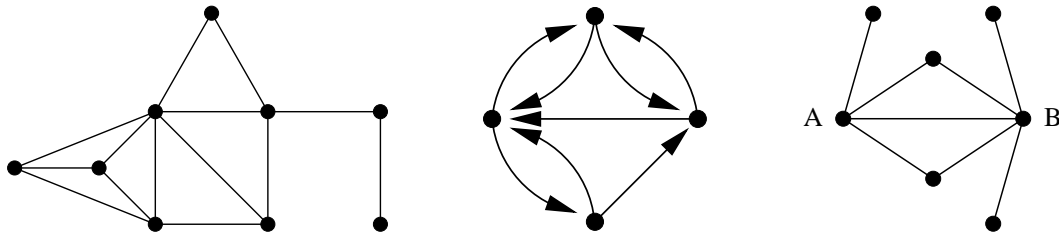


Complex Systems 535/Physics 508: Homework 2

1. Consider these three networks:



- (i) Find a 3-core in the first network.
 - (ii) What is the reciprocity of the second network?
 - (iii) What is the cosine similarity of vertices A and B in the third network?
2. Consider a k -regular undirected network (i.e., a network in which every vertex has degree k).
- (i) Show that the vector $\mathbf{1} = (1, 1, 1, \dots)$ is an eigenvector of the adjacency matrix with eigenvalue k .
 - (ii) By making use of the fact that eigenvectors are orthogonal (or otherwise), show that there is no other eigenvector that has all elements positive. The Perron–Frobenius theorem says that the eigenvector with all elements positive has the largest eigenvalue, and hence the eigenvector $\mathbf{1}$ gives, by definition, the eigenvector centrality of our k -regular network and the centralities are the same for every vertex.
 - (iii) Find the Katz centralities of all vertices in a k -regular network.
 - (iv) Name a centrality measure that could give different centralities for different vertices in a regular network.
3. A particular network is believed to have a degree distribution that follows a power law for degree greater than or equal to 10. A random sample of vertices is taken and their degrees measured. The degrees of the first twenty vertices with degrees 10 or greater are:

16	17	10	26	13
14	28	45	10	12
12	10	136	16	25
36	12	14	22	10

Estimate the exponent α of the power law and the error on that estimate.

Continued on the next page →

4. In a survey of couples in the city of San Francisco in 1992, Catania *et al.* recorded, among other things, the ethnicity of interviewees and calculated the fraction of couples whose members were from various ethnic groups. The fractions were as follows:

		Women				Total
		Black	Hispanic	White	Other	
Men	Black	0.258	0.016	0.035	0.013	0.323
	Hispanic	0.012	0.157	0.058	0.019	0.247
	White	0.013	0.023	0.306	0.035	0.377
	Other	0.005	0.007	0.024	0.016	0.053
Total		0.289	0.204	0.423	0.084	

Assuming the couples interviewed to be a representative sample of the edges in the undirected network of relationships, and treating the vertices as being of four types—black, hispanic, white, and other—calculate the numbers e_{rr} and a_r that appear in Eq. (7.76) of the book, for each type. Hence calculate the modularity of the network with respect to ethnicity. What do you conclude about homophily in this community?