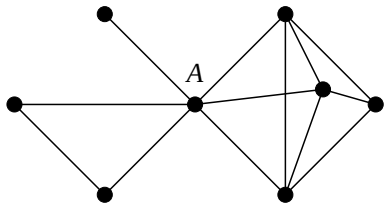
Complex Systems 535/Physics 508: Example midterm problems

1. Consider this small network:



   (a) [**2 points**] What is the diameter of the network?

   (b) [**2 points**] Is it planar? How would you prove this?

   (c) [**2 points**] Give a calculation of the closeness centrality of the vertex marked *A*.

   (d) [**2 points**] Give a calculation of the (unnormalized) betweenness centrality of the vertex marked *A*.

   (e) [**2 points**] Circle the 2-components in the network.

2. A directed acyclic network is one containing no directed loops of edges.

   (a) [**2 points**] Give two examples from different areas of study of real-world directed networks that are acyclic or approximately so.

   (b) [**3 points**] If $\mathbf{A}$ is the adjacency matrix of an acyclic network, prove that $\mathbf{A}^n = 0$, where n is the number of vertices in the network.

   (c) [**5 points**] A centrality measure for acyclic networks is defined as follows. Each vertex i has an "authority weight" x_i and a "hub weight" y_i. The hub weight is the sum of the authority weights of the vertices you point to. The authority weight is the sum of the hub weights of those who point to you *plus* a "free" extra amount α, similar to what you get in PageRank. Derive an expression for the vector $\mathbf{x}$ of authority weights in terms of the adjacency matrix. (The vector $\mathbf{x}$ should appear on only one side of your expression. You can neglect overall multiplicative constants.)

3. A network is created by taking n vertices and for each pair (i, j) of distinct vertices placing an undirected edge between them with probability p. You can assume n is large.

   (a) [**2 points**] What is the mean degree c of a vertex in the network?

   (b) [**3 points**] What is the mean degree of the neighbor of a vertex?

   (c) [**2 points**] What is the clustering coefficient of the network?

   (d) [**3 points**] What is the degree distribution, i.e., what is the probability p_k that a randomly chosen vertex will have degree k?

4. A network consists of a ring of n vertices, each of which is connected to the r vertices immediately clockwise from it around the ring and to the r vertices immediately anticlockwise. Thus each vertex has degree $k = 2r$. You can assume that $n \gg r$.

   (a) [**2 points**] How many "connected triples" are there in the network, i.e., a vertex plus an unordered pair of its neighbors?

   (b) [**4 points**] How many triangles are there in the network, i.e., trios of vertices each connected to both of the others?

   (c) [**4 points**] What is the clustering coefficient of the network?