

Cosmological Constraints on Scalar Field Dark Matter after PLANCK

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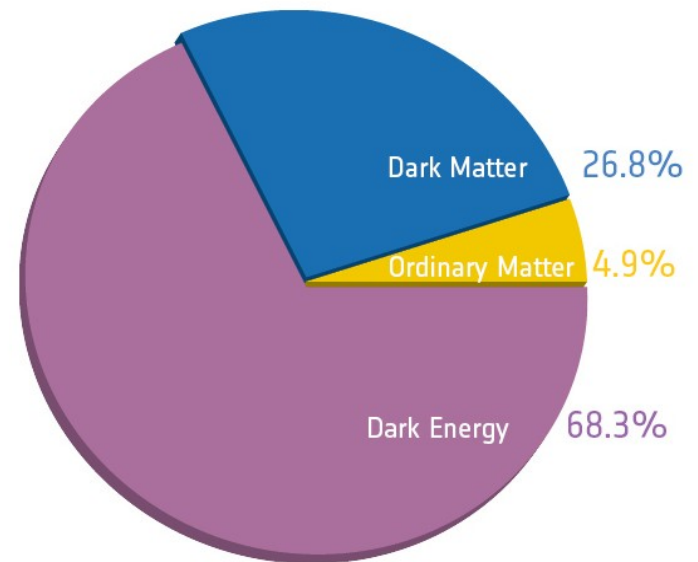
Cosmology after Planck Workshop

MCTP, Ann Arbor, Sep 25, 2013

Cosmic Inventory at the present epoch (after PLANCK)

- Dark Energy: cosmological constant $w = P/\rho = -1$
- Dark Matter: cold, “dust-like” $w = 0$
- Radiation (photons + SM neutrinos): $w = 1/3$
- Baryons: cold, “dust-like” $w = 0$

6-parameter base Λ CDM model
remains a best-fit model :
“concordance” model of modern
cosmology



HOWEVER:

***What is the* Dark Matter ?**

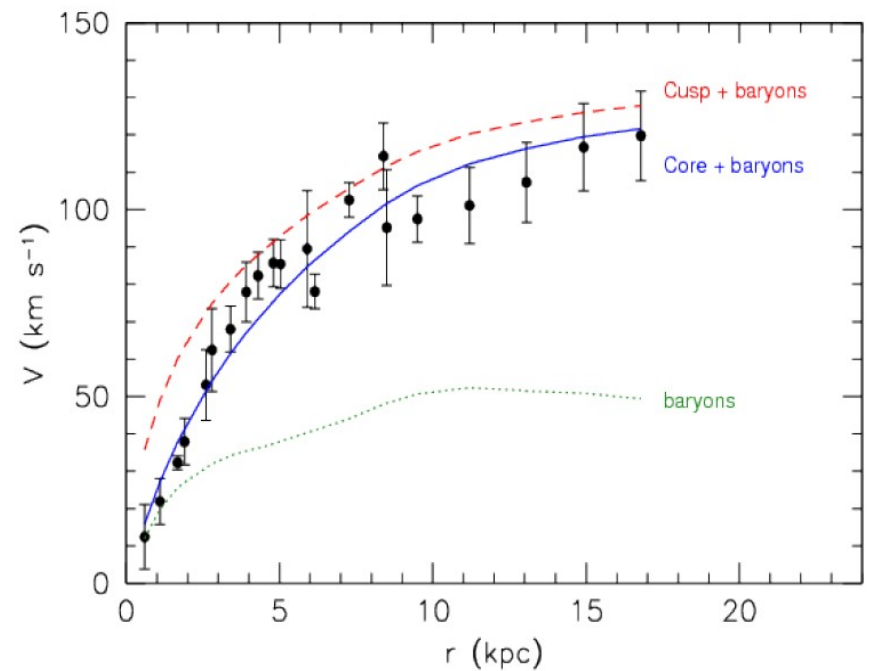
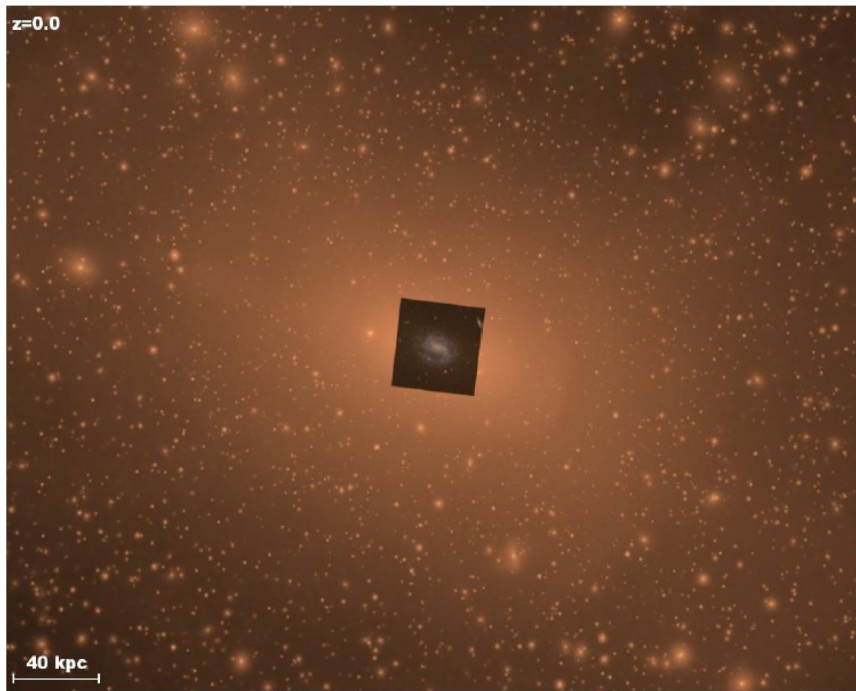
→ all particle candidates are beyond the SM
and none of them detected

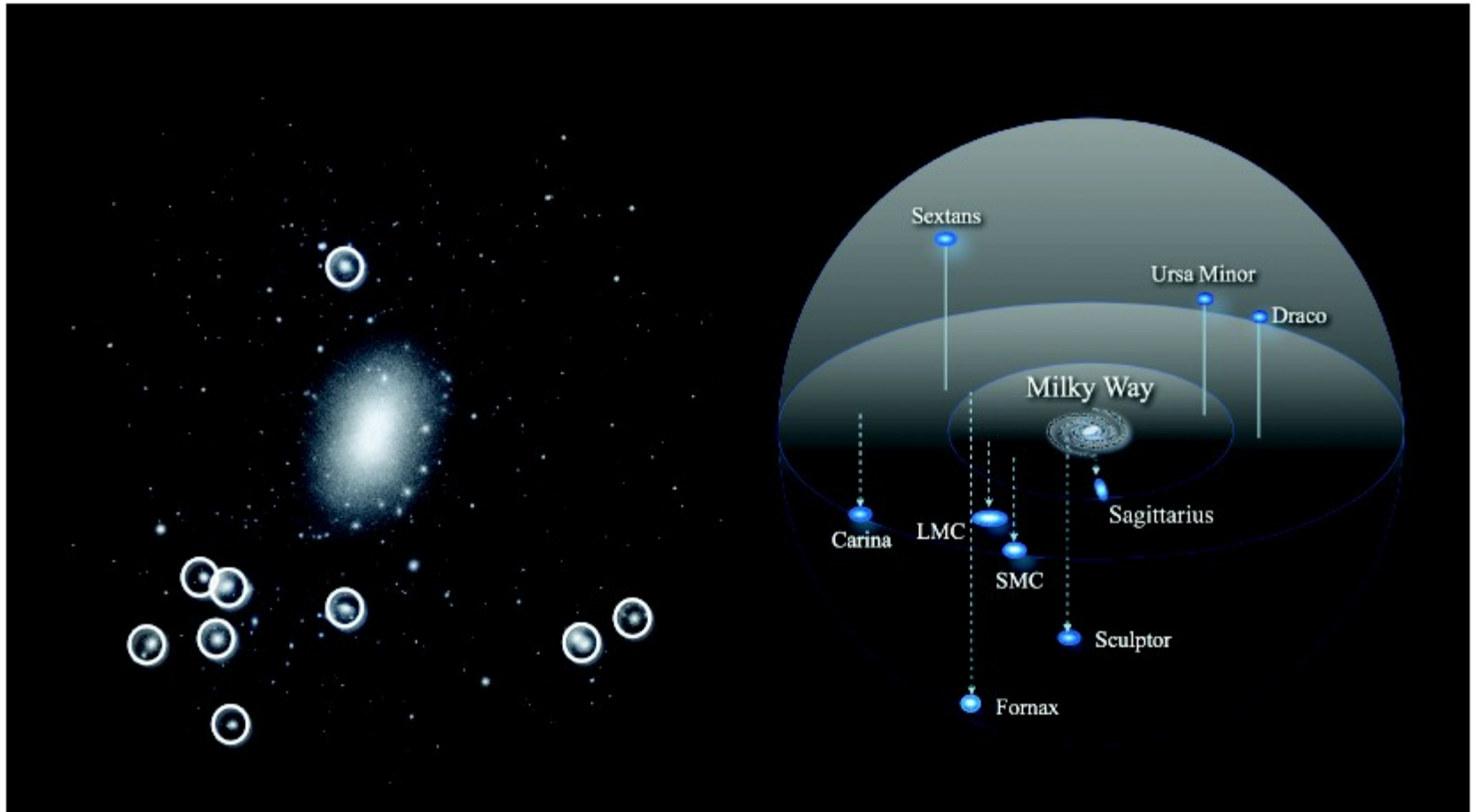
**Problems on small-scales with
standard Cold Dark Matter !?**

→ DM nature remains unsettled

cosmological N-body simulations:

discrepancies between theory and observations on galaxy scales
(cusp/core galactic profiles; overabundance of satellites)





Ultra-light scalars for DM: $m \sim (10^{-33} - 10^{-15}) \text{ eV}/c^2$

light and ultra-light axion-like particles (ALPs)

concept of QCD axion generalized to ALPs:

pseudo-Nambu-Goldstone bosons from the breaking of global symmetries of unification theories beyond the standard model (appear generically in string compactifications)

(„String Axiverse“: Arvanitaki et al 2010; Acharya, Bobkov & Kumar 2010; Marsh 2011; Higaki & Kobayashi, 2011)

gravitational excitons

in multidimensional cosmologies and braneworld models

(Günther & Zhuk 1997; Arkani-Hamed et al 1999; Matos 2001)

- ultra-light DM bosons described by (coherent) scalar field, even on galactic scales !

→ Provide natural cutoff scale („Jeans scale“) for the smallest structures in the Universe and shallower central core densities

e.g. $m \sim 10^{-22} \text{ eV}/c^2 \leftrightarrow R \sim \lambda_{\text{deB}} \sim 1 \text{ kpc}$
(no self-interaction)

$m \gg 10^{-22} \text{ eV}/c^2 \leftrightarrow \lambda_{\text{deB}} \ll R \sim 1 \text{ kpc}$
(high self-interaction pressure)

→ attracts attention of astrophysicists

Astrophysics literature on scalar field dark matter

(excluding the QCD axion)

Turner (1983); Khlopov, Malomed & Zeldovich (1985); Tkachev (1986)
Press, Ryden & Spergel (1990); Widrow & Kaiser (1993)
Sin (1994); Schunck (1994); Lee & Koh (1995)
Vilenkin and Peebles (1999); Hu, Barkana & Gruzinov (2000)
Goodman (2000); Peebles (2000); Riotto & Tkachev (2000)
Guzmán & Matos (2000); Barcelo, Liberati & Visser (2001)
Guzmán & Ureña-López (2003);
Arbey, Lesgourges & Salati (2002, 2003); Short & Coles (2006)
Fukuyama, Morikawa & Tatekawa (2008)
Woo & Chiueh (2009), Ureña-López (2009)
Lee & Lim (2010); Arvanitaki et al. (2010); Marsh (2011)
Suárez & Matos (2011); Harko (2011); Chavanis (2011);
TRD & Shapiro (2010, 2012); Slepian & Goodman (2012);

Scalar Field Dark Matter (SFDM)

Assume DM is composed of ultra-light, spin-0 bosons

$n \lambda_{\text{deB}} \gg 1 \rightarrow$ critical temperature of Bose-Einstein-condensation $T_c \gg \text{TeV}$

(Almost) all of the bosons are in the ground state

\rightarrow described by a single (coherent) classical field: scalar-field dark matter (SFDM)

We consider *complex scalar fields*

$$\mathcal{L} = \frac{\hbar^2}{2m} g^{\mu\nu} \partial_\mu \psi^* \partial_\nu \psi - V(\psi) \quad V(\psi) = \frac{1}{2} m c^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4$$

Complex field obeys

U(1)-symmetry, particle number conserved \rightarrow no self-annihilation !

Dilute gas: *2-body repulsive interactions only*

first Born approx: $\lambda \geq 0$ is an energy-independent coupling constant

Fundamental SFDM parameters: m and λ

Take the same cosmic inventory as the basic Λ CDM model, except that **CDM is replaced by SFDM** \rightarrow **Λ SFDM**

Cosmological parameters from **Planck results XVI (2013)**:

$\Omega_m = \Omega_b + \Omega_c$	Basic		Derived	
	h	0.673	$\Omega_m h^2$	0.14187
$\Omega_\Lambda = 1 - \Omega_m - \Omega_r$	$\Omega_b h^2$	0.02207	$\Omega_r h^2$	4.184×10^{-5}
	$\Omega_c h^2$	0.1198	z_{eq}	3390
	T_{CMB}/K	2.7255	Ω_Λ	0.687

assuming SM
neutrinos are
massless

TABLE I. Cosmological parameters. The values in the left column (‘Basic’) are quoted from the Planck collaboration: central values of the 68% confidence limits for the base Λ CDM model with Planck+WP+highL data, see Table 5 in [5]. We calculate those in the right column (‘Derived’).

Homogeneous background Universe

Scalar field of SFDM depends only on time

Energy-momentum tensor is diagonal \rightarrow perfect fluid description

$$(T_{\mu\nu})_{\text{SFDM}} = (\bar{\rho}_{\text{SFDM}} + \bar{p}_{\text{SFDM}})u_{\mu}u_{\nu}/c^2 - g_{\mu\nu}\bar{p}_{\text{SFDM}}$$

$$\bar{\rho}_{\text{SFDM}} = (T_0^0)_{\text{SFDM}} = \frac{\hbar^2}{2mc^2}|\partial_t\psi|^2 + \frac{1}{2}mc^2|\psi|^2 + \frac{\lambda}{2}|\psi|^4$$

$$\bar{p}_{\text{SFDM}} = -(T_i^i)_{\text{SFDM}} = \frac{\hbar^2}{2mc^2}|\partial_t\psi|^2 - \frac{1}{2}mc^2|\psi|^2 - \frac{\lambda}{2}|\psi|^4$$

KG equation of motion $\rightarrow \frac{\partial \bar{\rho}_{\text{SFDM}}}{\partial t} + \frac{3da/dt}{a}(\bar{\rho}_{\text{SFDM}} + \bar{p}_{\text{SFDM}}) = 0$

Basic behavior of scalar fields: oscillation over time, characterized by its changes in phase θ , and oscillation angular frequency $\omega = \partial_t \theta$

- Fast oscillation regime („oscillation“): $\omega / H \gg 1$

$$\omega = \frac{mc^2}{\hbar} \sqrt{1 + \frac{2\lambda}{mc^2} |\psi|^2}$$

$$\langle \bar{\rho} \rangle = mc^2 \langle |\psi|^2 \rangle + \frac{3}{2} \lambda \langle |\psi|^4 \rangle \approx mc^2 \langle |\psi|^2 \rangle + \frac{3}{2} \lambda \langle |\psi|^2 \rangle^2,$$

$$\langle \bar{p} \rangle = \frac{1}{2} \lambda \langle |\psi|^4 \rangle \approx \frac{1}{2} \lambda \langle |\psi|^2 \rangle^2.$$

→ **equation of state**

$$\langle \bar{p} \rangle = \frac{m^2 c^4}{18\lambda} \left(\sqrt{1 + \frac{6\lambda \langle \bar{\rho} \rangle}{m^2 c^4}} - 1 \right)^2$$

(1) CDM-like phase: non-relativistic $\langle \bar{w} \rangle = 0$

$$\frac{3}{2} \lambda \langle |\psi|^2 \rangle^2 \ll mc^2 \langle |\psi|^2 \rangle \quad \langle \bar{p} \rangle \approx \frac{\lambda}{2m^2 c^4} \langle \bar{\rho} \rangle^2 \approx 0$$

→ evolves like CDM $\langle \bar{\rho} \rangle \propto a^{-3}, \quad a \sim t^{2/3}$

(2) Radiation-like phase: relativistic $\langle \bar{w} \rangle = 1/3$

$$\frac{3}{2} \lambda \langle |\psi|^2 \rangle^2 \gg mc^2 \langle |\psi|^2 \rangle \quad \langle \bar{p} \rangle \approx \frac{1}{3} \langle \bar{\rho} \rangle \approx \frac{1}{2} \lambda \langle |\psi|^2 \rangle^2$$

→ evolves like radiation $\langle \bar{\rho} \rangle \propto a^{-4}, \quad a \sim t^{1/2}$

NOTE: SFDM without self-interaction ($\lambda = 0$) does *not* undergo this radiation-like phase !

- Slow oscillation regime („roll “) : $\omega / H \ll 1$

(1) Stiff phase: equation of state of „stiff matter“
relativistic limit ($w = 1$)

$$\bar{p} \approx \bar{\rho} \approx \frac{\hbar^2}{2mc^2} |\partial_t \psi|^2$$

→ evolves as $\langle \bar{\rho} \rangle \propto a^{-6}, \quad a \sim t^{1/3}$

Evolution of Λ SFDM

- Solve Friedmann equation coupled with energy conservation / equ. of motion and the EOS, by integrating from the present-day backwards to the point when $\omega/H = 200$ (i.e. still well into the fast-oscillation regime) at $a \sim 10^{-7}$
'Late-time solution': its initial conditions are from the Table (Planck data)
- At earlier times up to the Big Bang, solve the system exactly (i.e. no averaging over oscillation periods)
'Early-time solution': integration starts where we cease to apply the fast-oscillation approximation at $\omega/H = 200$, back to the Big Bang, in a way that it matches to the late-time solution

Can do that

for different choice of SFDM mass m and coupling strength λ

The other cosmic components are handled in the usual, standard way.

Fiducial SFDM Model

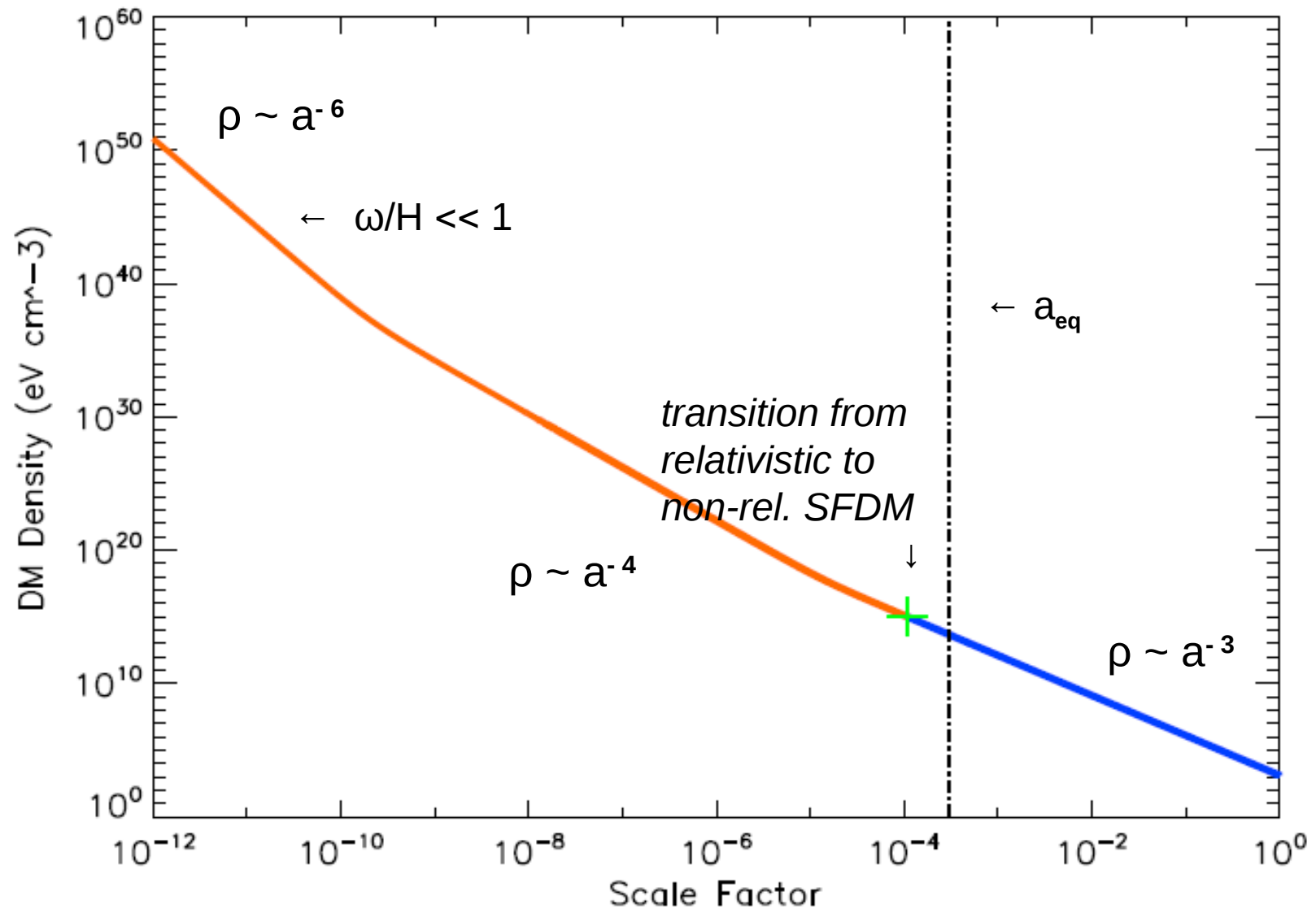
$$(m, \lambda)_{\text{fiducial}} = (3 \times 10^{-21} \text{ eV}/c^2, 1.8 \times 10^{-59} \text{ eV cm}^3)$$

$$\lambda/(mc^2)^2 = 2 \times 10^{-18} \text{ eV}^{-1} \text{ cm}^3$$

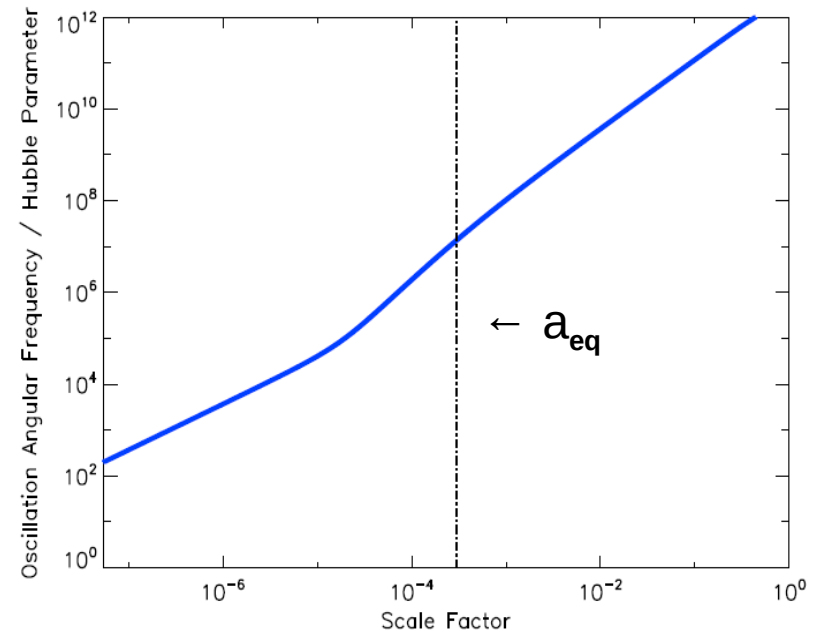
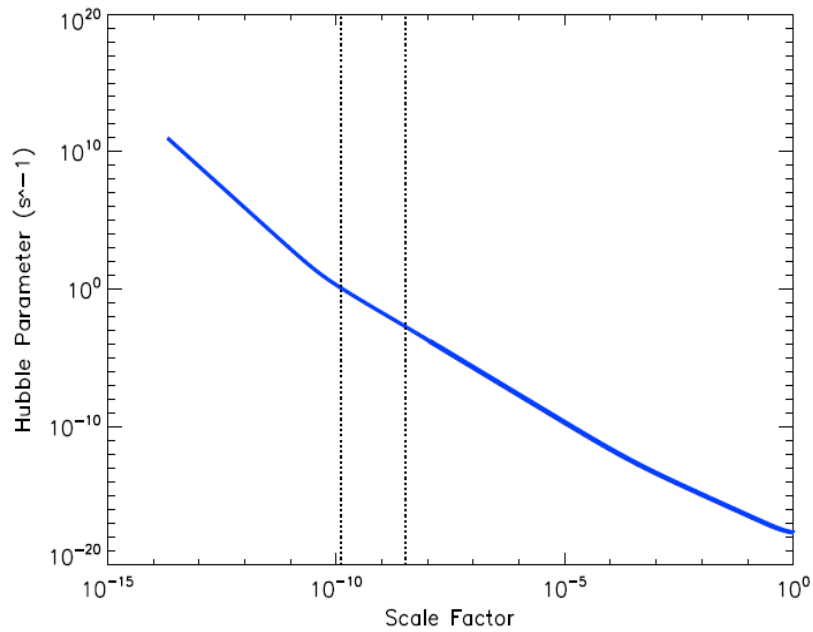
in natural units: $\hat{\lambda}_{\text{fiducial}} \simeq 10^{-83}$

For comparison $\hat{\lambda}_{\text{QCDaxion}} \simeq 10^{-57}$

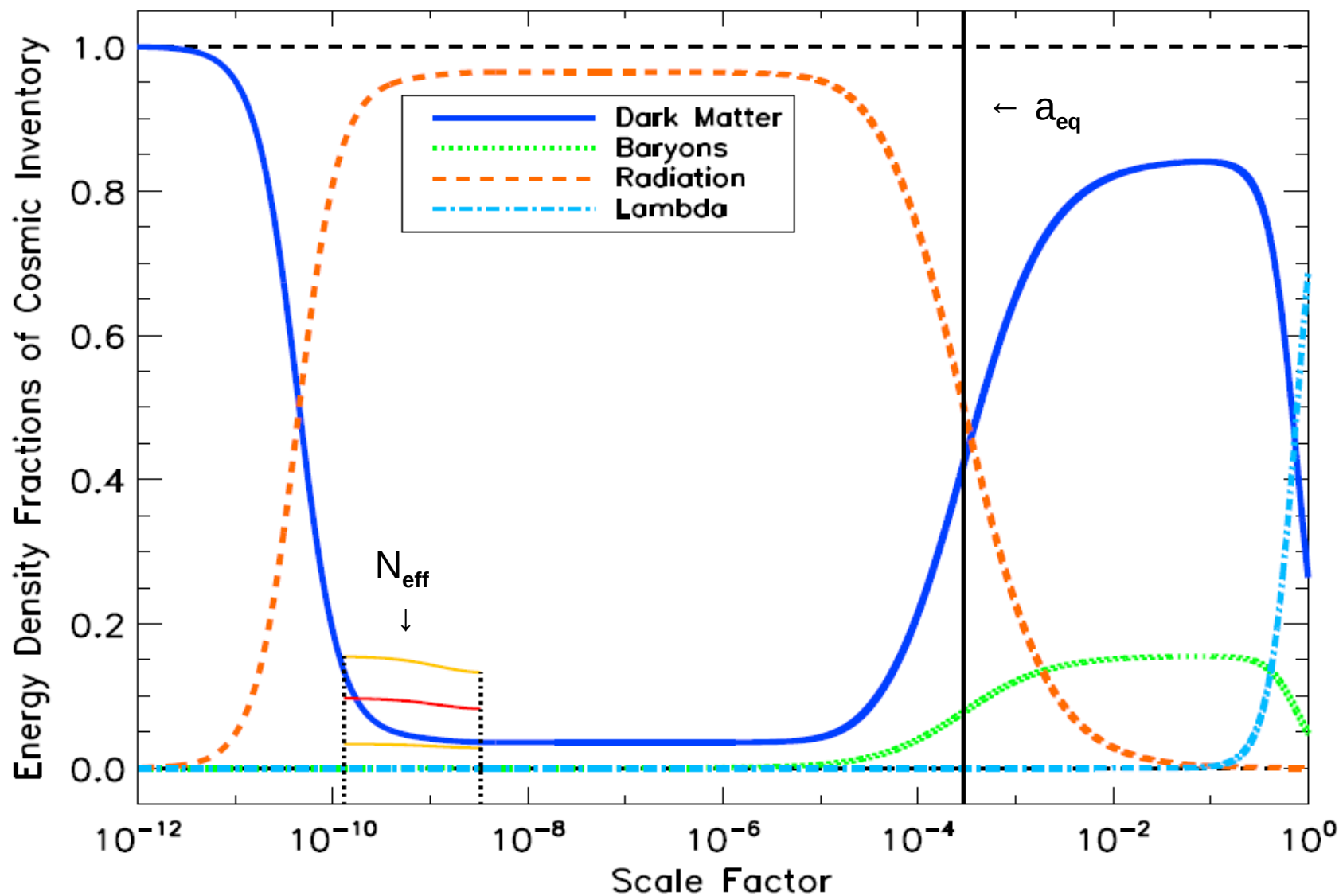
Evolution of SFDM energy density



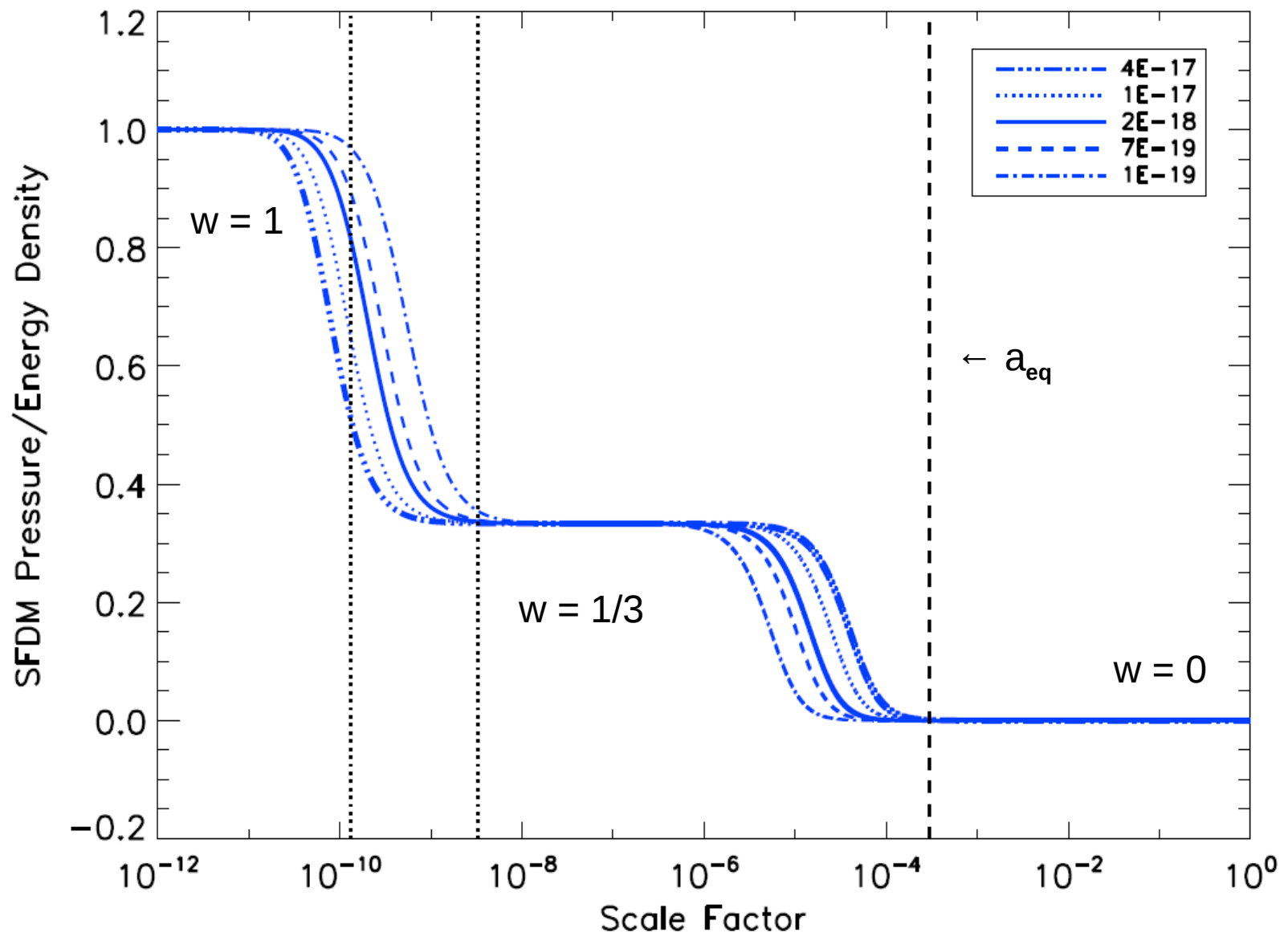
Evolution of H and ω/H



Evolution of Λ SFDM



The larger $\lambda/(mc^2)^2$, the longer lasts the radiation-like phase



Constraints on SFDM from the CMB

redshift of **matter-radiation equality** z_{eq} :

$$1 + z_{\text{eq}} = \frac{\Omega_b h^2 + \Omega_c h^2}{\Omega_r h^2}$$

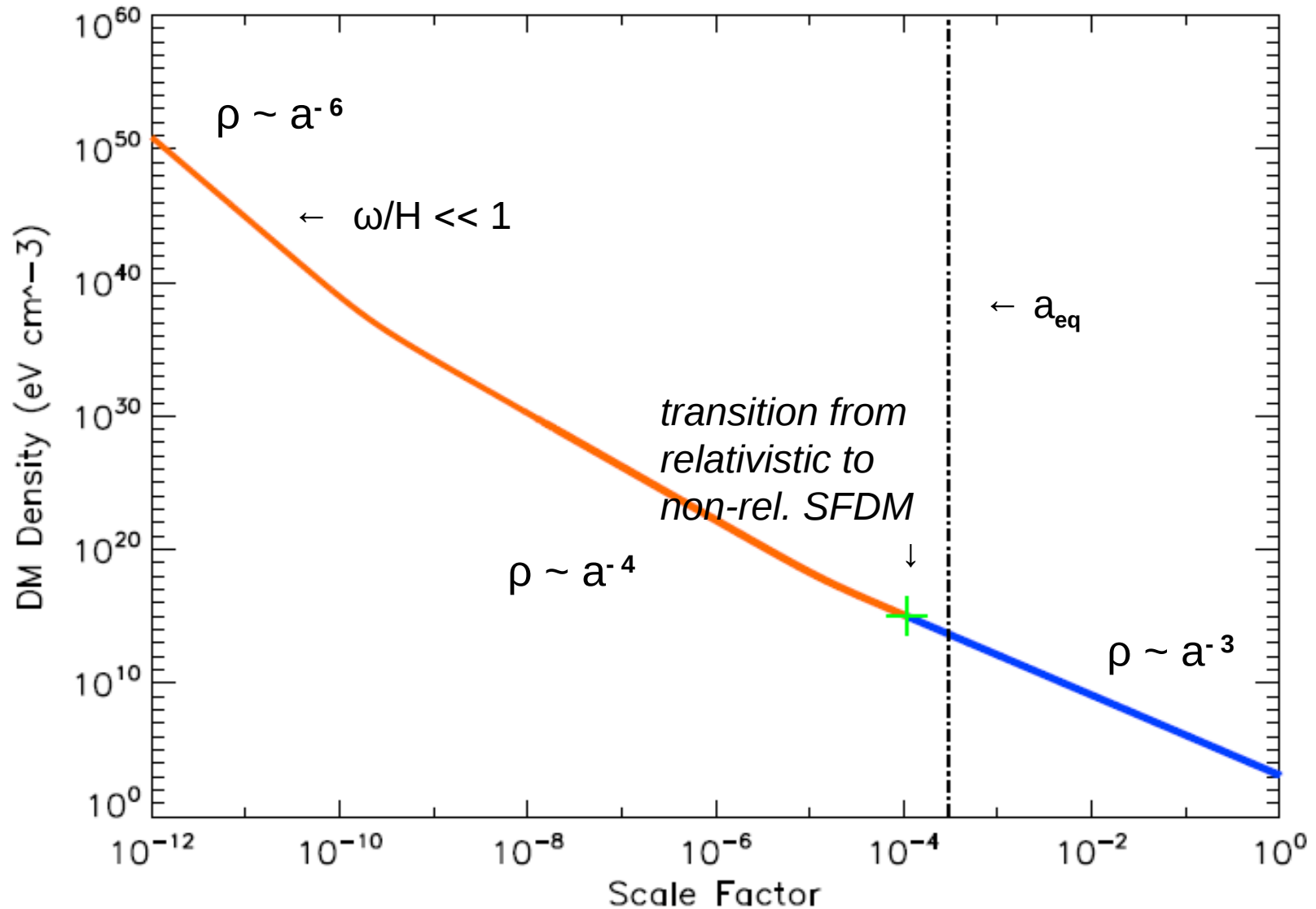
require SFDM to be fully non-relativistic at z_{eq}

i.e. the transition from the relativistic phase (‘radiation-like’) to the non-relativistic phase (‘CDM-like’) must happen early enough such that SFDM is cold at z_{eq}

→ *constraint on the ratio $\lambda/(mc^2)^2$*

$$\lambda/(mc^2)^2 \leq 4 \times 10^{-17} \text{ eV}^{-1} \text{ cm}^3 \text{ for a chosen threshold of } \langle \bar{w} \rangle = 0.001$$

Evolution of SFDM energy density



Constraints on SFDM from BBN

effective number of relativistic degrees of freedom / neutrinos: N_{eff}

in Λ CDM with SM neutrinos only: $N_{\text{eff,standard}} = 3.046$

in Λ SFDM: if SFDM is relativistic during BBN

→ contributes to N_{eff} as an extra relativistic component

$$\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff,standard}}$$

Therefore, constraints on N_{eff} from BBN allow control on SFDM parameters.

in Λ SFDM: ΔN_{eff} caused by SFDM is *changing with time* !

→ must study the evolution of N_{eff} throughout BBN

Important 2 stages in standard BBN:

- beginning of neutron/proton freeze-out around $T_{\text{n/p}} = 1.293 \text{ MeV}$: $a_{\text{n/p}}$
- beginning of nuclei production (D) around $T_{\text{nuc}} \sim 0.07 \text{ MeV}$: a_{nuc}

Constraints on SFDM from BBN

effective number of relativistic degrees of freedom / neutrinos: N_{eff}

SFDM is the only source for ΔN_{eff} , i.e. infer N_{eff} during BBN from

$$\frac{\Delta N_{\text{eff}}}{N_{\text{eff,standard}}} = \frac{\bar{\rho}_{\text{SFDM}}}{\bar{\rho}_{\nu}}$$

and compare the N_{eff} obtained this way to the measured value
(which is constant over time)

We impose a conservative constraint: the N_{eff} during BBN *be all the time* within the **1σ confidence limits** of

$$N_{\text{eff}} = 3.71^{+0.47}_{-0.45} \quad \text{or} \quad \Delta N_\nu = 0.66^{+0.47}_{-0.45} \quad (\text{Steigman 2012})$$

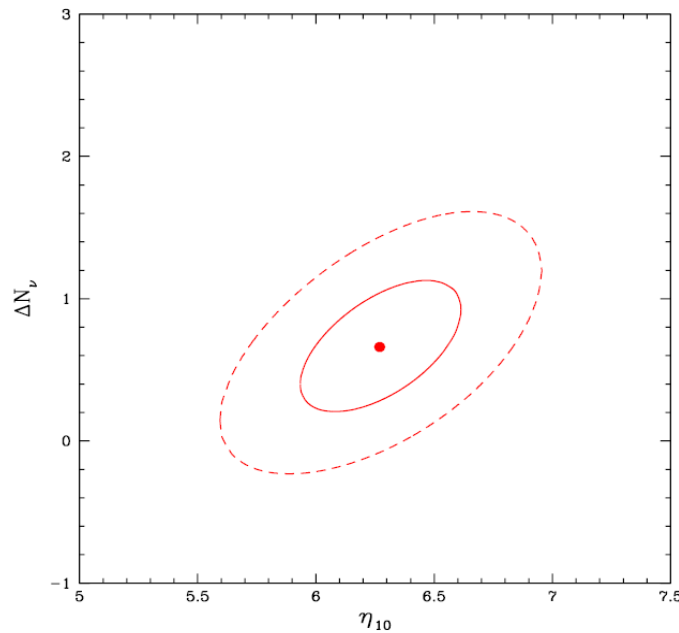
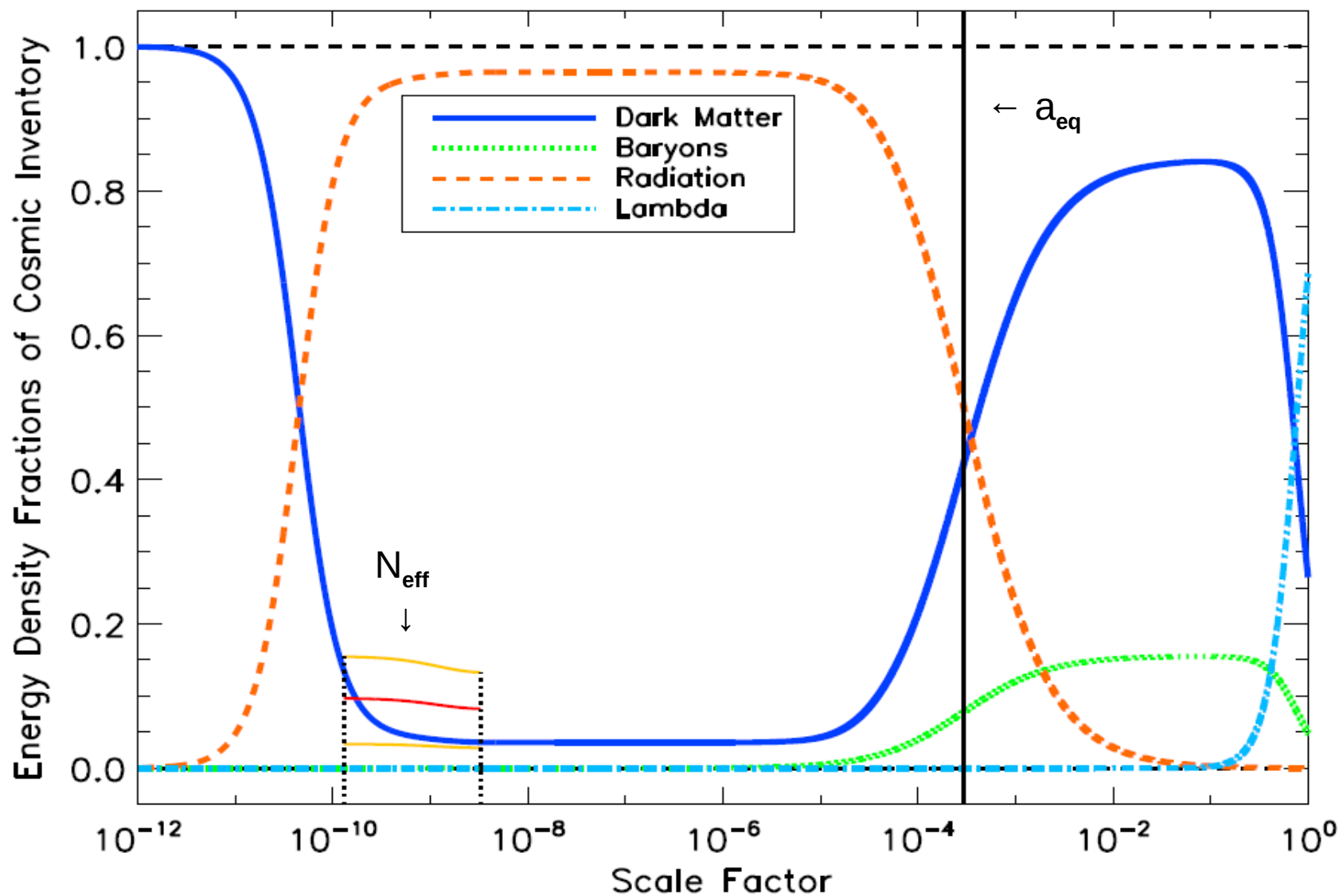
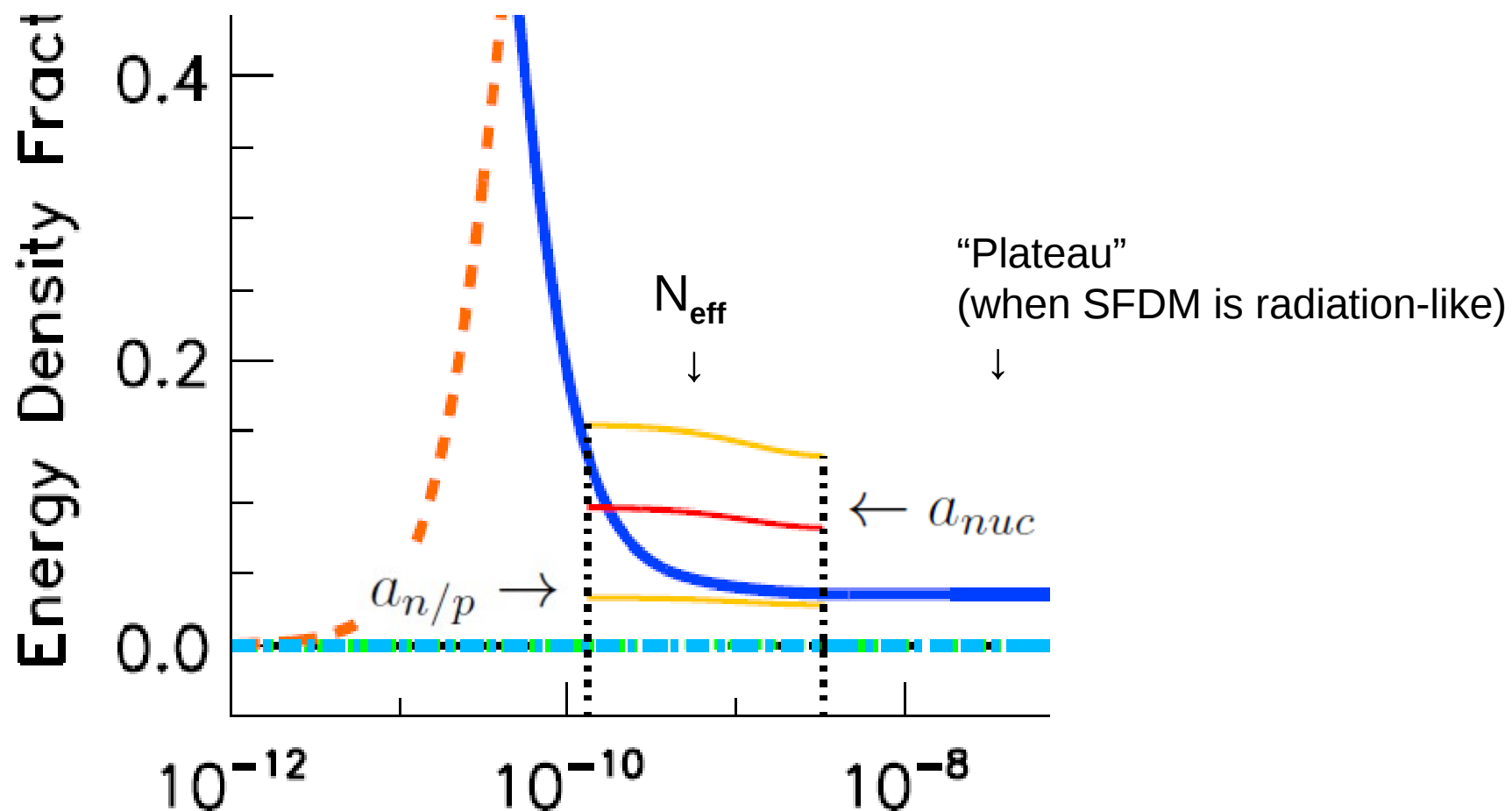


Figure 4: The BBN-inferred 68% (solid) and 95% (dashed) contours in the $\Delta N_\nu - \eta_{10}$ plane derived from D and ${}^4\text{He}$ assuming that $\xi = 0$.

Evolution of Λ SFDM



Evolution of Λ SFDM during BBN



Constraints on SFDM from BBN

effective number of relativistic degrees of freedom / neutrinos: N_{eff}

the relation between N_{eff} and Ω_{SFDM} is analytic during the „plateau“
(i.e. during the radiation-like phase) *if* SFDM reaches it before a_{nuc} :

$$N_{\text{eff}} = 3.71^{+0.47}_{-0.45} \quad \rightarrow \quad 0.028 \leq \Omega_{\text{SFDM,plateau}} \leq 0.132$$

the higher $\lambda/(mc^2)^2$, the higher the “plateau”

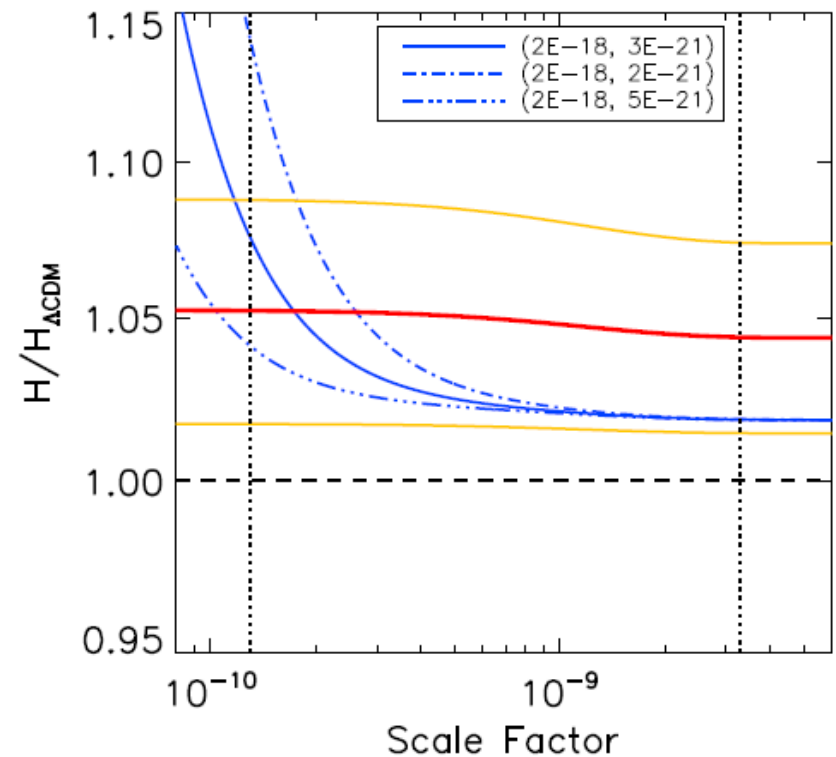
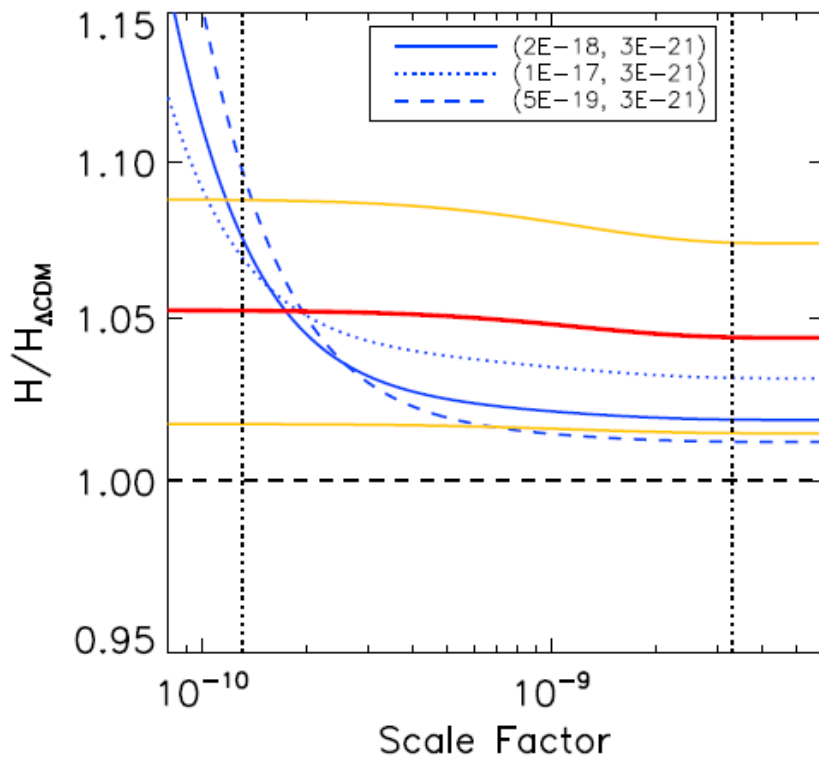
\rightarrow *constraint on $\lambda/(mc^2)^2$*

$$9.5 \times 10^{-19} \text{ eV}^{-1} \text{ cm}^3 \leq \lambda/(mc^2)^2 \leq 1.5 \times 10^{-16} \text{ eV}^{-1} \text{ cm}^3$$

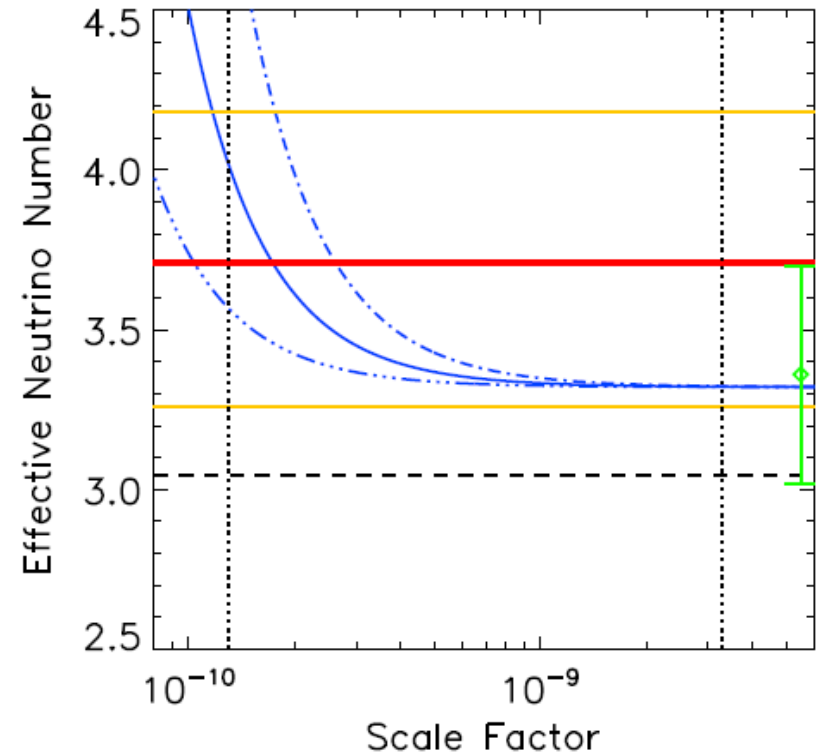
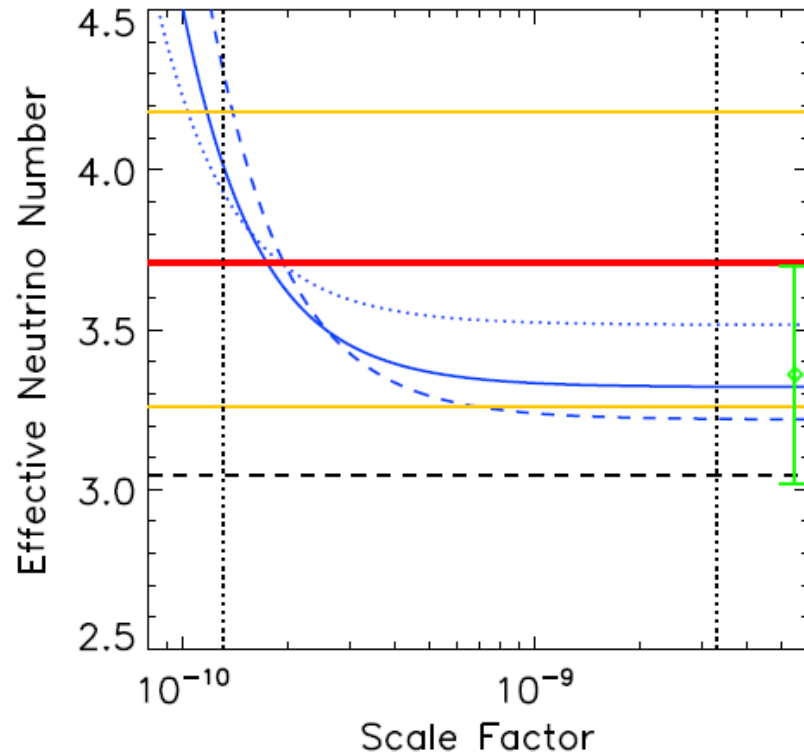
Constraints on SFDM from BBN

Meanwhile, earlier at $a_{n/p}$, the transition from the stiff to the radiation-like phase may not have finished and the value of N_{eff} **can be higher than at the plateau** – this is a function of *both* $\lambda/(mc^2)^2$ **and** m

→ *constraint on $\lambda/(mc^2)^2$ and m*

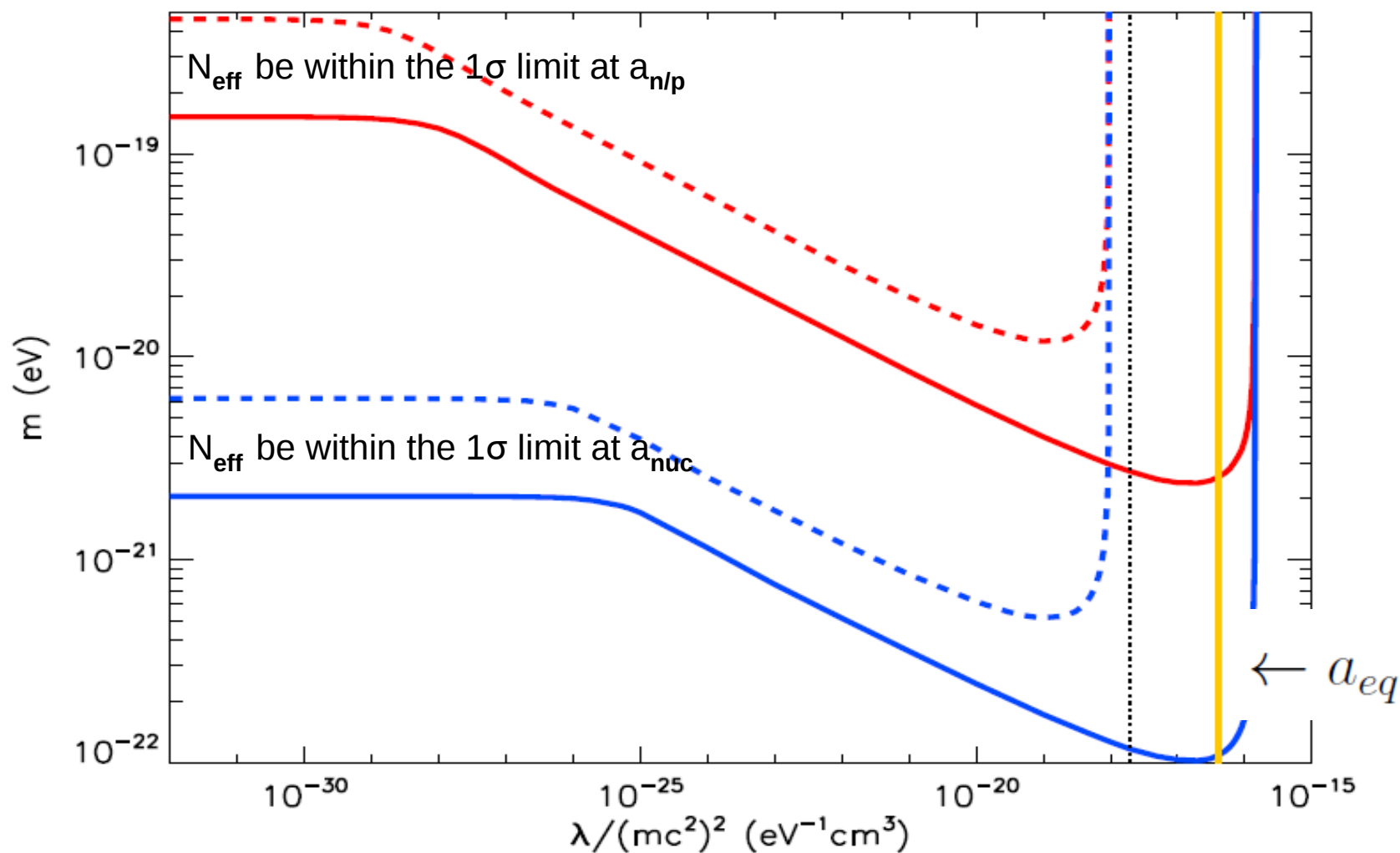


Constraints on SFDM from BBN

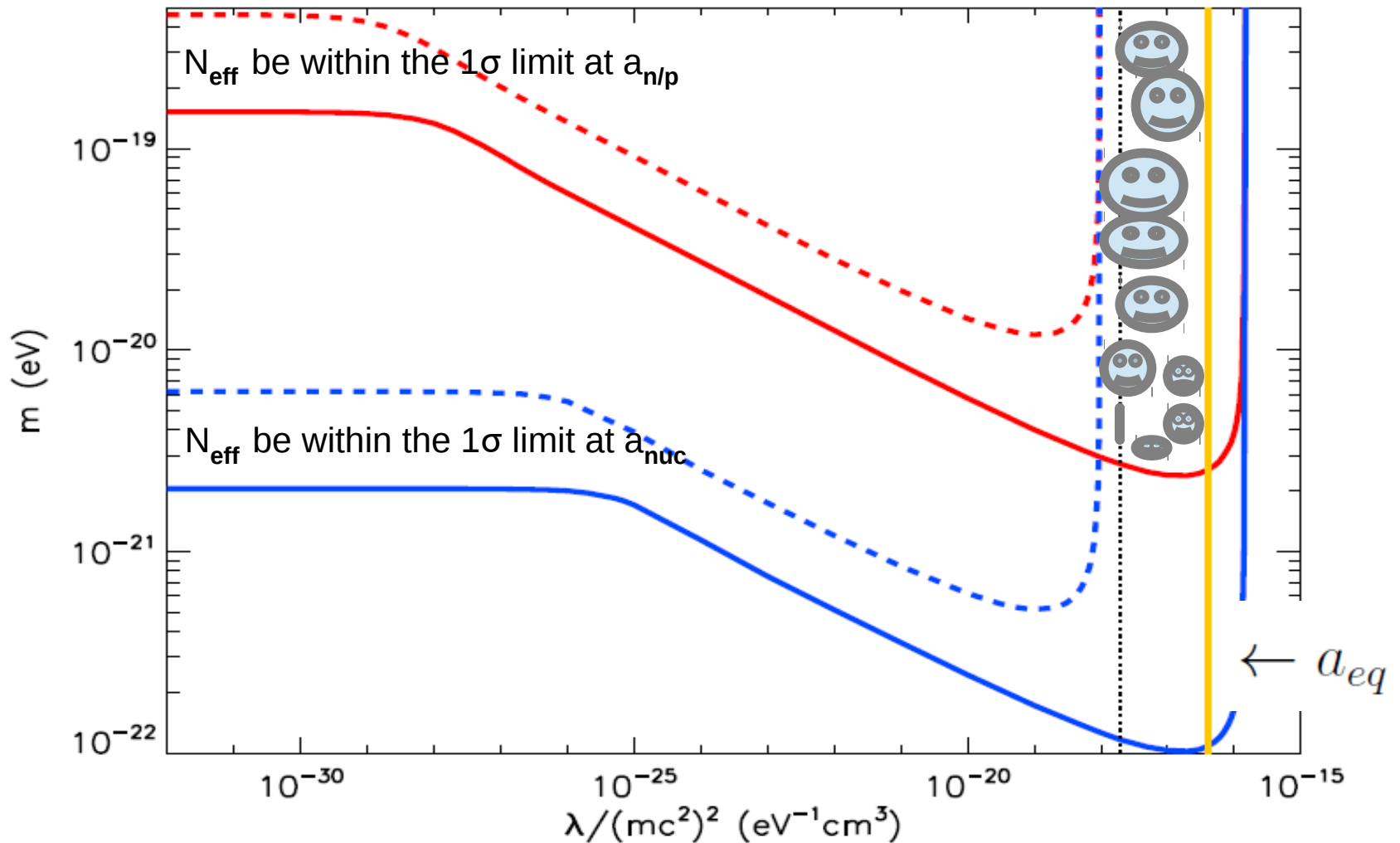


compare: $N_{\text{eff}} = 3.36 \pm 0.34$ (green bar) from CMB alone (Planck+WP+highL)

Parameter space for SFDM



Allowed parameter space for SFDM

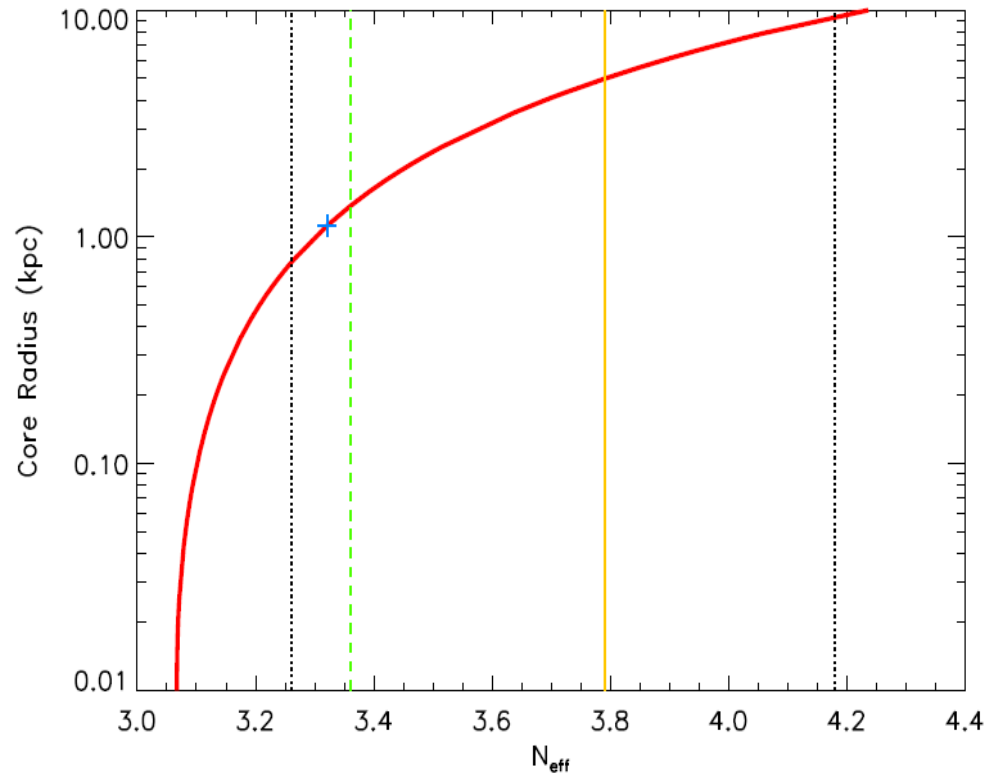


Models with $\lambda = 0$ (Fuzzy Dark Matter) are excluded within 1σ in any case: need high enough mass m !

N_{eff} and the minimum size of halos

- N_{eff} during the radiation-like phase of SFDM is solely determined by $\lambda/(mc^2)^2$
- The radius R of the smallest virialized object (core radius of SFDM halo) is also determined solely by $\lambda/(mc^2)^2$ for SFDM with $R \gg \lambda_{\text{deB}}$

$$R = \pi c^2 \sqrt{\frac{\lambda}{4\pi G (mc^2)^2}}$$



→ higher N_{eff} implies stronger self-interaction pressure,
hence larger minimum scale for Dark Matter structure !

Main take-away

- Scalar-field dark matter is a good dark matter candidate
- Were able to restrict the allowed parameter space severely (compared to previous literature), even by considering only the evolution of the background universe
- Nevertheless, there remains a semi-infinite stripe in the parameter space which is in accordance with current observations, including parameter sets which are able to resolve the small-scale problems of CDM
- The currently favored value of $N_{\text{eff}} > N_{\text{eff,standard}}$ from BBN and CMB would rule out the possibility of Fuzzy Dark Matter (as the only DM component)
- SFDM with self-interaction provides a natural explanation of why N_{eff} during BBN is higher than that inferred from the CMB from Planck data