Physics 411: Homework 1

1. **A ball dropped from a tower:** A ball is dropped from a tower of height h. It has initial velocity zero and accelerates downwards under gravity. Write a program that asks the user to enter the height in meters of the tower and then calculates and prints the time in seconds till the ball hits the ground, ignoring air resistance. Use your program to calculate the time for a ball dropped from a 100 m high tower.

2. **Special relativity:** A spaceship travels from Earth in a straight line at a speed v to another planet x light years away. Write a program to ask the user for the value of x and the speed v as a fraction of the speed of light, then print out the time in years that the spaceship takes to reach its destination (a) in the rest frame of an observer on Earth and (b) as perceived by a passenger on board the ship. Use your program to calculate the answers for a planet 10 light years away with $v = 0.99c$.

3. **Altitude of a satellite:** A satellite is to be launched into a circular orbit around the Earth so that it orbits the planet once every T seconds.

   (a) Show that the altitude h above the Earth's surface that the satellite must have is

   $$h = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} - R,$$

   where $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ is Newton's gravitational constant, $M = 5.97 \times 10^{24}\,\mathrm{kg}$ is the mass of the Earth, and $R = 6371\,\mathrm{km}$ is its radius.

   (b) Write a program that asks the user to enter the desired value of T and then calculates and prints out the correct altitude in meters.

   (c) Use your program to calculate the altitudes of satellites that orbit the Earth once a day (so-called "geosynchronous" orbit), once every 90 minutes, and once every 45 minutes. What do you conclude from the last of these calculations?

4. **Catalan numbers:** The Catalan numbers C_n are a sequence of integers 1, 1, 2, 5, 14, 42, 132... that play an important role in quantum mechanics and the theory of disordered systems. (They were central to Eugene Wigner's proof of the so-called semicircle law.) They are defined by

   $$C_0 = 1, \qquad C_{n+1} = \frac{4n+2}{n+2}\,C_n.$$

   Write a program that prints in increasing order all Catalan numbers less than or equal to one billion.

5. **The semiempirical mass formula:** In nuclear physics, the semiempirical mass formula is a formula for calculating the approximate nuclear binding energy B of an atomic nucleus

with atomic number Z and mass number A:

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}},$$

where, in units of millions of electron volts, the constants are $a_1 = 15.67$, $a_2 = 17.23$, $a_3 = 0.75$, $a_4 = 93.2$, and

$$a_5 = \begin{cases} 12.0 & \text{if } Z \text{ and } A-Z \text{ are both even,} \\ -12.0 & \text{if } Z \text{ and } A-Z \text{ are both odd,} \\ 0 & \text{otherwise.} \end{cases}$$

Write a program that takes as its input the values of A and Z, and prints out the binding energy for the corresponding atom. Use your program to find the binding energy of an atom with $A = 58$ and $Z = 28$. (The correct answer is around 490 MeV.)