

Complex Systems 535/Physics 508: Homework 4

1. What (roughly) is the time complexity of:

- (i) Vacuuming a carpet if the size of the input to the operation is the number n of square feet of carpet?
- (ii) Finding a word in a (paper) dictionary if the size of the input is the number n of words in the dictionary?

Explain your reasoning in each case.

2. For a network of n vertices show that:

- (i) It takes time $O(n^2)$ to multiply the adjacency matrix into an arbitrary vector if the network is stored in adjacency matrix form, but only time $O(m)$ if it is in adjacency list form.
- (ii) It takes time $O(n(n+m))$ to find the diameter of the network.
- (iii) It takes time $O(\langle k \rangle)$ to list the neighbors of a vertex, on average, but time $O(\langle k^2 \rangle)$ to list the second neighbors. You can assume the network is stored in adjacency list format. (In a network with a power-law degree distribution, where $\langle k \rangle$ is finite but $\langle k^2 \rangle$ formally diverges, this means the second operation is much more work than the first.)
- (iv) For a directed network in which in- and out-degrees are uncorrelated, it takes time $O(m^2/n)$ to calculate the reciprocity of the network. Why is the restriction to uncorrelated degrees necessary? What could happen if they were correlated?

3. What is the time complexity, as a function of the number n of vertices and m of edges, of the following network operations if the network in question is stored in adjacency list format?

- (i) Calculating the mean degree.
- (ii) Calculating the median degree.
- (iii) Calculating the air-travel route between two airports that has the shortest total flying time, assuming the flying time of each individual flight is known.
- (iv) Calculating the minimum number of nodes that would have to fail to disconnect two (otherwise functional) nodes on the Internet.

4. Consider the following centrality measure, which I just made up off the top of my head. With ordinary degree centrality, you get points for each person you are connected to. But perhaps you could get points for people you are two steps away from in the network, or three, or more, although probably you should get fewer points the further away someone is. So let us define the centrality x_i of vertex i to be a sum of contributions as follows: 1 for yourself, α for each person at distance 1 in the network, α^2 for each person at distance 2, and so forth.

- (i) Write an expression for x_i in terms of α and the geodesic distances d_{ij} between vertex pairs.
- (ii) Describe briefly an algorithm for calculating this centrality measure. What is the time complexity for calculating x_i for all i ?
- (iii) Suppose individuals in a network have about c connections on average, so that a person typically has about c first neighbors, c^2 second neighbors, and so on (ignoring the effects of transitivity). What happens to the contributions to the centrality when $\alpha \gtrsim 1/c$?