Complex Systems 535/Physics 508: Final review

The final exam is at **10:30am–12:30pm on Monday, December 21** in 455 Dennison (the usual room). Here is a list of the topics you should know about for the exam.

Empirical studies of networks:

1. The basic types of network: technological, social, information, and biological, and the principal examples of each
2. Techniques for measuring networks, such as:
   (a) Internet: traceroute sampling, BGP
   (b) Social networks: questionnaires, observation, archival records, affiliation networks, the small-world experiment, snowball sampling
   (c) The Web: web crawlers
   (d) Citation networks: citation databases, online citation crawlers
   (e) Metabolic networks: radiological tagging
   (f) Protein networks: co-immunoprecipitation, affinity purification, two-hybrid screens
   (g) Neural networks: slice electron microscopy

Basic graph theoretical concepts:

1. Vertices, directed and undirected edges
2. Undirected and directed networks, cocitation and bibliographic coupling
3. Bipartite networks, one-mode projections
4. Cyclic and acyclic networks
5. Adjacency matrices, incidence matrices
6. Degree, in-degree, out-degree, degree distributions
7. Paths, geodesics, Eulerian and Hamiltonian paths, diameter, vertex- and edge-independent paths, connectivity, Menger's theorem, the max-flow/min-cut theorem
8. Components, weakly connected and strongly connected, in- and out-components
9. The graph Laplacian

Measures and metrics:

1. Centrality, degree centrality, eigenvector centrality, Katz centrality, PageRank, HITS, closeness, betweenness
2. Groups of vertices: k-components, k-cliques, k-cores, etc.
3. Transitivity, clustering coefficients, reciprocity
4. Signed networks, structural balance, clusterability
5. Similarity: structural equivalence, cosine similarity, Euclidean distance, Pearson coefficient
6. Assortative mixing, disassortative mixing, modularity and correlation coefficients

Empirical properties of real-world networks:

1. Component structure (one large component, many small ones)
2. Path lengths and diameter, the small-world effect
3. Clustering/transitivity and the clustering coefficient
4. Degree distributions, especially the scale-free case of a power-law degree distribution
5. Assortative mixing

Algorithms:

1. Computational complexity of algorithms and how to calculate it
2. Data structures for storing networks
3. Complexity of simple operations such as adding and removing edges
4. Basic network algorithms, such as breadth first search, closeness centrality, betweenness centrality, Dijkstra's algorithm, and the Ford–Fulkerson maximum flow algorithm (the "augmenting path" algorithm), and their complexity
5. Matrix-based algorithms, such as calculation of the leading eigenvector and the algebraic connectivity
6. Graph partitioning and community detection, the Kernighan–Lin algorithm, spectral partitioning, modularity maximization, hierarchical clustering, single and complete linkage methods, dendrograms, betweenness-based algorithms

Models of networks:

1. The Poisson random graph $G(n,p)$, degree distribution, clustering coefficient, phase transition, giant component, size of the giant component, average size of a small component, diameter
2. The configuration model, generating functions and their properties, degree and excess degree, component sizes, giant component, position of the phase transition, criterion for existence of a giant component, the case of a power-law degree distribution
3. Models of network growth, Price's model, its solution and algorithm for computer simulation, the Barabási–Albert model, time evolution of preferential attachment models, models where edges can be added independently of vertex creation, non-linear preferential attachment
4. Vertex copying models, equivalence to preferential attachment
5. Network optimization models, the model of Solé and Ferrer i Cancho
6. The small-world model, scaling analysis, mean-field solution

Processes taking place on networks:

1. Percolation, its use as a model of network resilience, solution on the configuration model, position of the percolation threshold, vanishing of the threshold in the power-law case, size of the giant cluster
2. Epidemics on networks, SI and SIR models, epidemic threshold, mapping to bond percolation, dynamic equations for an epidemic on a network, approximate solution of SI and SIR models
3. Search processes on networks, web search, peer-to-peer networks, network navigation and Milgram's small-world experiment