Physics 406: Homework 1

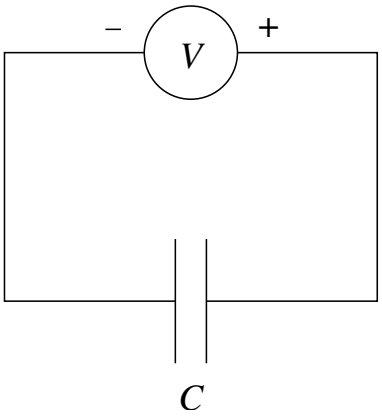
1. **Perfect gas:** The pressure p and volume V of one (gram) mole of a perfect gas obey the equation of state $pV = RT$, where $R = 8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$.

   (a) How much work must be done to compress a mole of this gas at constant temperature T from volume V_1 to volume V_2? Verify that your answer is indeed positive when the volume of the gas gets *smaller*.

   (b) How much work must be done to compress a mole of this gas at constant temperature T from pressure p_1 to pressure p_2?

   (c) How much work must be done to compress *half* a mole of this gas at constant temperature $T = 20°\mathrm{C}$ from one atmosphere pressure to two atmospheres? (Recall that $0°\mathrm{C} \equiv 273.15\,\mathrm{K}$ at atmospheric pressure.)

2. **Electrical work:** Suppose we have a circuit in which a voltage source is connected to a capacitor with capacitance C (which may depend on temperature):



   Starting with an uncharged capacitor and zero voltage, we isothermally increase the voltage, so as to charge the capacitor.

   (a) If the charge across the capacitor is q at some point, what is the voltage across it? (Note: we use lower-case q to avoid confusion with heat Q, but the charge is the extensive variable here.)

   (b) The definition of electrical work is the product of charge moved times the potential difference it moves across. So the amount of work đW that needs to be done by the voltage source in order to move an extra small amount of charge dq from one side of the capacitor to the other is đ$W = V\mathrm{d}q$. This is the equivalent of đ$W = -p\mathrm{d}V$ in our fluid system. If we charge the capacitor from empty up to a charge q, what is the total work done as a function of q and the capacitance C?

   (c) And as a function of the voltage V?

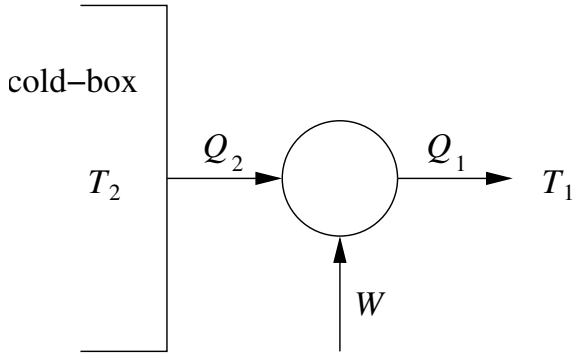
   (d) Capacitors heat up when you charge them. What is the differential form of the First Law for a change in internal energy of the capacitor (i.e., what is the expression for an infinitessimal change dU in the internal energy of the capacitor)? And what is the amount of heat absorbed đQ by the capacitor for a given change dU in internal energy and dq in charge?

(e) Hence write down an expression for the heat capacity c_q at constant charge of the capacitor, and another (more complicated) one for the heat capacity c_V at constant voltage. If the capacitance is independent of temperature, what is the expression for c_V?

(f) An infinitessimal change in the enthalpy of a pressure/volume system is, as we have seen, $\mathrm{d}H = đQ + V\mathrm{d}p$. What is the corresponding expression for the enthalpy of a capacitor?

3. **Heat capacity of copper:** A 0.1 kg piece of copper metal is heated to 100°C and dropped into a thermally insulated vessel containing 0.2 liters of water at 15°C. After the system has come to equilibrium, the temperature is measured again and both copper and water are found to be at 18.8°C. Assuming the heat capacities of both water and copper to be constant over the range of temperatures involved, calculate the ratio of the specific heat capacities of copper and water.

4. **Refrigerator:** An improbable domestic refrigerator, which contains a quart of milk, a bowl of petunias, and an unidentified green object left there by the previous tenant, is perfectly efficient, i.e., it is as efficient as any refrigerator can be. A refrigerator's efficiency is the ratio between the amount of heat Q_2 extracted from the cold-box and the amount of work W done: $\eta = Q_2/W$.



(a) What is the efficiency of the refrigerator in terms of the temperatures T_1 and T_2 of the kitchen and the cold-box?

(b) If the cold-box is at -4°C and the kitchen is at 23°C, what is the efficiency?

(c) If the refrigerator consumes 100 Watts when running, at what rate does heat come out the back of the refrigerator?

(d) A real refrigerator is of course not perfectly efficient. If a refrigerator with only 20% the efficiency is used to cool the same selection of objects to the same temperature, how much power will the fridge consume when running? And how much heat will come out the back?