Physics 411: Homework 1

You have one week to do the homework. It is due, in class, on **Thursday, January 24**. For each problem, the materials you are required to turn in are indicated next to the check symbol: ☑

Many people find it convenient to create a document using Word (or any other word processor) and then copy and paste programs, output, figures, and so forth into the document. When you're finished, you can print out the document and hand it in.

1. **Altitude of a satellite:** A satellite is to be launched into a circular orbit around the Earth so that it orbits the planet once every T seconds.

   (a) Treating the Earth as a perfect sphere (which is only approximately correct), show that the altitude h above the Earth's surface that the satellite must have is

   $$h = \left(\frac{GMT^2}{4\pi^2} \right)^{1/3} - R,$$

   where $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ is Newton's gravitational constant, $M = 5.97 \times 10^{24}\,\mathrm{kg}$ is the mass of the Earth, and $R = 6371\,\mathrm{km}$ is its radius.

   (b) Write a program that asks the user to enter the desired value of T and then calculates and prints out the corresponding altitude in meters.

   (c) Use your program to calculate the altitudes of satellites that orbit the Earth once a day (so-called "geosynchronous" orbit), once every 90 minutes, and once every 45 minutes. What do you conclude from the last of these calculations?

   ☑ **For full credit** turn in a printout of your finished program, a printout of the three runs of the program showing the three answers it produces, and your answer to the question in part (c).

2. **Special relativity:** A spaceship travels from Earth in a straight line at a speed v to another planet x light years away. Write a program to ask the user for the value of x and the speed v as a fraction of the speed of light, then print out the time in years that the spaceship takes to reach its destination (a) in the rest frame of an observer on Earth and (b) as perceived by a passenger on board the ship. Use your program to calculate the answers for a planet 10 light years away with $v = 0.99c$.

   ☑ **For full credit** turn in a printout of your program plus a printout of the program in action showing the answers it produces.

3. **The Madelung constant:** In condensed matter physics the *Madelung constant* gives the total electric potential felt by an atom in a solid. It depends on the charges on the other atoms nearby and their locations. Consider for instance solid sodium chloride—table salt. The sodium chloride crystal has atoms arranged on a cubic lattice, but with alternating sodium and chlorine atoms, the sodium ones having a positive charge $+e$ and the chlorine ones a negative charge $-e$, where e is the charge on the electron. If we label each position

on the lattice by three integer coordinates (i, j, k), then the sodium atoms fall at positions where $i + j + k$ is even, and the chlorine atoms at positions where $i + j + k$ is odd.

Consider a sodium atom at the origin, $i = j = k = 0$, and let us calculate the Madelung constant. If the spacing of atoms on the lattice is a, then the distance from the origin to the atom at position (i, j, k) is

$$\sqrt{(ia)^2 + (ja)^2 + (ka)^2} = a\sqrt{i^2 + j^2 + k^2},$$

and the electric potential at the origin created by such an atom is

$$V(i, j, k) = \pm \frac{e}{4\pi\epsilon_0 a\sqrt{i^2 + j^2 + k^2}},$$

with ϵ_0 being the permittivity of the vacuum and the sign of the expression depending on whether $i + j + k$ is even or odd. The total potential felt by the sodium atom is then the sum of this quantity over all other atoms. Let us assume a cubic box around the sodium at the origin, extending L atoms in all directions. Then

$$V_{\text{total}} = \sum_{\substack{i,j,k=-L \\ \text{not } i=j=k=0}}^{L} V(i, j, k) = \frac{e}{4\pi\epsilon_0 a} M,$$

where M is the Madelung constant. Technically, in fact, the Madelung constant is the value of M when $L \to \infty$, but one can get a good approximation just by using a large value of L.

Write a program to calculate and print the Madelung constant for sodium chloride. Use as large a value of L as you can, while still having your program run in reasonable time—say in a minute or less.

☑ **For full credit** turn in a printout of your finished program plus a printout of the program in action showing the answer it produces.

4. **Prime numbers:** Example 2.8 in the book gives a program for finding prime numbers but that program is not particularly efficient: it checks each number to see if it is divisible by any number less than itself. We can develop a much faster program for prime numbers by making use of the following observations:

   (a) A number n is prime if it has no prime factors less than n. Hence we only need to check if it is divisible by other primes.

   (b) If a number n is non-prime, having a factor r, then $n = rs$, where s is also a factor. If $r \geq \sqrt{n}$ then $n = rs \geq \sqrt{n}s$, which implies that $s \leq \sqrt{n}$. In other words, any non-prime must have factors (and hence also prime factors) less than or equal to $\sqrt{n}$.

   Thus to determine if a number is prime we have to check its prime factors only up to and including $\sqrt{n}$—if there are none then the number is prime.

   Write a Python program that prints all the primes up to 10 000. Create a list to store the primes, which starts out with just the one prime number 2 in it, then for each number n

from 3 to 10 000 check whether the number is divisible by any of the primes in the list up to and including $\sqrt{n}$. As soon as you find a single prime factor you can stop checking the rest of them—you know n is not a prime. If you find no prime factors $\sqrt{n}$ or less, then n is prime and you should add it to the list. You can print out the list all in one go at the end of the program, or you can print out the individual numbers as you find them.

☑ **For full credit** turn in a copy of your program along with the last five (largest) primes it finds. No need to turn in a printout of the whole list of primes—it would be very long.

5. **The semi-empirical mass formula:** In nuclear physics, the semi-empirical mass formula is a formula for calculating the approximate nuclear binding energy B of an atomic nucleus with atomic number Z and mass number A:

$$B = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A} + \frac{a_5}{A^{1/2}},$$

where, in units of millions of electron volts, the constants are $a_1 = 15.67$, $a_2 = 17.23$, $a_3 = 0.75$, $a_4 = 93.2$, and

$$a_5 = \begin{cases} 0 & \text{if } A \text{ is odd,} \\ 12.0 & \text{if } A \text{ and } Z \text{ are both even,} \\ -12.0 & \text{if } A \text{ is even and } Z \text{ is odd.} \end{cases}$$

(a) Write a program that takes as its input the values of A and Z, and prints out the binding energy for the corresponding atom. Use your program to find the binding energy of an atom with $A = 58$ and $Z = 28$. (Hint: The correct answer is around 490 MeV.)

(b) Modify your program to print out not the total binding energy B, but the binding energy per nucleon, which is B/A.

(c) Now modify your program so that it takes as input just a single value of the atomic number Z and then goes through all values of A from $A = Z$ to $A = 3Z$, to find the one that has the largest binding energy per nucleon. This is the most stable nucleus with the given atomic number. Have your program print out the value of A for this most stable nucleus and the value of the binding energy per nucleon.

(d) Modify your program again so that, instead of taking Z as input, it runs through all values of Z from 1 to 100 and prints out the most stable value of A for each one. At what value of Z does the maximum binding energy occur? (The true answer, in real life, is $Z = 26$, which is iron. You should find that the semi-empirical mass formula gets the answer roughly right, but not exactly.)

☑ **For full credit** turn in a printout of your final program from part (d), a printout of the program in action showing the output it produces, and your answer to the question in part (d).