Comments on multi-loop calculations of the lightest Higgs mass in supersymmetry

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Higgs boson mass from theory workshop

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The calculation of multi-loop corrections to the Higgs mass in SUSY has a long history with contributions from many people.

Publicly available programs:

`FeynHiggs, CPSuperH, H3m, Softsusy, SuSpect, Spheno, ISASUSY,`...?

Today I will mostly confine comments to some results that are not yet incorporated in these public programs, as far as I know.

To appear someday: SUSANA (David Robertson and SPM)

2014? 2015?

Pole mass, schematically:

$M_h^2 = m^2_{h,\rm tree} + \Pi_{hh}(M_h^2)$ from self-energy 4×4 matrix $\Pi_{\phi_i^0\phi_j^0}(p^2)$.

Three distinct approaches:

- Effective potential approximation: $\Pi(p^2) \approx \Pi(0)$.

  Fully known but not fully implemented (?) at 2 loops.

- Feynman diagrams

  2-loop: $\mathcal{O}(y_t^2\alpha_S)$, $\mathcal{O}(y_b^2\alpha_S)$, $\mathcal{O}(y_t^4)$, $\mathcal{O}(y_t^2 y_b^2)$, $\mathcal{O}(y_b^4)$, $\mathcal{O}(g^2\alpha_S)$,
  some $\mathcal{O}(y_t^2 g^2)$, $\mathcal{O}(y_b^2 g^2)$

  3-loop: $\mathcal{O}(y_t^2\alpha_S^2)$ (Kant, Harlander, Mihaila, Steinhauser, `H3m`)

- Effective field theory + RG running

  Best bet for future progress?

  3-loop $\mathcal{O}(y_t^4\alpha_S)$ not negligible

Effective potential approximation

$$V_{\text{eff}}(v_u, v_d, g_3, g, g', y_t, y_b, y_\tau, \mu, b, m^2_{H_u}, m^2_{H_d}, m^2_{\tilde{Q}_{1,2,3}}, \ldots)$$
$$= V^{(0)} + V^{(1)} + V^{(2)} + \ldots$$

In the absence of CP violation:

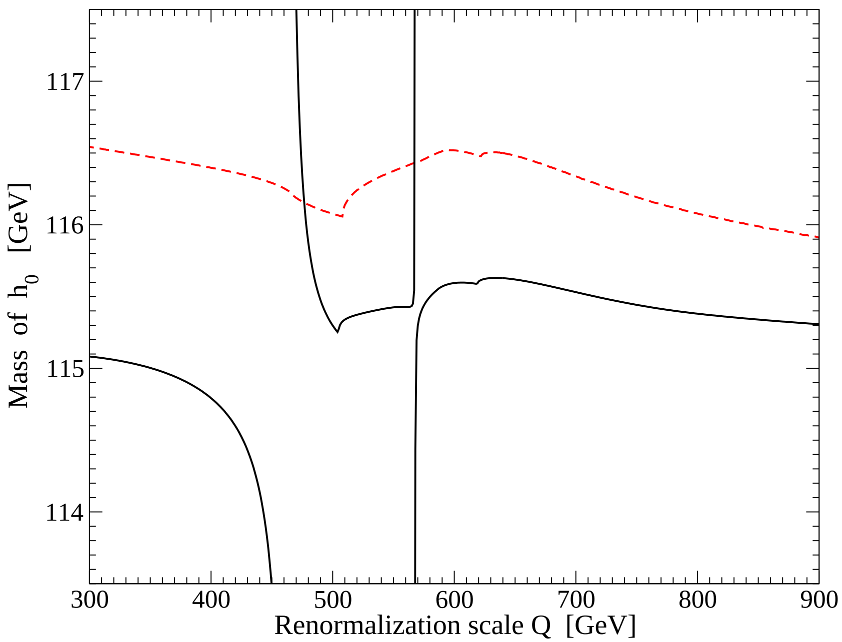
$$\begin{pmatrix} \Pi^{(n)}_{hh}(0) & \Pi^{(n)}_{hH}(0) \\ \Pi^{(n)}_{Hh}(0) & \Pi^{(n)}_{HH}(0) \end{pmatrix} = \frac{1}{2} R^T \begin{pmatrix} \frac{\partial^2 V^{(n)}}{\partial v_u^2} & \frac{\partial^2 V^{(n)}}{\partial v_u \partial v_d} \\ \frac{\partial^2 V^{(n)}}{\partial v_d \partial v_u} & \frac{\partial^2 V^{(n)}}{\partial v_d^2} \end{pmatrix} R,$$
$$R = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix}.$$

The full expression for $V^{(2)}$ is known (`hep-ph/0206136`), but only approximations neglecting some electroweak parts are implemented in public programs, as far as I know.

Derivatives of the effective potential can be taken numerically by finite differences, rather than analytically. Much easier!

Sample 2-loop effective potential result `hep-ph/0211366`

(model choice was pre-LHC; now completely ruled out!)



Red dashed: neglect electroweak effects in $V^{(2)}$.

Black solid: full $V^{(2)}$.

- 2-loop EW effects are not negligible.
- What's with the blowing up at special values of renormalization scale Q?

  At Q =568 GeV: $m_{G^0}, m_{G^\pm} = 0$.

  At Q =463 GeV: $m_{h^0} = 0$.

  This reflects a "flaw" in the effective potential approximation. . .

Reason for the blow-ups:

$$
\begin{aligned}
V^{(2)} &= m_h^2 \left[c_2 \ln^2(m_h^2/Q^2) + c_1 \ln(m_h^2/Q^2) \right] + \\
&\quad m_G^2 \left[c_1' \ln(m_G^2/Q^2) \right] + \ldots
\end{aligned}
$$

So, $V^{(2)}$ is finite, but derivatives of $V^{(2)}$ will diverge, whenever $m_G^2 = 0$ or $m_h^2 = 0$.

Also occurs in the Standard Model, and gets worse at higher loop orders, as we will see.

For the purposes of computing M_h^2, one has two choices:

- Avoid choices of Q where m_G^2 is small.
  Counter-intuitive! Might have expected $m_G^2 = 0$ to be a **good** scale choice.
- Avoid the effective potential approximation altogether.
  Keeping the p^2-dependent contributions to $\Pi(p^2)$ eliminates the singularities in M_h.

Feynman diagram approach to $\Pi(p^2)$

My versions of this are in hep-ph/0312092, 0405022. Some features:

- Purely $\overline{\mathrm{DR}}'$ scheme (no on-shell input parameters)
- On-shell top, bottom, gluino, squark, neutralino, chargino, heavy Higgs masses calculated separately and similarly.
- Couplings, masses from expanding around minimum of the 2-loop effective potential. (Tadpoles cancel.)
- Includes all contributions in the union of:
  - diagrams that involve α_S
  - diagrams that do not vanish when g, g' are set to 0
  - diagrams with at most one massive vector boson W, Z (in principle)
- No public code, yet. . .

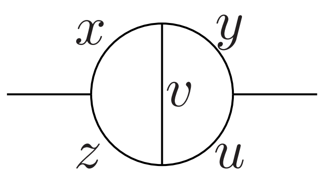
Evaluation of 2-loop self-energy basis integrals with arbitrary masses:

Values at $s = 0$ are known analytically, in terms of logs, polylogs.

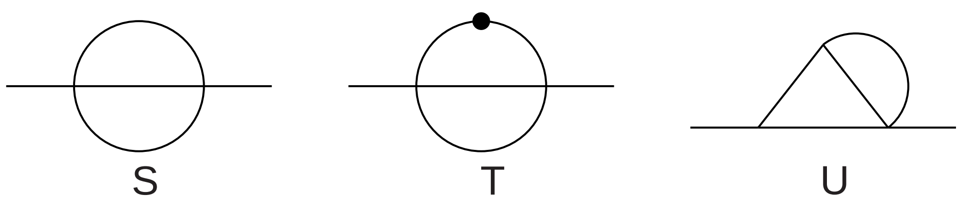
$$\frac{\partial}{\partial s}\text{(basis integral)} = \text{(another self-energy integral)}$$
$$= \text{(linear combination of basis integrals)}$$

So, we have a set of coupled, first-order, linear differential equations.

Consider the Master integral $M(x, y, z, u, v)$:



and 13 basis integrals obtained from it by removing propagators:



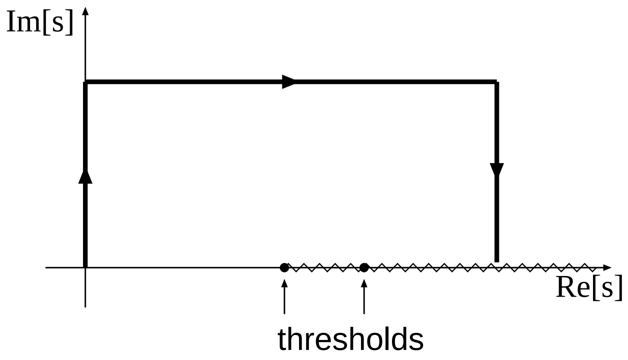
Call these 13 integrals I_n, $(n = 1, \ldots, 13)$.

Differential equations method for basis integrals

$$\frac{d}{ds} I_n = \sum_m K_{nm} I_m + C_n$$

Here K_{nm} are rational functions of s and $x, y, z \ldots$, and C_n are one-loop integrals. These are obtained by using Tarasov's recursion relations.

Solve for basis integrals I_n using Runge-Kutta integration in the complex s-plane, starting from known values at $s = 0$.



TSIL $=$ **T**wo-Loop **S**elf-energy **I**ntegral **L**ibrary

David G. Robertson, SPM, hep-ph/0501132

Program written in C, callable from C++, Fortran

- Basis integrals computed for any values of all masses and s.
- All 13 integrals from a given master topology obtained simultaneously in a single numerical computation.
- Checks on the numerical accuracy follow from changing choice of contour.
- Computation times of order 0.01 second for 13 basis integrals on modern hardware.
- TSIL knows all special cases that have been done analytically in terms of polylogarithms

In the Hopi culture native to the American southwest, Tsil is the Chili Pepper Kachina.

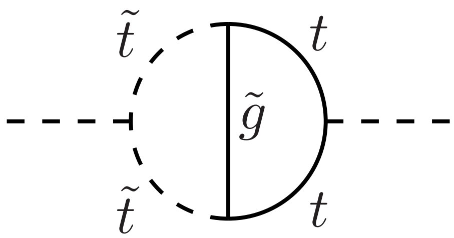
The Kachina are supernatural spirits, represented by masked figurines and impersonated by ceremonial dancers.

TSIL is a runner Kachina. If he beats you in a race, he may stuff your mouth with hot chili peppers.

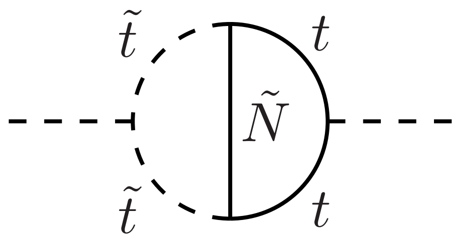
Back to the calculation of M_h in hep-ph/0312092 and hep-ph/0405022:

Sample diagrams included:

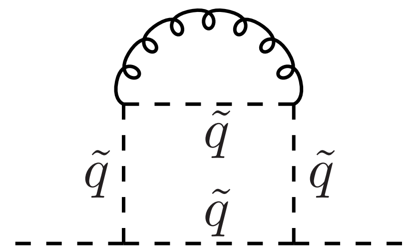
$\mathcal{O}(y_t^2 \alpha_S)$



$\mathcal{O}(y_t^4)$

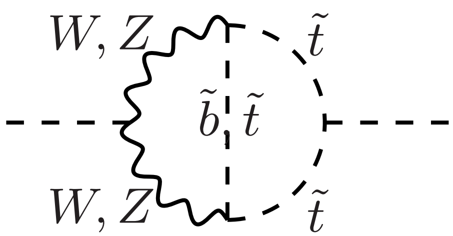


$\mathcal{O}(g^2 \alpha_S)$

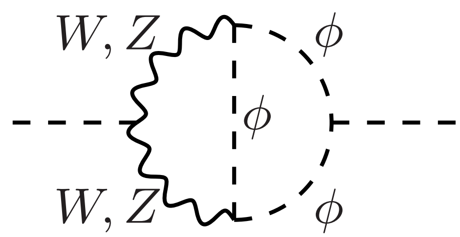


Not included:

$\mathcal{O}(g^2 y_t^2)$



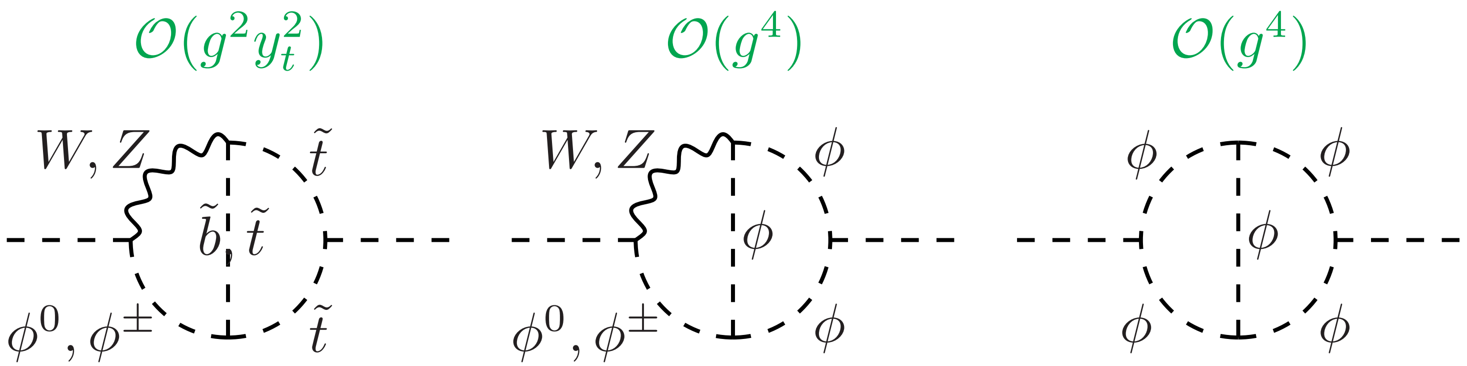
$\mathcal{O}(g^4)$



As far as I know, no calculation includes these. (?)

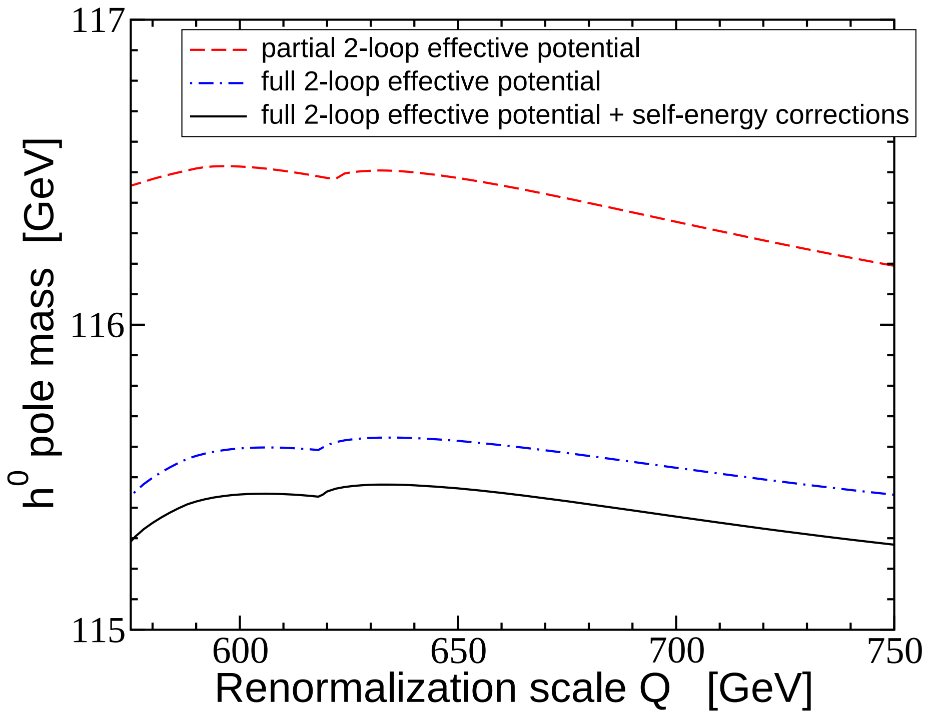
The effective potential approximation (and p^2 expansions) fails badly for graphs like the second one, when $\phi = G^0, G^\pm, h^0$.

Included in principle (formulas for generic theories in hep-ph/0312092) but not in practice (no explicit MSSM formulas in hep-ph/0405022):



Again, the effective potential approximation is a random number generator for these, when $\phi = G^0, G^\pm, h^0$.

No point in including these, until diagrams with multiple W, Z lines are included.



The difference between the full effective potential and the diagrammatic self-energy calculation seems to be of order 0.1 to 0.2 GeV.

However:

- Above is a pre-LHC model with light squarks
- The largest deviations should be expected for diagrams in which a momentum routing through the graph encounters only light (G^0, $G^\pm$, W, Z, h^0, b, $\ldots$) particles, where the effective potential approximation is bad. Those are exactly the 2-loop diagrams that have never (?) been calculated.

Question: How does one define the VEVs and $\tan\beta$?

Unfortunately, the SUSY Les Houches Accords (SLHA) standards leave this completely ambiguous.

`BLOCK HMIX` contains $\tan\beta(Q)$ and $v(Q)$, but this could mean several things:

- The minima of the **tree-level** potential, evaluated at Q.
  Need to include tadpole diagrams.
  Depending on choice of Q, may not be a proper minimum at all.
- The minima of the **2-loop** effective potential, evaluated at Q.
  No need to include tadpole diagrams; they cancel.
  Specifically Landau gauge.
  This is what I use.
- Some other determination (from Z mass)?

(This is one of several things that makes comparisons between approaches problematic. Which data are inputs; physical top mass or top Yukawa coupling?)

What we learned since July 4, 2012:

- A Higgs scalar exists near $M_H = 126$ GeV
- Consistent with Standard Model Higgs
- No new physics that could be associated with non-Standard-Model-ness of the EWSB sector is apparent, including SUSY

If we still believe in SUSY, we're probably looking at very heavy gluino and squarks, at least.

Dear Susy,

Lately, you seem very distant. You’ve ignored all my plans and invitations to get together. Sometimes I wonder if you are there for me at all. You are very beautiful and I do love you but maybe I should date other models for a while.

P.S. In a couple of years, maybe we can try again?

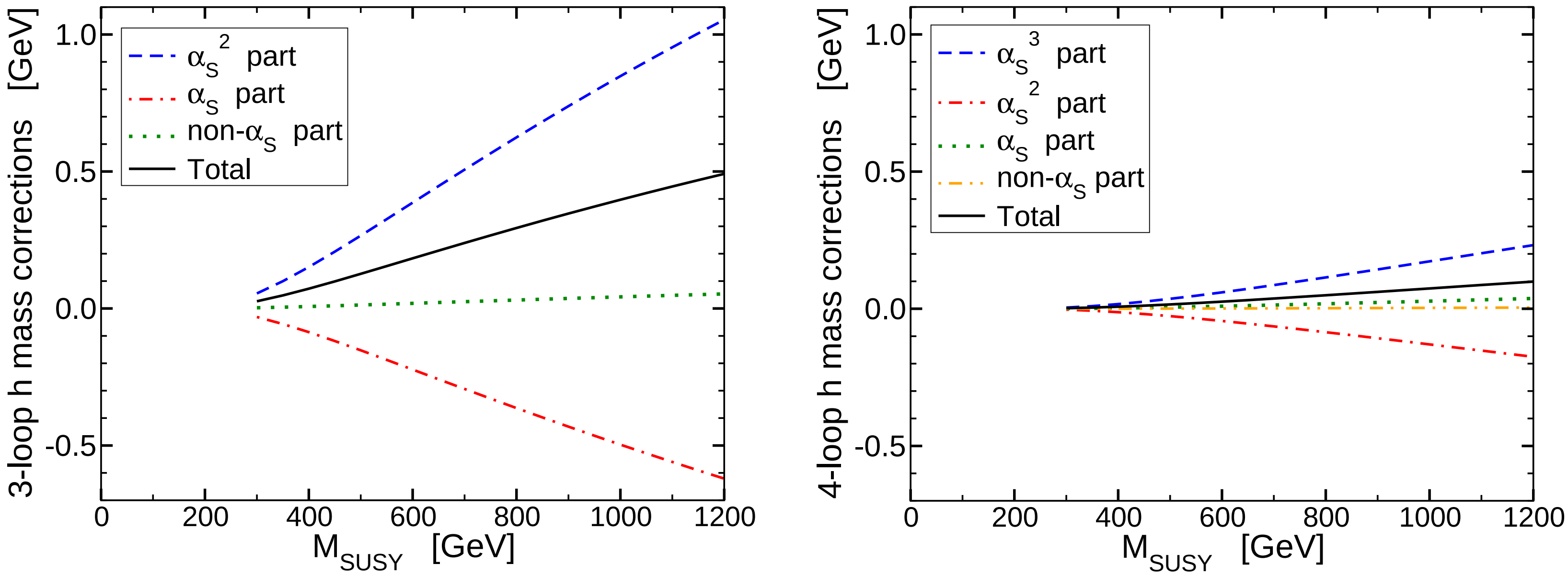
The data suggests that maybe the Higgs mass calculation, and electroweak symmetry breaking, should be analyzed within the context of the Standard Model, even if supersymmetry is correct.

Strategy:

- Matching of MSSM parameters to Standard Model parameters at the Lagrangian level, as an effective theory. Perform at multi-loop order, including all thresholds.
- Renormalization group running to, say, $Q = 175$ GeV.
- Calculate v, M_h, M_t, M_W, M_Z strictly in terms of the Standard Model parameters: m_H^2, λ, g_3, g, g', y_t, y_b.

Also need non-renormalizable couplings in the Standard Model effective theory, if some superpartners are light?

Leading-log 3-loop and 4-loop contributions to M_h, for common superpartner mass $M_{\rm SUSY}$, $\tan\beta \gg 1$, $M_t = 172$ GeV. (From hep-ph/0701051)



The NLO in QCD tends to cancel about half of the LO in QCD.

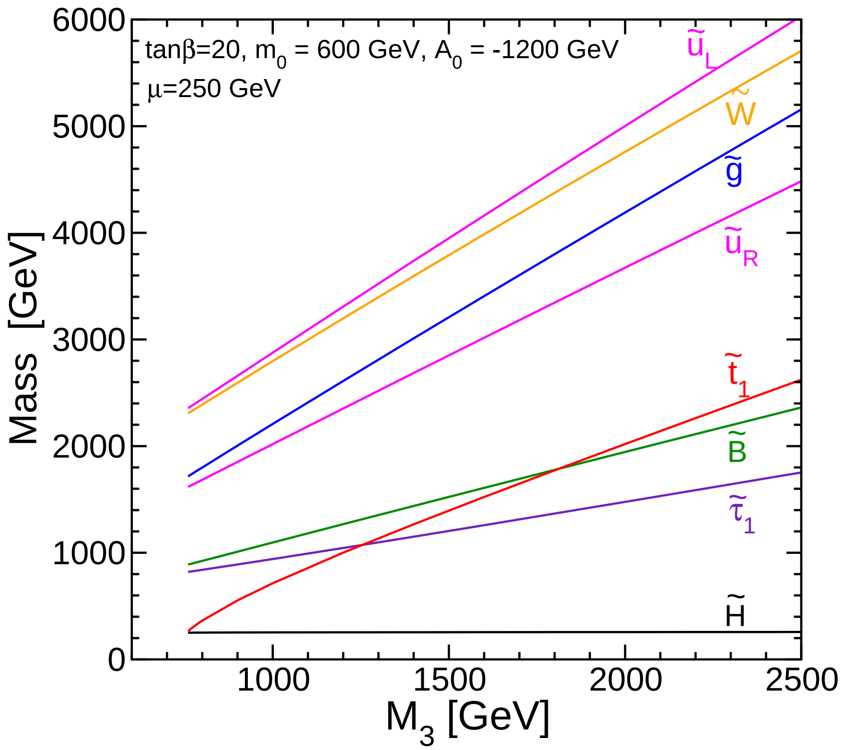
Caveat: depends on how perturbation theory is organized. Your mileage may vary!

Practical Lesson: Theoretical errors are still significant.

The thresholds could be highly non-degenerate.

“Semi-natural” supersymmetry models = light Higgsinos, everything else heavy

Gordy Kane, James Wells, Howie Baer, and others have explored this sort of thing.
(“Light Higgsino World”)



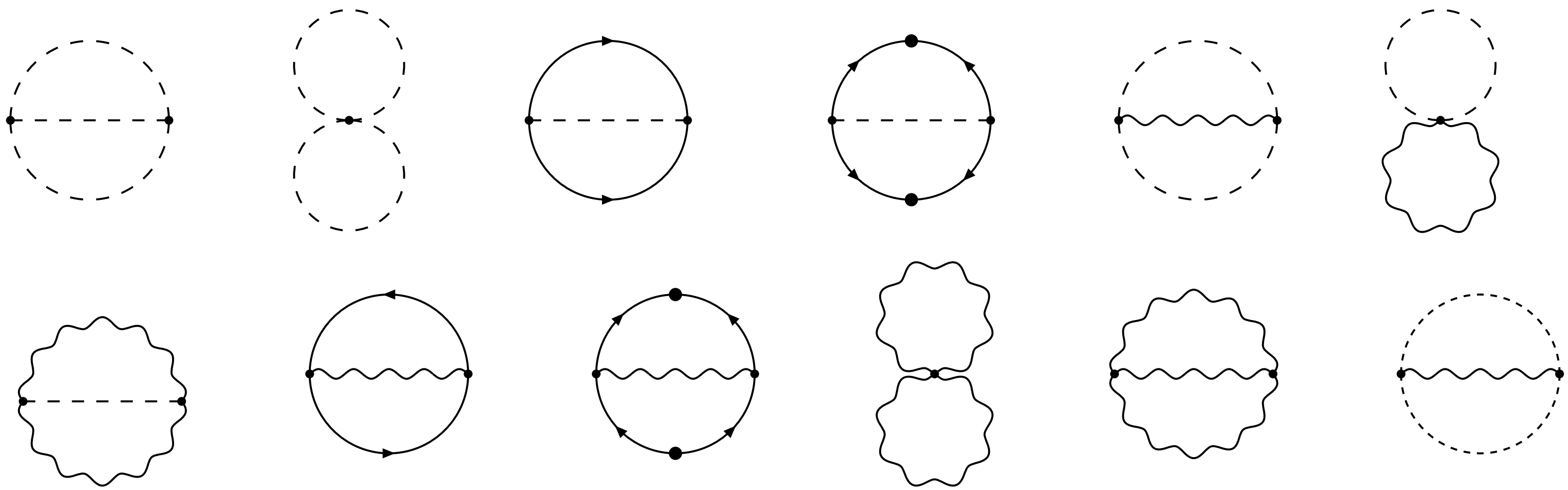
Non-universal gaugino masses
(SPM 1312.0582)

Could be extremely hard to detect **any** superpartners at LHC.

Hierarchy of effective theories.

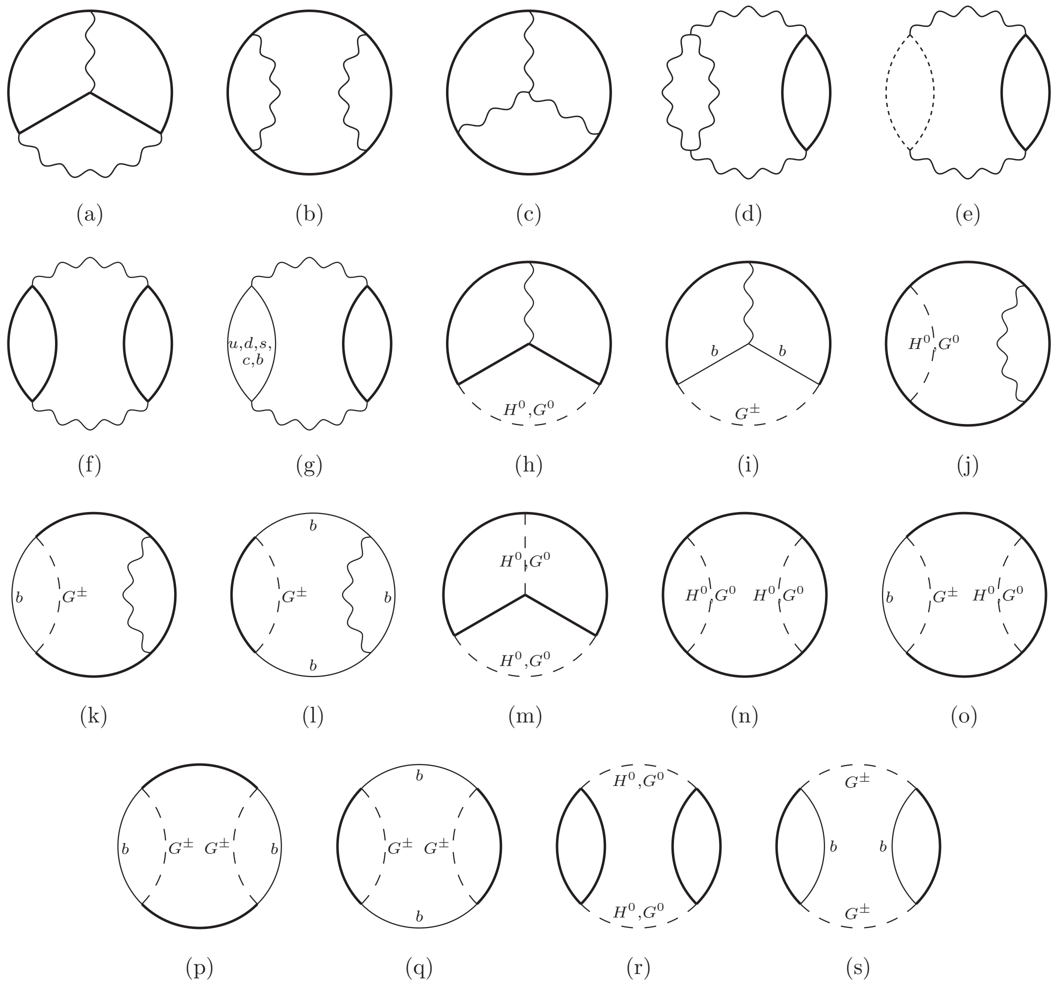
Recently, I worked out the leading contributions to the 3-loop $V_{\rm eff}$ within the Standard Model. That's what I will spend the rest of my time discussing.

The effective potential is computed as the sum of 1-particle irreducible vacuum graphs. At 2-loops, this was done in a classic paper of Ford, Jack, and Jones Nucl.Phys. B387 (1992).



Results are reduced to dilogarithms, for any mass ratios. Any theory uses the same 12 functions, up to subtleties involving dimensional reduction for SUSY. (hep-ph/0111209).

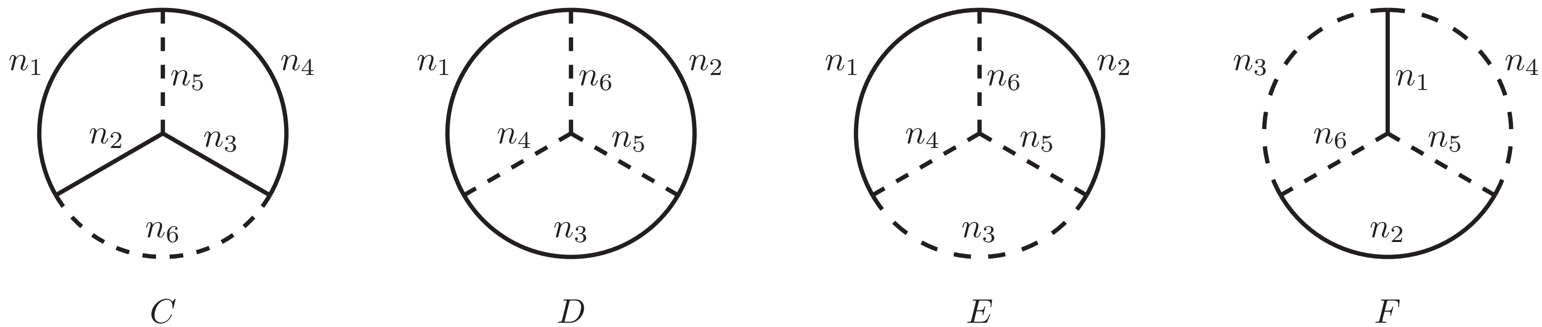
For the 3-loop Standard Model contribution, take the leading order diagrams: $g_3^4 m_t^4$ and $g_3^2 y_t^2 m_t^4$ and $y_t^4 m_t^4$:



How to calculate these?

Most only involve one scale: the field-dependent, running, top quark (mass)$^2 = y_t^2 v^2/2$.

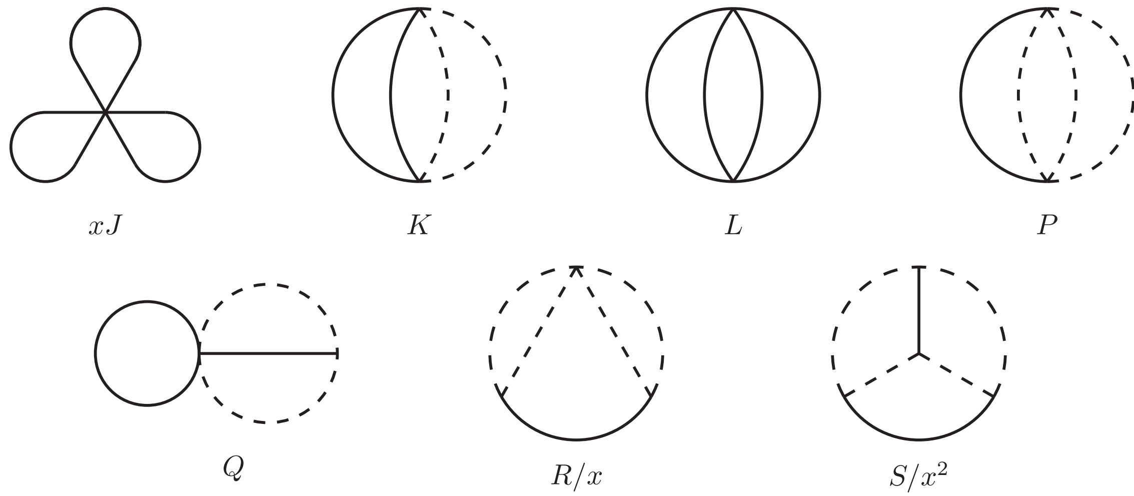
By elementary algebra, can reduce the necessary integrals to 4 types:



The n_i are the powers to which the propagators are raised.
Solid line = top (mass)^2, dashed = massless.

Now use recursion relations obtained by dimensional analysis and integration by parts, to reduce to only 7 “master” integrals. . .

The 7 master integrals:



These were evaluated by D. Broadhurst in Z. Phys. C 54, 599 (1992), Eur. Phys. J. C 8, 311 (1999) [hep-th/9803091]. Examples:

$$
\begin{aligned}
J &= A^3/x, \\
K &= \frac{A^3}{x}\Big(-\frac{1}{3}-\frac{\epsilon}{6}+\frac{5\epsilon^2}{12}+\left[\frac{79}{24}-\frac{8\zeta(3)}{3}\right]\epsilon^3+\left[\frac{685}{48}+\frac{2\pi^4}{15}-\frac{4\zeta(3)}{3}\right]\epsilon^4+\dots\Big),
\end{aligned}
$$

where $x =$ (mass)^2, and

$$
A \equiv \int_p \frac{1}{p^2+x} = \frac{\Gamma(1-d/2)}{(4\pi)^{d/2}} x^{d/2-1}.
$$

Also need some 2-scale integrals, for the diagrams that have virtual Goldstones and Higgs. They can be evaluated in terms of the 1-scale integrals, by differential equations method. For example:

$$I_2(x,y) \equiv \int_p \int_q \frac{1}{(p^2+x)q^2[(p+q)^2+y]},$$

$$I_3(x,y) \equiv \int_p \int_q \int_k \frac{1}{[(p-k)^2+x][(q+k)^2+x]p^2q^2[k^2+y]}.$$

From integration by parts, these satisfy:

$$(x-y)^2 \frac{d}{dy} I_2 = (3-d)(x-y)I_2 + (1-d/2)(1-y/x)(y/x)^{d/2-1}A^2,$$

$$2y(y-x)\frac{d}{dy} I_3 = (dx - 2x + 3dy - 10y)I_3 + (2d-4)AI_2 + (8-3d)K.$$

Evaluate by expanding in small $y =$ Goldstone or Higgs $(\text{mass})^2$ and $d = 4 - 2\epsilon$.

Now, after renormalization, obtain for the Standard Model $V_{\rm eff} = \sum_\ell \frac{V^{(\ell)}}{(16\pi^2)^\ell}$:

$$\begin{aligned}
V^{(3)} \;=\;& g_3^4 T^2 \Big\{ -184\,\overline{\ln}^3(T) + 868\,\overline{\ln}^2(T) - 1842.2\,\overline{\ln}(T) + 1957.3 \Big\} \\
&+ g_3^2 y_t^2 T^2 \Big\{ 60\,\overline{\ln}^3(T) - 360\,\overline{\ln}^2(T) + 1220.9\,\overline{\ln}(T) - 780.3 \Big\} \\
&+ y_t^4 T^2 \Big\{ 24.75\,\overline{\ln}^3(T) + [81 + 81\ \ln(H/T) + 27\ \ln(G/T)]\overline{\ln}^2(T) \\
&+ [-971.6 - 54\ \ln(H/T) - 54\ \ln(G/T)]\overline{\ln}(T) \\
&+ 504.5 + 9\ \ln(H/T) + 27\ \ln(G/T) \Big\}.
\end{aligned}$$

where the field-dependent squared masses for top, Higgs, and Goldstone are:

$$\begin{aligned}
T &= y_t^2 \phi^2/2, \\
H &= m^2 + 3\lambda\phi^2, \\
G &= m^2 + \lambda\phi^2,
\end{aligned}$$

and

$$\overline{\ln}(X) = \ln(X/Q^2).$$

Some comments (see 1310.7553 for more details):

- Keeping all Lagrangian input parameters fixed, the impact of including the 3-loop potential on the VEV is:

  $\Delta_{3-\mathrm{loop}}\, v(M_t) = -0.34$ GeV $\quad (-0.14\%)$

  However, in the real world, Lagrangian λ and m^2 are not directly accessible.

- There is a remarkable cancellation in $\Delta_{3-\mathrm{loop},g_3^4}\lambda$. It is more than a factor of 30 smaller smaller than individual contributions.

- Effect on $\Delta_{3-\mathrm{loop}}\lambda$ is comparable to parametric error from a 100 MeV uncertainty on the Higgs mass.

- Need to compute full 3-loop self-energy for real estimate of $\Delta_{3-\mathrm{loop}}\lambda$.

- The 3-loop effective potential is singular in the limit of $G \to 0$.

The Goldstone Boson Catastrophe

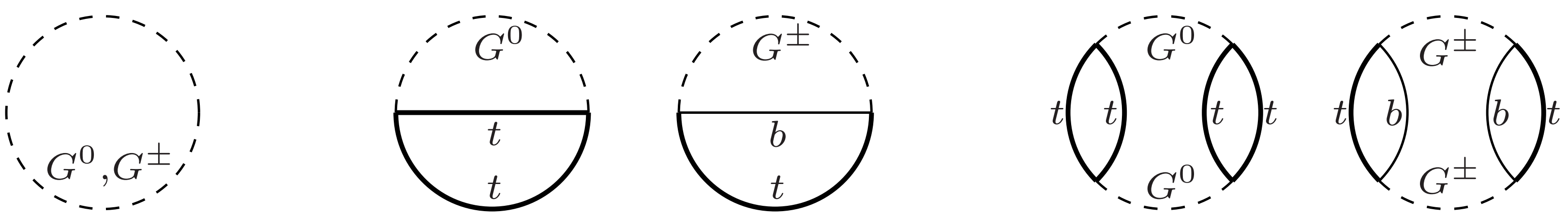
With $T, G =$ squared masses of top, Goldstone:

$$V^{(1)} \sim \frac{3}{4} G^2 \,\overline{\ln} G,$$ 2nd derivative singular as $G \to 0$

$$V^{(2)} \sim -3N_c y_t^2 T \left[\overline{\ln} T - 1\right] G \,\overline{\ln} G,$$ 1st derivative singular as $G \to 0$

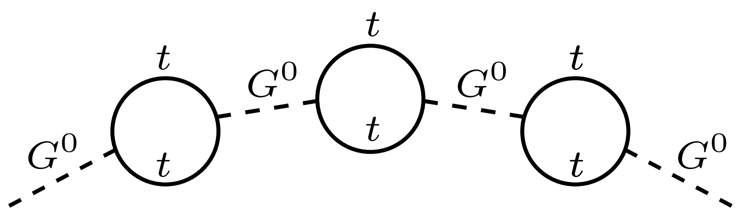
$$V^{(3)} \sim 3\left[N_c y_t^2 T (\overline{\ln} T - 1)\right]^2 \,\overline{\ln} G.$$ singular as $G \to 0$

These come from diagrams:

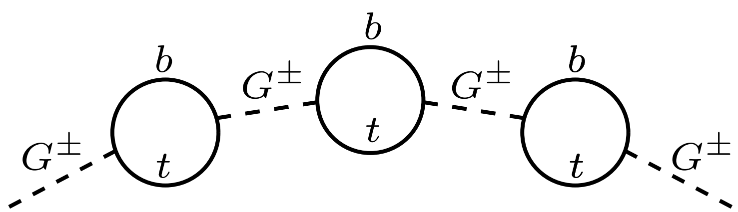


At higher loop orders, the $G \to 0$ singularities should get worse. . .

From ℓ-loop diagrams with $\ell - 1$ top or top/bottom one-loop subdiagrams:



$$V^{(\ell)} \sim (N_c y_t^2)^{\ell-1} T^2 \left(\frac{T}{G}\right)^{\ell-3} \overline{\ln} G$$



For $\ell \geq 4$,
power-law singularity as $G \to 0$.

For a generic choice of renormalization scale, $G \neq 0$, and there is no true singularity.

However, one might have expected that choosing Q so that $G = 0$ would be a *good* choice, since, at all orders, the Goldstone is massless in Landau gauge. (In the Standard Model, this choice is roughly $Q = 110$ GeV.)

In reality, this is the one choice one must not make!

Can some kind of resummation or RG improvement “fix” this?

Outlook

- Different approaches are useful
- Plenty to do
- My opinion: data is pushing us towards an effective field theory approach to calculating M_h
  - Calculate M_h just within Standard Model
    Useful even if SUSY is wrong...
  - Match SUSY to the Standard Model (and very possibly intermediate effective theories) and run
- Plenty to do at the level of the Standard Model (3-loop M_h, 4-loop V_{eff})
- Is there a neat resolution of the Goldstone Boson Catastrophe?