Physics 406: Summary of important results

This is a list of important equations and other results that you should know. You may take this list into the final exam with you.

Partial derivatives:

$$\mathrm{d}f = \left.\frac{\partial f}{\partial x}\right|_y \mathrm{d}x + \left.\frac{\partial f}{\partial y}\right|_x \mathrm{d}y, \qquad \left.\frac{\partial x}{\partial y}\right|_z = \left[\left.\frac{\partial y}{\partial x}\right|_z\right]^{-1}, \qquad \left.\frac{\partial x}{\partial y}\right|_z = -\left.\frac{\partial x}{\partial z}\right|_y \left.\frac{\partial z}{\partial y}\right|_x.$$

Internal energy: $\mathrm{d}U = đQ + đW$ with $đQ = T\,\mathrm{d}S$ and:

$đW = -p\,\mathrm{d}V$	(fluid pressure/volume system)
$đW = f\,\mathrm{d}L$	(spring or wire with force f and length L)
$đW = V\,\mathrm{d}q$	(capacitor with voltage V and charge q)
$đW = -\mathbf{B}\cdot\mathrm{d}\mathbf{m}$	(magnet with magnetization $\mathbf{m}$ in field $\mathbf{B}$)
$đW = \gamma\,\mathrm{d}A$	(surface with surface tension γ and area A)

Thus for example, in a pressure/volume system $\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V$. This applies for irreversible as well as reversible changes, but the individual equalities $đQ = T\,\mathrm{d}S$ and $đW = -p\,\mathrm{d}V$ only apply for reversible ones. Heat capacity at constant x (where x is any variable) is in general given by

$$C_x = T\left.\frac{\partial S}{\partial T}\right|_x, \qquad \text{e.g.,}\quad C_V = T\left.\frac{\partial S}{\partial T}\right|_V \quad \text{and} \quad C_p = T\left.\frac{\partial S}{\partial T}\right|_p.$$

Potential functions and Maxwell relations: For pressure volume system

$$H = U + pV \quad \text{(enthalpy)}, \qquad F = U - TS \quad \text{(free energy)}, \qquad G = U + pV - TS \quad \text{(Gibbs energy)}.$$

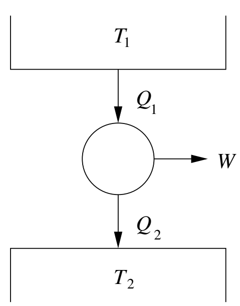
Similar expressions apply for other types of systems (non-pressure/volume systems). There is one Maxwell relation for each potential function, derived by equating partial second derivatives. For instance

$$\frac{\partial^2 U}{\partial V \partial S} = \frac{\partial^2 U}{\partial S \partial V} \qquad \Rightarrow \qquad \left.\frac{\partial T}{\partial V}\right|_S = -\left.\frac{\partial p}{\partial S}\right|_V.$$

Each heat capacity is the derivative of the corresponding potential function:

$$C_V = \left.\frac{\partial U}{\partial T}\right|_V, \qquad C_p = \left.\frac{\partial H}{\partial T}\right|_p.$$

Heat engines:



Efficiency of a reversible engine ($T_1 > T_2$):

$$\eta = \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}, \qquad \eta_R = \frac{Q_2}{W} \quad \text{(refrigerator)}, \qquad \eta_H = \frac{Q_1}{W} \quad \text{(heat pump)}.$$

Isolated systems: All microstates equally likely. Most likely macrostate maximizes the Boltzmann entropy $S = k \ln g$, where g is the multiplicity. When non-interacting systems are combined, entropy is additive (i.e., extensive); multiplicity is multiplicative.

Fixed temperature systems: States s appear with Boltzmann probability

$$p(s) = \frac{\mathrm{e}^{-\varepsilon_s/\tau}}{Z}, \qquad Z = \sum_s \mathrm{e}^{-\varepsilon_s/\tau},$$

where $\tau = kT$ and $k = 1.38 \times 10^{-23}\,\mathrm{JK}^{-1}$. Macroscopic thermodynamic quantities are then given by

$$U = \tau^2 \frac{\partial \ln Z}{\partial \tau}\bigg|_V, \qquad F = -\tau \ln Z, \qquad C = \frac{\partial U}{\partial \tau}\bigg|_V, \qquad \sigma = -\frac{\partial F}{\partial \tau}\bigg|_V, \qquad p = -\frac{\partial F}{\partial V}\bigg|_\tau.$$

Sterling's approximation: $\ln k! \simeq k \ln k - k$.

Perfect gas: Density of states in three dimensions is

$$n(\varepsilon) = \frac{V(2I+1)}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2},$$

where I is the spin of the particles. ($I = \frac{1}{2}$ for fermions.)

$$Z = \frac{1}{N!} Z_1^N, \qquad Z_1 = \frac{V}{(2\pi\hbar^2/m\tau)^{3/2}}, \qquad pV = N\tau, \qquad \sigma = N(\tfrac{5}{2} - \ln[(2\pi\hbar^2/m\tau)^{3/2}\rho]),$$

where $\rho = N/V$ is the number density.

Photons and phonons: Density of states is

$$n(\omega) = \frac{V}{\pi^2 c^3}\,\omega^2\,\mathrm{d}\omega \quad \text{(photons, } c \text{ is speed of light)}, \quad n(\omega) = \frac{3V}{2\pi^2 v^3}\,\omega^2\,\mathrm{d}\omega \quad \text{(phonons, } v \text{ is speed of sound)}.$$

Systems with variable numbers of particles: Grand ensemble:

$$\mathcal{Z} = \sum_s \mathrm{e}^{-(\varepsilon_s - \mu N_s)/\tau}, \qquad U = \tau^2 \frac{\partial \ln \mathcal{Z}}{\partial \tau}, \qquad \Omega = -\tau \ln \mathcal{Z},$$

$$\sigma = -\frac{\partial \Omega}{\partial \tau}\bigg|_{V,\mu}, \qquad p = -\frac{\partial \Omega}{\partial V}\bigg|_{\tau,\mu}, \qquad N = -\frac{\partial \Omega}{\partial \mu}\bigg|_{V,\tau}.$$

Quantum gases: Number of particles in single-particle state with energy ε is

$$f(\varepsilon) = \frac{1}{\mathrm{e}^{(\varepsilon-\mu)/\tau} + 1} \quad \text{(fermions)}, \qquad f(\varepsilon) = \frac{1}{\mathrm{e}^{(\varepsilon-\mu)/\tau} - 1} \quad \text{(bosons)}.$$