}{4}F^2$ so ~~conformal~~

$f \propto a^4 \rightarrow \frac{d\rho_B}{d\ln k}$ scale invariant and frozen, used for magnetogenesis Ratra '92

Strong coupling pbm., since $\alpha \propto f^{-1} \gg 1$ at early times Demozzi, Mukhanov, Rubinstein '09

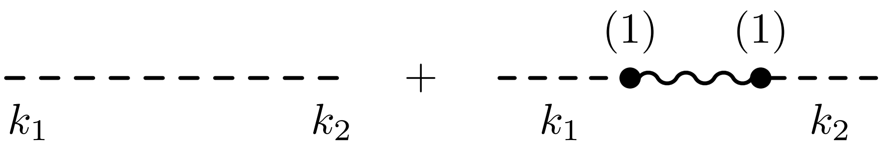
Duality: $f \propto a^{-4} \rightarrow$ scale invariant "electric" component (weak coupling)

$\mathcal{L} = \mathcal{L}_\phi - \frac{f\left(\phi\right)}{4}F^2$ with $f\left(\phi\left(t\right)\right) \propto a^{-4}$ supports $\vec{A}\left(t\right) \neq 0$

Watanabe, Kanno, Soda '09

# Perturbations

Expanding $f(\phi)\,F^2$, $\mathcal{L}_{\text{int}} \supset a^4 \left[4\vec{E}^{(0)} \cdot \delta\vec{E}\,\zeta + 2\delta\vec{E} \cdot \delta\vec{E}\zeta\right]$

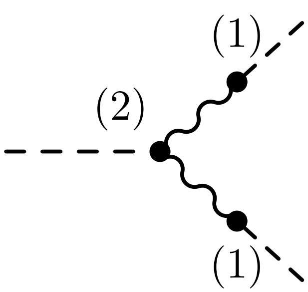



Dulaney, Gresham '10;
Gumrukcuoglu, Himmetoglu, MP '10
Watanabe, Kanno, Soda '10

$$P\left(\vec{k}\right) \simeq P(k)\left[1 + g_* \cos^2\theta_{\vec{k},\,\vec{E}^{(0)}}\right]$$

$$g_* \simeq -\frac{48}{\epsilon} N_{\text{CMB}}^2 \frac{2\rho_{E^{(0)}}}{V(\phi)}$$

$g_* = 0.1$ for $\frac{\Delta H}{H} \sim \frac{\rho_{E^{(0)}}}{V(\phi)} \sim 10^{-8}$



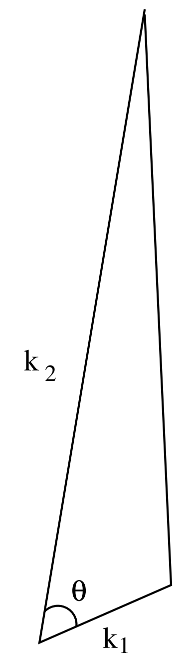

Strict relation between $P_\zeta\left(\vec{k}\right)$ and $B_\zeta$

$$B_\zeta \to \frac{\text{O}(1)}{k_1^3 k_2^3} \quad , \quad k_1 \ll k_2, k_3$$

Bartolo, Matarrese,
MP, Ricciardone '12

$$f_{\text{NL}}^{\text{eff.local}} \sim 25\,\frac{|g_*|}{0.1}$$

$$B_\zeta \propto \frac{1-\cos^2\theta_{\hat{k}_1,\,\hat{E}^{(0)}} - \cos^2\theta_{\hat{k}_2,\,\hat{E}^{(0)}} + \cos\theta_{\hat{k}_1,\,\hat{E}^{(0)}}\cos\theta_{\hat{k}_2,\,\hat{E}^{(0)}}\cos^2\theta_{\hat{k}_1,\,\hat{k}_2}}{k_1^3\,k_2^3} \quad , \quad k_1 \ll k_2,\, k_3$$




Peaked as local in squeezed limit

Nontivial angular dependence

$$B_\zeta\Big|_{\text{isotropic measurement}} \propto \frac{1+\cos^2\theta_{\hat{k}_1,\,\hat{k}_2}}{k_1^3\,k_2^3}$$

Angular modulation in standard scalar field models from gradient of longest mode, subdominant in $\frac{k_1}{k_i} \ll 1$ Lewis '11

Motivation for studying

$$B_\zeta(k_1,k_2,k_3) = \sum_L c_L P_L(\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)P_\zeta(k_1)P_\zeta(k_2) + (2\text{ perm})$$

Shiraishi, Komatsu, MP, Barnaby, '13

$fF^2$ model : $c_0 = \frac{6}{5}f_{\rm NL} \simeq 32\frac{|g_*|}{0.1}$ , $c_2 = \frac{c_0}{2}$

solid inflation : $c_2 \gg c_0$

$$B_\zeta(k_1, k_2, k_3) = \sum_L c_L P_L(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2) P_\zeta(k_1) P_\zeta(k_2) + (2 \text{ perm})$$

$$c_0 = \frac{6}{5} f_{\rm NL}$$

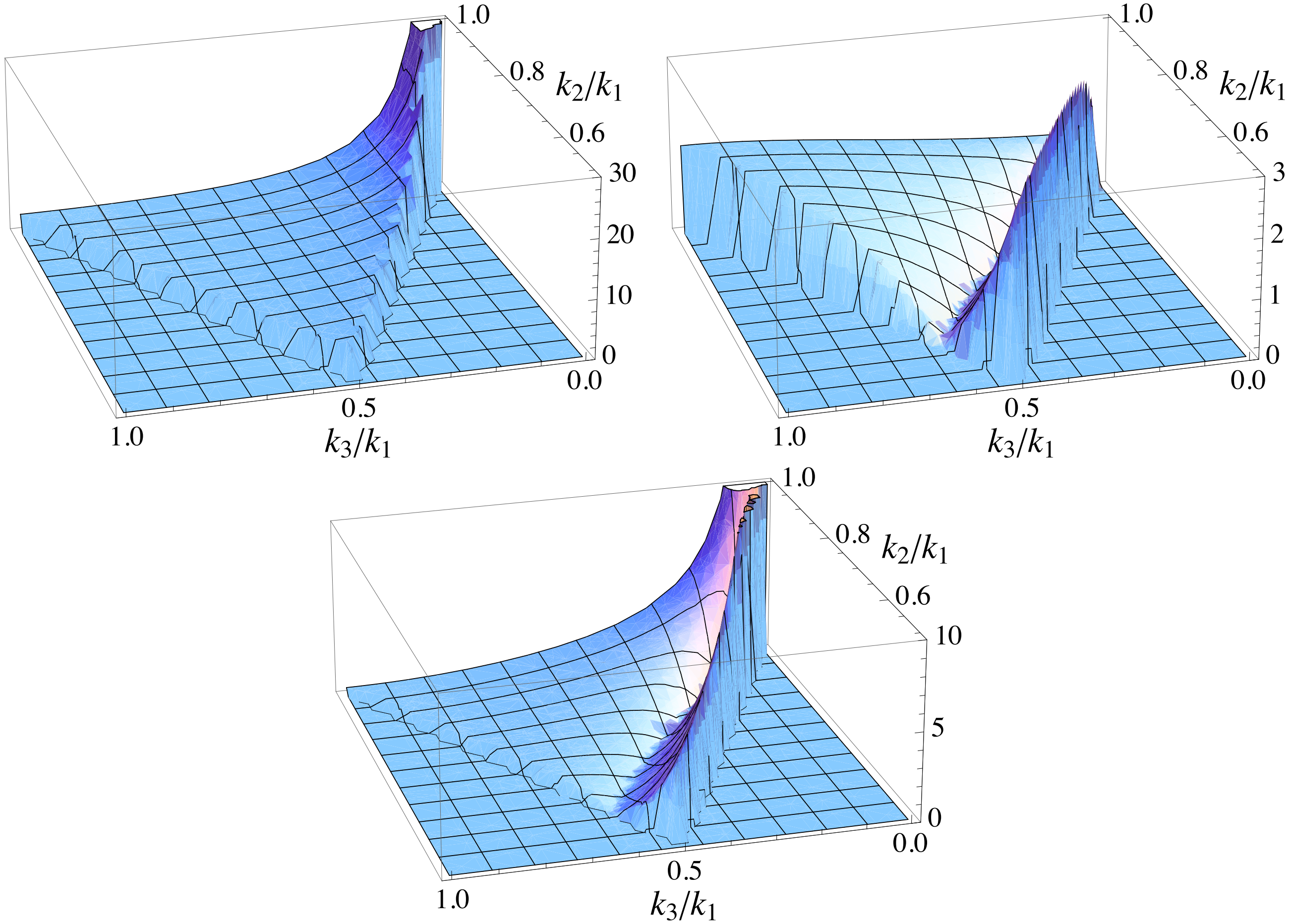



**Figure 1**. Absolute values of the shape function of $L = 0$, $(k_1k_2k_3)^2 S_0$ (top left panel), that of $L = 1$, $(k_1k_2k_3)^2 S_1$ (top right panel), and that of $L = 2$, $(k_1k_2k_3)^2 S_2$ (bottom panel). We restrict the plot range to $k_3 \le k_2 \le k_1$ and $|k_1 - k_2| \le k_3 \le k_1 + k_2$ for symmetry and the triangular condition. The

Full sky

CV-limited up

to $\ell_{\rm max} = 2000$:

$\delta c_0 = 4.4$ , $\delta c_1 = 61$

$\delta c_2 = 13$ (68%CL)

Planck (68%CL)

$c_0 = 3.2 \pm 7$

$c_1 = 11 \pm 113$

$c_2 = 3.8 \pm 27.8$

$\Downarrow$

$|g_*| < 0.05$

- Other analogies besides $B_{\text{squeezed}}$?
- Recall $X\,F_X, X^2\,F_{XX} \ll F$ (very weak reaction to volume expansion), and $F_Y + F_Z \ll F$ (very weak reaction to sound deformations).

Prolonged anisotropy $$\frac{\Delta H}{H} \propto \mathrm{e}^{-\frac{4}{3}c_T^2 \epsilon H t}$$

$P_\zeta$ on this background: $$g_* = \mathcal{O}\left(\frac{\Delta H}{\epsilon H}\right) \gg \mathcal{O}\left(\frac{\Delta H}{H}\right)$$

Bartolo, Matarrese, MP, Ricciardone '13

Wald's isotropization theorem ('83): an anisotropic universe with a cosmological constant + a 2nd source with

$$\mathcal{D} \;\equiv\; t^\mu t^\nu T^{\text{2nd}}_{\mu\nu} > 0 \qquad\qquad \mathcal{S} \;\equiv\; t^\mu t^\nu \left(T^{\text{2nd}}_{\mu\nu} - \frac{T^{\text{2nd}}}{2} g_{\mu\nu}\right) > 0$$

isotropizes on a $\sim \sqrt{1/\Lambda}$ timescale

solid : $$T_{\mu\nu} = g_{\mu\nu} F(t) - 2\partial_\mu \phi^i \partial_\nu \phi^j \frac{\partial F}{\partial B^{ij}} \equiv g_{\mu\nu} F(t_{\text{in}}) + T^{\text{2nd}}_{\mu\nu}$$

$$\mathcal{D} = -\left\{-F(t_{\text{in}}) - [-F(t)]\right\} < 0$$

## Is $\Delta H$ to be expected ?

- In $f(\phi)F^2$, frozen and scale invariant $\delta\vec{E}_k$
- Modes produced in first $N_{\text{TOT}} - 60$ e-folds randomly add up to $\vec{E}_{\text{IR}}$ observed as classical, homogeneous and anisotropic by CMB modes
- Value $\vec{E}_{\text{IR}}$ in our realization drawn by a gaussian of mean 0 and variance

  $\left\langle \vec{E}_{\text{IR}}^2 \right\rangle \sim H^4 \left( N_{\text{tot}} - 60 \right)$

- $|g_*|_{\text{expected}} \gtrsim 0.1 \ \dfrac{N_{\text{tot}} - N_{\text{CMB}}}{37}$ (generically, too anisotropic)

  If this model is realized in nature, either $N_{\text{tot}} \sim N_{\text{CMB}}$, or we live in a patch where $\vec{E}_{\text{IR}} \ll \sqrt{\left\langle \vec{E}_{\text{IR}}^2 \right\rangle}$

- Same in solid inflation ?

# Conclusions

- Two distinct models, characterized by breaking of spatial symmetry

- Common features (anisotropic background, $B_{\text{squeezed}}$, growing $\zeta$), most of which at odds with simplest inflationary expectations

- New class, with perhaps additional new signatures