Complex Systems 535/Physics 508: Homework 1

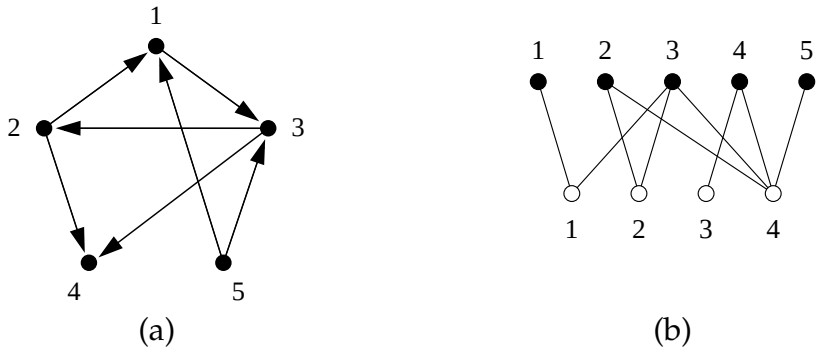
1. Give one real-life example of each of the following types of networks:

   (a) An acyclic (or approximately acyclic) directed network
   (b) A cyclic directed network
   (c) A tree (or approximate tree)
   (d) A planar (or approximately planar) network
   (e) A bipartite network

   Describe briefly one empirical technique that could be used to measure the structure of each of the following networks (i.e., to fully determine the positions of all the edges):

   (f) The world wide web
   (g) The internet
   (h) A food web
   (i) A network of friendships between a group of coworkers
   (j) A power grid

2. Consider the following two networks:



(a) (b)

   Network (a) is a directed network. Network (b) is undirected but bipartite. Write down:

   (a) the adjacency matrix of network (a);
   (b) the incidence matrix of network (b);
   (c) the adjacency matrix for the network created when we project network (b) onto its black vertices.

3. Consider a network which is simple (i.e., it contains no multiedges or self-edges) and consists of n nodes in a single component. What is the maximum possible number of edges it could have? What is the minimum possible number of edges it could have? Explain briefly how you arrive at your answers.

4. Demonstrate the following for undirected networks:

   (a) A 3-regular graph must have an even number of nodes.
   (b) The average degree of a tree is strictly less than 2.

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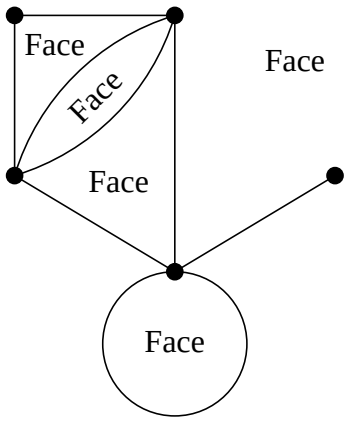
5. Consider a connected planar network with n vertices and m edges. Let f be the number of "faces" of the network, i.e., areas bounded by edges when the network is drawn in planar form. The outside of the network, the area extending to infinity on all sides, is also considered a face. The network can have multiedges and self-edges:



(a) How do n, m, and f change when we add a single vertex to such a network along with a single edge attaching it to an existing vertex?

(b) How do n, m, and f change when we add a single edge between two existing vertices (or a self-edge attached to just one vertex), in such a way as to maintain planarity of the network?

(c) What are the values of n, m, and f for a network with a single vertex and no edges?

(d) Hence by induction prove a general relation between n, m, and f for all connected planar networks.

(e) Now suppose that our network is simple. Show that the mean degree c of a simple, connected, planar network is strictly less than six.