Discrete Symmetries of Quiver Theories and Wrapped Branes
hep-th/0602094

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Great Lakes Strings
April Fool's Day

Overview

Pattern first recognized in hep-th/9811048:
(Gukov, Rangamani, Witten)

- D3 on Orbifold 6D Backgrounds → Quiver Gauge Theories
- Orbifold $\mathbb{Z}_n$ Backgrounds → Cycles Valued in $\mathbb{Z}_n$
- Branes may wrap these cycles.
- Number Operators of Wrapped Branes have AdS/CFT Dual
- Quiver Gauge Theories Have $\mathbb{Z}_n$ symmetries
- Discrete Symmetries → NONCOMMUTATIVE

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$Y^{p,q}$ Geometries/CFT Duals

New infinite class of theories $Y^{p,q}$ geometries (Gauntlett, Martelli, Sparks, Waldram (0403002))

$$ds_10^2 = H^{-\frac{1}{2}} dx^\mu dx_\mu + H^{\frac{1}{2}} \left(dr^2 + r^2 \left(ds^2_{Y^{p,q}} \right) \right) \quad (1)$$

When $\mathrm{GCD}(p, q) = a \neq 1$ these are orbifold geometries. Quiver diagram given by (Martelli, Sparks (0411238))

$$\underbrace{\underbrace{(\sigma\tilde{\sigma}\tau \cdots\cdots\cdots\cdots\cdots\cdots\cdots)}_{((p-q)/a)\ \tau-\text{type},\ (q/a)\ \sigma-\text{type}}(\cdots)(\cdots)\cdots}_{a-\text{times}} \quad (2)$$

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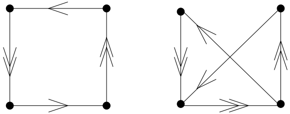
Unit cells



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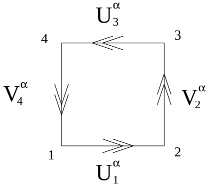
Our Work

Example: $Y^{2,0}$: Diagram



- Symmetries $A : (1,2,3,4) \to (3,4,1,2)$
  $B : (1,1,\omega,\omega^{-1})$ and $C : (\omega,\omega^{-1},\omega^{-1},\omega)$ with $\omega^{2N} = 1$
- These satisfy (up to the COGG)

$$A^2 = B^2 = C^2 = 1, \quad AB = BAC, \quad C \text{ commutes} \qquad (3)$$

and is a finite Heisenberg Group

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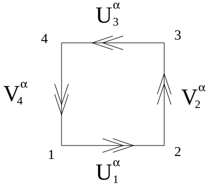

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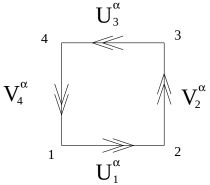

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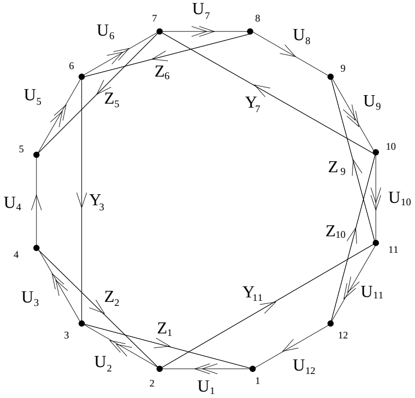
General $Y^{p,q}$ (p, q not coprime)

We find this to be a general pattern,
even for complicated $Y^{p,q}$!
We work out explicitly:



Conclusions

For a large class of theories, we find that
Wrapped Brane Number Operators
DO NOT COMMUTE!
(Worked on by D. Belov and G. Moore)

We later generalize this to even the non-conformal case!
(hep-th/0603114)
(also, see hep-th/0412193 Herzog, Ejaz, Klebenov for non-conformal generalizations)