The structure of solutions of pure N=4 supergravity in five dimensions

by

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Motivation

Supergravity solutions are important

- describe low mass d.o.f. of super-string/M theory
- Gauge-gravity correspondence
- Classical solutions such black holes, black rings, p-branes and pp waves

Two popular methods

- Make ansatz for the metric based on isometries.
- Analyze G-structure (when a Killing spinor is present)

For large number of supersymmetries

- G-structure method more involved
- New solutions

Simple case: N=4, D=5 supergravity

{Awada and Townsend NPB255(1985)617}

Method applied to N=2, D=5 case by

{Gauntlett, Martelli, Sparks, Waldram: Class. Quan. Grav. **20**(2003)4587}

Generalize method to N=4, D=5 case with Lagrangian (Bosonic part)

$$\mathcal{L} = -\frac{1}{2}R - \frac{1}{4}e^{2\sigma/\sqrt{3}}F^{ij}_{\mu\nu}F^{\mu\nu}_{ij} - \frac{1}{4}e^{-4\sigma/\sqrt{3}}G_{\mu\nu}G^{\mu\nu} - \frac{1}{2}(\partial_\mu\sigma)^2$$

Content

R symmetry : USp(4)

Content:	e_μ^m	$(A_\mu^{ij},$	$B_\mu)$	σ	$\Psi_\mu^i,$	χ^i
USp(4) rep.	$\mathbf{1}$	$\mathbf{5}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{4}$	$\mathbf{4}$

$$f^{[ij]} = i\bar{\epsilon}^i\epsilon^j \qquad f, f^a \qquad \mathbf{1}+\mathbf{5}$$

$$V_\mu^{[ij]} = \bar{\epsilon}^i\gamma_\mu\epsilon^j \qquad K_\mu, V_\mu^a \qquad \mathbf{1}+\mathbf{5}$$

$$\Phi^{(ij)} = i\bar{\epsilon}^i\gamma_{\mu\nu}\epsilon^j \qquad \Phi^{ab} \qquad \mathbf{10}$$

Related by $\delta\Psi_\mu^i = 0$, $\delta\chi^i = 0$ and Fierz relations.

Properties

Killing vector $$\boxed{K_\mu = \frac{1}{4}\Omega_{ij} V^{ij}_\mu}$$

$$K_\mu K^\mu = -f^{a2} \quad \begin{array}{ll} = 0 & \text{null case} \\ < 0 & \text{time-like case} \end{array}$$

Identification of a Killing vector naturally separates the metric into a Killing direction and a 3(4)-dimensional base in (null) time-like case

The base possesses

- R^3 structure in null case.
- $SU(2)$ structure in a timelike case ($f^{a2} = f^2$).
- $SO(4)$ structure in general timelike case.
- Holonomy not preserved in general.

Null case: R^3 structure

$$ds^2 = H^{-1}du(2dv + \mathcal{F}du) + H^2 h_{mn}(dx^m + a^m du)(dx^n + a^n du)$$

$$G = G_{+m} e^+ \wedge e^m - H^{-2} *_3 d\mathcal{H}_1$$

$$F^a = F^a_{+m} e^+ \wedge e^m + \frac{1}{\sqrt{2}} H^{-2} *_3 [u^a d\mathcal{H}_2 - \mathcal{H}_2 du^a]$$

$$V^a_\mu = u^a K_\mu \qquad e^{\sqrt{3}\sigma} = \frac{\mathcal{H}_1}{\mathcal{H}_2}$$

$\mathcal{H}_1$ and $\mathcal{H}_2$ are harmonic.

u^a points out $SO(5) \supset SO(4) \simeq SU(2)_L \times SU(2)_R$.

Work in progress

- Characterize better the 3(4) dim base by identifying their holonomies from SUSY variation integrability conditions.
- Further constrain yet undermined functions using Bianchi identities and Einstein equation.
- Construct particular solutions in various cases.