Chapter 1

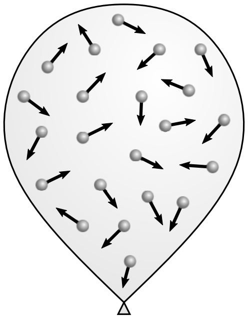
Sound

We begin our study of the science of music by asking "What is sound?" What is sound made of? What properties does it have? And what do they tell us about music? At the most basic level, sound is a wave of pressure, moving through the air. It consists of tiny changes in air pressure generated by the objects around us as they move. Let's take a look at how this happens.

1.1 Air

Air is the gas that makes up the atmosphere of planet Earth. It is all around us and we breath it in to get the oxygen we need to live. It is transparent and invisible, although you can feel it: the wind on your face is air hitting your skin. And tiny vibrations of the air, like ripples on the surface of a pond, are picked up by our ears as sound.

Air, like all matter, is made of up of atoms and molecules. Air is a mixture of two gases, about 80% nitrogen and 20% oxygen (plus trace amounts of others). Nitrogen and oxygen molecules are light, but they are not weightless. Air has a weight, though we are not usually aware of it. One cubic meter of normal air weighs 1.204 kilograms, or about two pounds, a fact that will be important to us in our discussions.



The air molecules inside a party balloon move around and collide with the walls of the balloon, producing air pressure on the walls.

Unlike the molecules in solid matter, which are fixed in place, molecules in gases are free to move around, jostling one another and bouncing off objects. The molecules in air are continually moving, and moving fast: a typical speed is about 500 meters per second (about 1800 kilometers per hour or 1100 miles per hour). This motion is responsible for the phenomenon of air pressure. Imagine the air inside a party balloon. As the molecules move around they bounce against the insides of the walls of the balloon and push them outward. You cannot feel the force of individual collisions—they are too small—but there are trillions of them every second and collectively they produce a force that you can feel. If you squeeze a balloon it pushes back against you firmly.

The air around us is constantly exerting pressure on its surroundings in exactly this way. There is air pressure on walls and floors and ceilings and furniture and people, the collective effect of many many molecules bouncing off them all the time. Technically, pressure is defined as the amount of force per square meter of surface. Force is measured in scientific units of *newtons*, so pressure is measured in newtons per square meter (written N/m^2), also called *pascals* (and denoted Pa). Normal atmospheric pressure is 101 000 Pa, although this is perhaps easier to envisage if translated into more familiar units: that's equivalent to about 15 lbs per square inch, one kilogram per square centimeter, or ten metric tonnes of force per square meter. That's a large amount of force, and seems perhaps surprising at first. If the air is exerting that much force on us, why do we not feel it? The answer is that the force is exerted in all directions and so balances out. Pressure does not intrinsically have a direction: pressure acts on any surface that you present to it. If you hold out your hand flat, for instance, then the pressure of the air above it exerts a force of one kilogram per square centimeter downwards on it. But the air below also exerts one kilogram per square centimeter upward from underneath your hand, so the net force on your hand is zero and you don't feel anything. If the air pressure changes—as when a plane lands, for instance—then you may notice it, but most of the time we are simply oblivious to the tremendous pressure around us. One way in which we do notice it, however, is in the pressure waves we call sound.

For reference, one newton is about the weight of an average apple, which is an easy way to remember it if you're familiar with the story of Isaac Newton, who is said to have invented his theory of gravity after seeing an apple fall from a tree.

To convert from newtons to kilograms, you divide by 9.81, although often it's good enough to just divide by 10, which gives roughly the same answer and is a lot easier to do.

1.2 Pressure waves

Sound consists of changes in the pressure of the air around us. The changes are tiny—a few parts per million or less for typical musical sounds. It is a testament to the remarkable sensitivity of our ears that we can hear sound at all.

To get an idea of what we are talking about, consider a long tube or pipe, as shown in Fig. 1.1a, closed off at both ends. Initially it contains air at normal atmospheric pressure, but now suppose we inject some extra air though a small hole at one end of the pipe, as indicated by the arrow in the figure. This will drive up the pressure at that end of the pipe so there is a pressure imbalance in the tube, but this imbalance cannot last. Over time the air will flow down the tube away from the hole and the pressure will equalize. For an analogy, imagine pouring water in at one side of a pool or pond. Temporarily this will make the water higher at that side, but very quickly water will flow from the higher parts to the lower parts and the water level will equalize everywhere.

The equalization of air pressure is rapid but not instantaneous. Rather, it happens as depicted in Fig. 1.1 panels (b) to (d), the region of higher pressure spreading out until the pressure is the same everywhere along the tube. The speed at which the front travels along the tube is an intrinsic property of air and is found to be 343 meters per second, which is equivalent to about 1200 kilometers per hour or 770 miles per

For other gases the speed would be different, but we are only interested in air here.

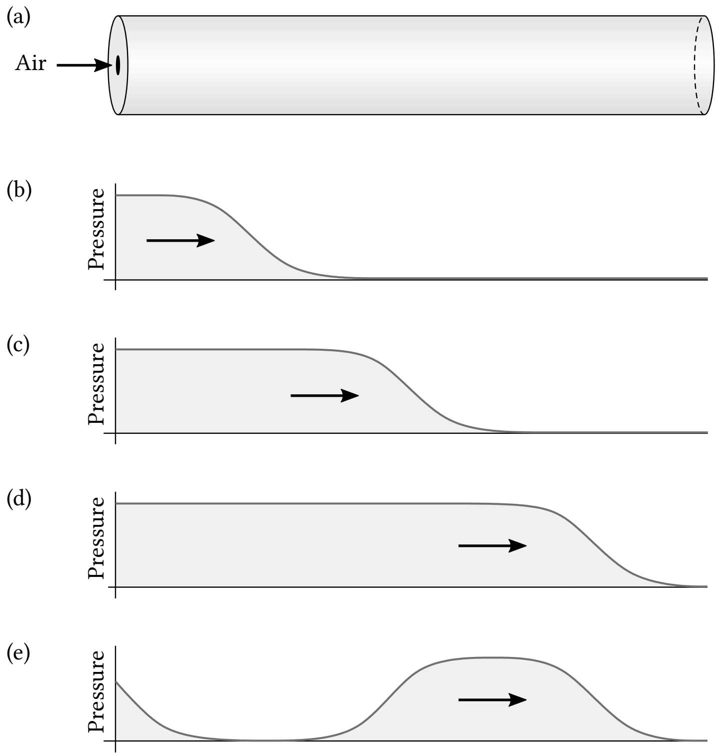


Figure 1.1: The motion of a pressure wave in a tube. (a) Air injected through a small hole at the end of an otherwise closed tube will push the pressure up inside the tube. Initially the pressure will increase near the end where the air is injected. (b) to (d) Over time the higher pressure will spread down the tube, moving at the speed of sound. (e) If we change the pressure in a more complicated pattern, raising it and lowering it over time, these disturbances will also travel down the tube at the speed of sound. Any changes we make in the pressure at one end will travel down the tube and be copied at other points a short time later.

hour.

Now suppose we take our experiment a step further. Instead of just injecting extra air one time, we first inject air then, a fraction of a second later, we remove some, driving the pressure down again. The first act, of injecting the air, will cause a front of higher pressure to travel down the tube as we have seen. The second act, of removing it, will cause another traveling front, but this time of lower pressure, with the opposite shape to the first front, as shown in Fig. 1.1e. The result is a pulse of pressure that travels down the tube. We can repeatedly raise and lower the pressure at the start of the tube as often as we like and each time the change the pressure travels down the tube at the same speed of 343 m/s, so that a point some distance down the tube experiences the same variations in pressure that are injected at the start of the tube, just a fraction of a second later because of the time the disturbance takes to travel.

This is what we call a sound wave. It is a change of pressure that travels through the air, so that whatever pattern of pressure we start off with is felt, some time later, at a distance from where it started. The pattern travels through the air with fixed speed 343 m/s, which we call the *speed of sound*.

We have described a wave traveling down a tube, which is an easy case to visualize, but sound waves do not need a tube to guide them. They will travel through air wherever it is available. The air is constantly filled with sound waves traveling in all directions, which our ears detect as the amazing array of sounds around us: the sound of people talking, of music, of birds and cars and footsteps and rain falling on the roof. All of these produce tiny variations in pressure which travel though the air as sound.

1.2.1 Sound pressure

As we have said, there is always pressure in the air, the prevailing atmospheric pressure. A sound wave consists of variations around this prevailing value: sometimes the pressure is slightly greater, sometimes slightly less. We can write the total pressure, including the regular atmospheric pressure and the extra part due to sound, as

$$P = P_0 + p. \tag{1.1}$$

Here P is the total pressure, P_0 is the atmospheric pressure, and p is the *sound pressure*, the additional pressure due to the sound wave. Almost always in this book it will be the sound pressure that we are concerned with. The atmospheric pressure will not usually matter—it is simply the background over which the sound pressure is superimposed. The sound pressure p is the part of the pressure that is important in our consideration of musical sound.

1.3 Sound production

In our discussion above we envisaged a pressure wave produced by injecting air into a tube. This is basically the way wind instruments like the clarinet work. A clarinet has a reed that moves back and forth rapidly, opening and closing an aperture and injecting bursts of air into the instrument's pipe. The human voice also works in essentially the same way, with the vocal cords playing the role of the reed. We look at the workings of wind instruments and the voice in Chapters 11 and 12 of this book.

Most sound, however, is produced in a different manner, by the motion or vibration of objects, like the floor when you walk across it, the strings and soundboard of a guitar when you strum a chord, or a drum head when you hit it with a drumstick.

Take the example of the guitar, for instance. Most of the sound from an acoustic guitar is produced not directly by the strings but by the soundboard, which is the wooden front panel of the instrument. When you pluck a string it vibrates and causes the soundboard to also vibrate, which in turn moves air back and forth and produces a sound wave. The situation is sketched in Fig. 1.2.

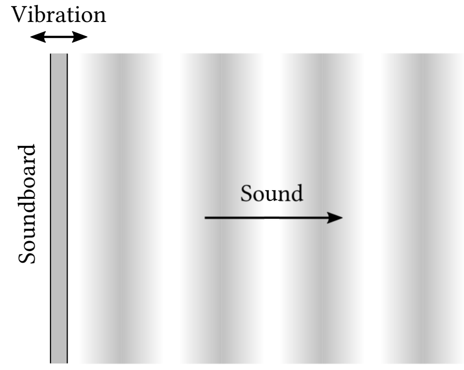


Figure 1.2: Sound production by a vibrating surface. A surface such as the soundboard of a guitar vibrates back and forth and alternately pushes on the air next to it then backs off, producing waves of pressure that travel outward in the form of sound.

When the soundboard moves outward, it pushes on the nearby air and compresses it, temporarily driving up the pressure in the immediate vicinity. When it moves back again the pressure goes back down. In Section 1.6.1 it is shown that the sound pressure p produced by a moving object is proportional to the velocity u of the object according to the formula

$$p = zu, \tag{1.2}$$

where the constant of proportionality z is called the *acoustic impedance* of air and has a value of 413 Pa s/m. Because the pressure and velocity are proportional to one another, the sound pressure just copies the pattern of movement of the object—the soundboard in this case—and then this pattern of pressure travels outward, producing the wave that we call sound. This is how a guitar makes music.

We study the workings of the guitar in more depth in Section 10.2.

In principle, any moving object can produce sound by pushing on the air around it. Musical instruments do this, but so also does the moving diaphragm of a loudspeaker, the metal of a ringing bell, the bottom of a bucket when you drum on it, or virtually any object when it is struck or brushed or scraped—a dinner plate, the floor under your feet, a door as you knock on it.

Example 1.1: Sound pressure from a guitar

Equation (1.2) tells us that the sound pressure produced by a guitar depends on the velocity of movement of the soundboard as it vibrates. As we will see in Chapter 2, the vibrations we are talking about in the case of a guitar have a rate of about 200 oscillations per second—the soundboard moves back and forth 200 times each second.

The movement of the soundboard is very small—completely invisible to the eye. A typical range of motion might be a thousandth of a millimeter or one micrometer. If the soundboard moves back and forth by 0.001 mm, then it travels a total of 0.002 mm in one round trip, and hence a total of $200 \times 0.002\,\text{mm} = 0.4\,\text{mm}$ per second or 0.0004 m/s. This gives us a rough estimate of the velocity of the soundboard. Plugging this figure into Eq. (1.2) and using $z = 413\,\mathrm{Pa\,s/m}$, we then find a rough estimate of the magnitude of the sound pressure:

$$p = 413 \times 0.0004 = 0.16\,\mathrm{Pa}. \tag{1.3}$$

This is a very small amount of pressure, almost a million times smaller than the 101 000 pascals of regular atmospheric pressure, but it is typical of musical sounds. Musical sounds are made up of very tiny, one-part-in-a-million variations in the air pressure around us.

1.4 Mathematics of sound waves

In Section 1.6 we show that a sound wave can be represented mathematically by an equation like this:

$$p(x,t) = f(x - ct). \tag{1.4}$$

In this equation $p(x,t)$ is the sound pressure at position x and time t, the quantity c is a constant (which we'll give a value for in a moment), and f is a mathematical function that represents the shape of the wave. A sound wave can take any shape at all—different shapes correspond to different sounds—so f can take any shape too.

What does this equation actually mean? Let us look at it in detail. First of all, the variables x and t measure position and time. We set up a distance scale that measures distance along the direction the sound wave is traveling in and we call this the x axis. Any position can be described by giving the appropriate value of x in meters. Where does the axis start? It does not matter. The x axis doesn't really exist. It is purely for our convenience, so we can choose the $x = 0$ point to be at any place we like, such as the middle of the laboratory, or the stage in a concert hall, or a speaker on a podium. Similarly we measure time t in seconds and we can describe any moment in time by saying what the value is of t at that moment. Again we can choose the start of the time-scale to be at any moment we like—it's just for our convenience, so we could for example choose $t = 0$ to represent the time at which an instrument plays a note, or the start of an experiment, or the time at which we start making measurements. Equation (1.4) applies no matter what choice we make.

Now consider how the pressure looks at time $t = 0$. Putting $t = 0$ in the equation we get

$$p(x,0) = f(x). \tag{1.5}$$

In other words at time $t = 0$ the pattern of pressure along the x axis of the wave is just equal to the function $f(x)$. An example is shown in the top frame of Fig. 1.3. If we were doing an experiment we could measure the pressure at all points along the x-axis and that would tell us the shape of $f(x)$ for that particular sound wave.

But once we know $f(x)$, then we can use it to calculate the pressure at any other time t. For instance, suppose we want to know the pressure one second later at

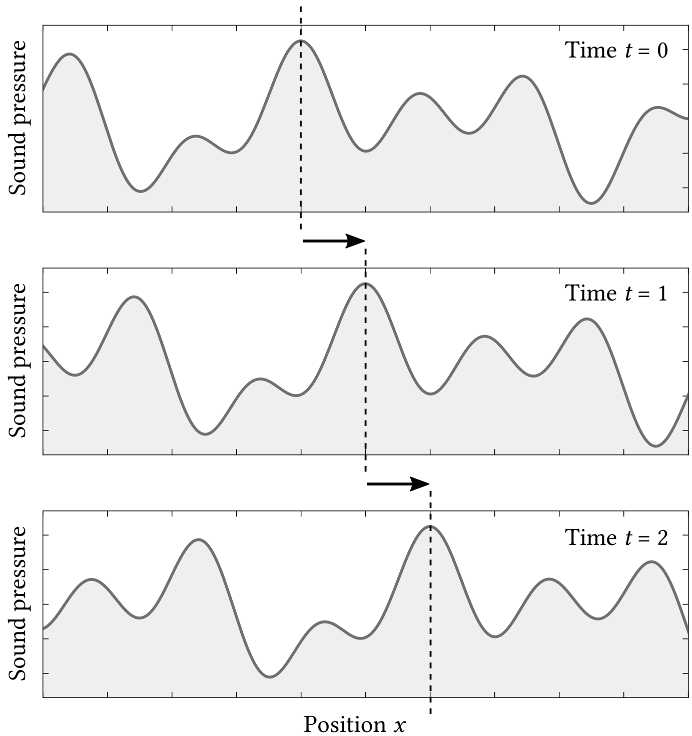


Figure 1.3: The motion of a sound wave over time. Top: at time $t = 0$ there is some wave of pressure spread out along the x-axis, as shown by this graph. Middle: one second later at $t = 1$ the wave has moved over to the right by a distance c. So, for instance, the peak marked by the vertical dashed line moves over as shown by the arrow. Bottom: another second after that it has moved a further distance c.

time $t = 1$. Putting $t = 1$ in Eq. (1.4), we find that

$$p(x, 1) = f(x - c). \tag{1.6}$$

This equation says that the pressure at position x at time $t = 1$ is $f(x - c)$. In other words it is the same as the pressure was at time $t = 0$, but at position $x - c$, which is c meters further back. Whatever the pressure *was* at $x - c$, that *same pressure* is now to be found at x. In other words the pressure has "moved" forward a distance c. If we wait another second, until $t = 2$, the pressure will move forward another distance c, and so forth.

This process is illustrated in Fig. 1.3: observe how the peak marked by the vertical dashed line in the first frame of the figure at time $t = 0$ has moved to the right in the second frame at time $t = 1$. In the third frame, a second later at $t = 2$, it has moved the same distance again. Every peak and trough in the wave moves in this same way, so the entire wave just travels through the air, moving a distance c every second.

This is the miracle of sound waves: whatever pressure you start with, that exact same pressure is felt at points further and further away as time goes by. The air transmits whatever pattern of pressure you make in one place to other surrounding places. Since the pressure moves c meters in every second, that means it has speed c, so c is the speed of the sound, which is 343 m/s, as we have said.

This almost magical mechanism is what makes all sound possible: music, speech, the barking of dogs, a car going by on the street. Any pattern of pressure changes gets transmitted through the air and if you have the means to detect it—ears or a microphone—then you can hear all the things going on around you. There are limits to how far sound will travel—it gets diluted with distance for reasons we discuss in Section 3.5—but sound in air provides us with a remarkable vehicle for transmitting thoughts and creative performances from one person to another.

Equation (1.4) is one of two possible mathematical forms for a sound wave. The other is

$$p(x,t) = f(x+ct). \tag{1.7}$$

The only difference between the two is that this one has a plus sign instead of a minus, which changes the direction of travel of the wave. In Eq. (1.4) and Fig. 1.3 the wave is traveling to the right (towards greater values of x) but in Eq. (1.7) it is traveling to the left. The mathematics tells that both are possible—sound will travel either way. Furthermore, though we have pictured sound traveling along a single "x" axis, there is no limit to what direction it can travel in. Sound will travel through open air in any direction. If we make a sound by playing a musical instrument, for instance, that sound will travel outward in all directions from the player and can be heard by people all around, provided they are not too far away.

1.5 Waveforms

Any pattern of pressure variation in the air gets carried away from its source at the speed of sound and can be picked up by a listener's ears. The pattern of the pressure changes that make up a sound are called the *waveform* of the sound. The standard way to represent a sound waveform is as a graph like the one shown in Fig. 1.4. The horizontal axis of the graph represents time in seconds and the vertical axis shows how the sound pressure varies. As the sound wave passes the listener, the pressure will go up and down according to the pattern in this graph.

The waveform determines everything about the way a sound sounds. If you know the waveform then you know the complete sound. For instance, sound recording works precisely because it captures a waveform and then reproduces it later on: if

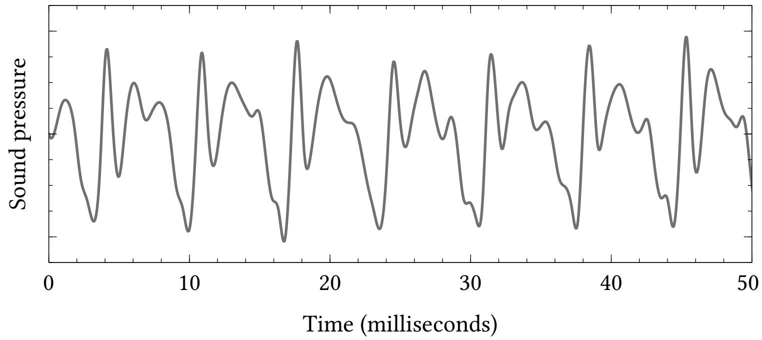


Figure 1.4: A sound waveform. The waveform of a sound is the pattern of pressure variation over time, conventionally depicted as a graph like this of sound pressure against time.

you can recreate the waveform exactly as it was, you will hear the exact original sound.

Waveforms come in an infinite variety of shapes and sizes, each of them corresponding to its own unique sound. There are deep rumbles and high-pitched squeals. There are sounds that are ear-splittingly loud and sounds so quiet we can barely hear them. There are bright sounds and warm sounds and tinny sounds and dull sounds. By studying the correspondence between the waveform and the sound we hear we will come to understand, in scientific terms, where all of this amazing variety comes from.

1.5.1 Waveforms of musical sounds

Not all sounds find use in music. Music, as it is played in most traditions and cultures around the world, focuses primarily on a subset of sounds, the *periodic* waveforms. A periodic waveform is one that repeats the same shape over and over again, as in Fig. 1.4 for example. In musical terms, the sound of a periodic waveform corresponds to a note with a clear pitch. Not all sounds have a clear pitch. What note does running water make, for instance? Or a thunderclap? Or a door slamming? These sounds do not have periodic waveforms and hence they are not "notes" in the way we commonly understand the word. To get a distinct note you need periodicity.

In practice, musical waveforms are not always exactly periodic. Musical instruments are not perfect machines, and slight variation in the waveform is normal and can even add interesting character to the sound. If you look closely at Fig. 1.4 you can see small changes in the shape from one cycle of the wave to the next. Nonetheless, to produce a useful musical note the waveform needs to be close to periodic.

An exception to the use of periodic waveforms in music is the sound of unpitched percussion instruments such as drums and cymbals, which have aperiodic waveforms and don't possess a clear pitch. We discuss the special characteristics of these instruments in Chapter 13. For the moment, however, we will focus on the periodic waveforms, which account for the sounds of most of the instruments you are probably familiar with, including the instruments of the orchestra, marching band, and jazz, rock, and pop music, with the exception of percussion.

1.5.2 Characteristics of waveforms

A periodic, repeating waveform has three essential characteristics that define its musical properties: frequency, amplitude, and shape.

Frequency: A periodic waveform consists of the same pattern of pressure variation repeated over and over again. The *frequency* of the waveform tells us how often it repeats, measured as the number of cycles per second. In musical terms, the frequency determines the pitch of the note we hear. Higher frequencies (more cycles per second) correspond to higher notes and lower frequencies to lower notes. We will see shortly exactly how this correspondence works. The frequencies of musical notes range from about 20 cycles per second to about 4000.

Amplitude: The amplitude of a waveform is the range of pressure variation from the highest highs to the lowest lows. Musically speaking, the amplitude corresponds to the loudness of the sound. Waveforms with more variation in pressure sound louder; those with less sound quieter. Amplitude could be measured in terms of the number of pascals of variation in the pressure of the wave, but in practice it is measured using a different scale, the decibel scale, and we will see shortly how this works.

Wave shape: The third property of the waveform is its shape. The shape controls the quality or "timbre" of the sound. Is it bright? Is it warm? Is it intense or jagged or soft or jangly? Two waves may have the same frequency and amplitude, and hence produce sounds with the same pitch and loudness, and yet they can sound completely different because they have different shapes. A violin and a trumpet can play the same note, but no one would mistake one for the other. In order to understand the link between waveform shape and timbre, we will learn about the tools of spectral analysis, which allow us to break a waveform into its component parts and understand exactly what each part contributes to the sound.

We will look in detail at each of these three properties of musical waveforms, frequency, amplitude, and shape, over the course of the next three chapters.

Advanced material

1.6 The wave equation

So far we have only described how sound behaves in rough terms, but it is possible describe its behavior precisely using differential equations, and specifically the *wave equation* for sound, which is derived from two basic physical facts: Newton's second law of motion and the compressibility of air.

Consider a sound wave traveling through the air in a tube or pipe as shown in Fig. 1.5, where the shading represents the changing sound pressure. Suppose the tube has cross-sectional area A and let us measure position along the tube by x. Consider a narrow slice of the air in the tube, between positions x and $x + \mathrm{d}x$. The pressure of the air to the left of the slice exerts a force on the slice equal to $P(x,t)A$, where $P(x,t)$ is the total pressure at the left side at time t. At the same time, the pressure at the right side is $P(x + \mathrm{d}x, t)$ and exerts a force on the slice in the opposite direction equal to $P(x + \mathrm{d}x, t)A$. So the net force on the slice is

$$\begin{aligned} F &= P(x,t)A - P(x + \mathrm{d}x, t)A \\ &= \frac{P(x,t) - P(x + \mathrm{d}x, t)}{\mathrm{d}x} A\,\mathrm{d}x \\ &= -\frac{\partial P}{\partial x} V, \end{aligned} \tag{1.8}$$

where $V = A\,\mathrm{d}x$ is the volume of the slice.

Another way of looking at our sound wave is in terms of the movement of the air. The molecules in air are moving all the time, as described in Section 1.1, bumping into one another and into walls and objects and people, but these motions are random so on average we can ignore them: on average the air is not moving anywhere, like ants milling around in an anthill, each individually moving even though the anthill overall isn't going anywhere. The sound wave, however, changes that. When sound pressure is present, increasing or decreasing the pressure from its normal atmospheric pressure, the air flows around to equalize that pressure, giving it a net overall motion. Let us denote the overall displacement of the air from its position at rest by $\xi(x,t)$, and its acceleration is the second derivative of this displacement $a = \partial^2\xi/\partial t^2$.

Now suppose that the density of the air is ρ. This means that the mass of our slice of air is $m = \rho V$, where V is again the volume. So we now have expressions for the force F on our slice (from Eq. (1.8)) and its mass m and acceleration a. Newton's second law of motion tells us that these three quantities are related by $F = ma$, and hence we have

$$-\frac{\partial P}{\partial x} V = \rho V \frac{\partial^2 \xi}{\partial t^2}. \tag{1.9}$$

We cancel a factor of V throughout and use Eq. (1.1), which says that $P = P_0 + p$ where the atmospheric pressure P_0 is a constant, and we get

$$-\frac{\partial p}{\partial x} = \rho \frac{\partial^2 \xi}{\partial t^2}. \tag{1.10}$$

This is one of the two results we will need for deriving the wave equation. The other concerns the compressibility of the air. As we have said, air is intrinsically squishy—think again of a party balloon. Exactly how squishy air is is measured by the *bulk modulus*. If we take a volume V of air at normal atmospheric pressure and squash it so that its volume decreases by a small amount ΔV, we will get a small increase in the pressure. This pressure increase is

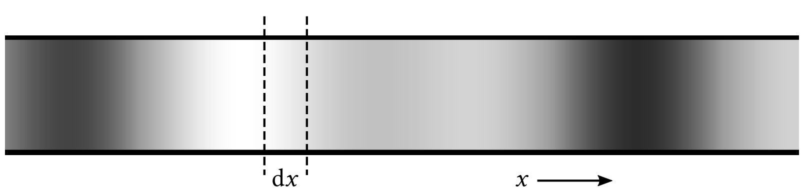


Figure 1.5: A sound wave traveling in a tube. Distance along the tube is measured by x and the shading represents variation in the pressure. We consider a small slice of air as shown, with width dx, and calculate the force of the air pressure acting on it from either side.

precisely what we call the sound pressure p—the excess pressure that comes from the compression of the air. The extra pressure is proportional to the fractional change in volume $\Delta V/V$ thus:

$$p = -B\frac{\Delta V}{V}. \tag{1.11}$$

The minus sign arises because the pressure goes up when the volume goes down. The constant of proportionality B is called the bulk modulus and is an intrinsic property of air. The bulk modulus has units of pressure (i.e., pascals) and for normal air at atmospheric pressure takes the value 1.42×10^5 Pa.

When a sound wave passes through our slice of air, the air at the left end of the slice at position x moves a distance $\xi(x,t)$, reducing the volume of the slice by an amount $\xi(x,t)A$. At the same time, the air at the right end of the slice moves a distance $\xi(x+\mathrm{d}x,t)$, increasing the volume by $\xi(x+\mathrm{d}x,t)A$. Thus the net increase in volume is

$$\begin{aligned}\Delta V &= \xi(x+\mathrm{d}x,t)A - \xi(x,t)A \\ &= \frac{\xi(x+\mathrm{d}x,t)-\xi(x,t)}{\mathrm{d}x}A\,\mathrm{d}x \\ &= \frac{\partial\xi}{\partial x}V. \end{aligned} \tag{1.12}$$

Substituting this expression for ΔV into Eq. (1.11) we get

$$p = -B\frac{\partial\xi}{\partial x}. \tag{1.13}$$

This is the second result we needed. Now we are ready to derive the wave equation.

Differentiating Eq. (1.13) twice with respect to t, we get

$$\frac{\partial^2 p}{\partial t^2} = -B\frac{\partial^2}{\partial t^2}\frac{\partial\xi}{\partial x} = -B\frac{\partial}{\partial x}\frac{\partial^2\xi}{\partial t^2} = \frac{B}{\rho}\frac{\partial^2 p}{\partial x^2} \tag{1.14}$$

where we have used Eq. (1.10) in the last equality. This result can be rearranged into the form

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = 0, \tag{1.15}$$

where

$$c = \sqrt{\frac{B}{\rho}}. \tag{1.16}$$

Equation (1.15) is the wave equation for sound waves. It describes how the sound pressure p varies in space and time.

The wave equation has two general solutions:

$$p(x,t) = f(x-ct), \tag{1.17}$$

$$p(x,t) = f(x+ct), \tag{1.18}$$

where $f(x)$ is any function of x. We can easily verify that these are solutions by substituting into the wave equation. For instance, taking the first solution, Eq. (1.17), we have

$$\frac{\partial^2 p}{\partial x^2} = f''(x-ct), \tag{1.19}$$

where $f''(x)$ is the second derivative of f, and

$$\frac{\partial^2 p}{\partial t^2} = c^2 f''(x-ct). \tag{1.20}$$

Substituting these into Eq. (1.15) then gives

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} = f''(x-ct) - \frac{1}{c^2}c^2 f''(x-ct) = 0, \tag{1.21}$$

as required. It is left as an exercise for the reader to verify that Eq. (1.18) is also a solution of the wave equation.

As discussed in Section 1.4, Eqs. (1.17) and (1.18) represent sound waves traveling in the positive and negative x directions respectively. The function f specifies the shape of the waveform and c is the speed with which the wave travels. In other words c is the speed of sound.

Equation (1.16) thus tells us that the speed of sound is $c = \sqrt{B/\rho}$ and given the value $B = 1.42 \times 10^5$ Pa of the bulk modulus, along with the measured density of air at standard pressure and room temperature, which is $\rho = 1.204\,\mathrm{kg/m^3}$, we find that

$$c = \sqrt{\frac{1.42\times 10^5}{1.204}} = 343\,\mathrm{m/s}. \tag{1.22}$$

The speed of sound has been measured many times in experiments and agrees well with this theoretical value.

Note, however, that the density ρ varies with temperature, so the actual value of the speed of sound may be slightly different from place to place and from day to day.[1]

[1] The bulk modulus does not depend on temperature so long as atmospheric pressure stays the same. In fact a standard result from thermodynamics says that the bulk modulus is equal to $\frac{7}{5}$ times the prevailing atmospheric pressure. For a standard atmospheric pressure of 1.013×10^5 Pa this gives a fixed value of $B = 1.418 \times 10^5$ Pa for the bulk modulus.

Temperature (°C)	Density ρ (kg/m^3)	Speed of sound c (m/s)	Acoustic impedance z (Pa s/m)
−25	1.422	315.8	449.2
−20	1.394	319.0	444.8
−15	1.367	322.1	440.4
−10	1.341	325.2	436.2
−5	1.316	328.3	432.2
0	1.292	331.3	428.2
5	1.269	334.3	424.3
10	1.247	337.3	420.5
15	1.225	340.3	416.9
20	1.204	343.2	413.3
25	1.184	346.1	409.8
30	1.164	349.0	406.4
35	1.146	351.9	403.1

Table 1.1: Variation of the speed of sound with temperature. As temperature goes up, air expands and its density ρ decreases, so the speed of sound c, given by Eq. (1.16), is higher at higher temperatures. At a typical room temperature of 20°C we have $c = 343$ m/s and we will use this value in many calculations in this book, but the actual value may be slightly higher or lower in practice. The acoustic impedance z, Eq. (1.30), also varies with temperature, going down as temperature goes up. All values are at standard atmospheric pressure of $101\,325 \times 10^5$ Pa.

The value of $c = 343$ m/s above is for a standard room temperature of 20°C. Table 1.1 gives values for a range of other temperatures. As we can see the speed of sound can be as low as 316 m/s at −25°C or as high as 352 m/s at 35°C. This variation has some practical consequences. For instance, it causes woodwind instruments to go out of tune as they warm up (see Section 11.1.3) and it requires sound level meters to be recalibrated when the temperature changes (Section 3.2.2). For general purposes, however, a value of 343 m/s is a good average figure for the speed of sound and we will use this value in this book.

1.6.1 Pressure, velocity, and acoustic impedance

The velocity $u(x,t)$ of the air at position x and time t is given in terms of the displacement $\xi(x,t)$ by

$$u = \frac{\partial \xi}{\partial t}. \tag{1.23}$$

Differentiating this equation with respect to time and using Eq. (1.10), we get

$$\frac{\partial u}{\partial t} = \frac{\partial^2 \xi}{\partial t^2} = -\frac{1}{\rho}\frac{\partial p}{\partial x}. \tag{1.24}$$

For a sound wave traveling in air, the pressure is given by the solution to the wave equation $p = f(x-ct)$ (Eq. (1.17)). Substituting into (1.24) and performing the derivative, we get

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} f'(x-ct), \tag{1.25}$$

and integrating again with respect to time we get

$$u = \frac{1}{\rho c} f(x-ct) + A = \frac{p}{\rho c} + A, \tag{1.26}$$

where A is an integration constant and we have used $p = f(x-ct)$ again.

We know that the velocity u is zero in undisturbed air where $p = 0$, so $u = 0$ when $p = 0$, which implies that the constant $A = 0$ and thus we find that the pressure and velocity in a sound wave are related by

$$u = \frac{p}{\rho c}. \tag{1.27}$$

In other words the pressure and velocity are proportional to one another. The quantity ρc is called the *acoustic impedance* of the air, denoted z:

$$z = \rho c, \tag{1.28}$$

and hence we can also write

$$p = zu. \tag{1.29}$$

Using Eq. (1.16) the acoustic impedance can also be written

$$z = \frac{B}{c}, \tag{1.30}$$

so acoustic impedance is essentially just a rescaled version of the bulk modulus and, like the bulk modulus, it is a measure of the stiffness of air, the extent to which it resists compression.

The density of air is $\rho = 1.204\,\text{kg/m}^3$, so with $c = 343\,\text{m/s}$ we have an acoustic impedance of

$$z = \rho c = 1.204 \times 343 = 413\,\text{Pa s/m}, \tag{1.31}$$

and we will use this value in this book. Note, however, that since, as we have seen, both the density of air and the speed of sound vary with temperature, the acoustic impedance does too, so somewhat higher or lower values are possible. In general acoustic impedance is lower for higher temperatures. Table 1.1 lists values for a range of temperatures.

We used Eq. (1.29) in Section 1.3 when we considered sound production by a moving object, like the soundboard of a guitar. In that case the soundboard moves with a certain velocity u and it causes the air in immediate contact with it to move at the same velocity. Once we know the velocity of the air, then Eq. (1.29) tells us the corresponding pressure and in this way we can calculate the sound pressure produced by a moving object. See Example 1.1 on page 5 for a demonstration.

Finally, on a technical note, we should point that strictly speaking z is the *specific* acoustic impedance. There is another quantity called acoustic impedance, denoted by capital Z, which is defined as the ratio $Z = F/u$ of the *force* F on an object and its velocity u. The two quantities are closely related. Pressure is force per unit area $p = F/A$, so

$$z = \frac{p}{u} = \frac{F/A}{u} = \frac{F/u}{A} = \frac{Z}{A}, \tag{1.32}$$

so the specific impedance is equal to the regular impedance divided by the area over which the force is exerted.

In some cases it is convenient to use the regular impedance. We will use it for instance in our discussion of the properties of the soundboard of a string instrument in Section 10.5. When talking about air, however, it makes more sense to work with pressure rather than force, which leads us to the specific impedance $z = p/u$. Usually in this book we will be talking about specific impedance, so we drop the word “specific,” except on the rare occasions where it is important to make the distinction.

Chapter summary:

- Sound is a **pressure wave**, moving in air.
- A disturbance in the air pressure will spread out from its source at a speed of 343 m/s, denoted in our equations by the letter c and commonly called the **speed of sound**.
- The speed of sound can be calculated from the formula

$$c = \sqrt{\frac{B}{\rho}},$$

where B is the bulk modulus of air (a measure of how much the air resists being compressed) and ρ is its density.

- The pattern of variation of pressure over time is called the **waveform** of a sound. The waveform determines everything we hear—if we know the waveform then we know the sound.
- Musical sounds often have **periodic waveforms**, ones that repeat the same pattern of pressure variation over and over again, although non-periodic waveforms are found in the sounds of percussion instruments. Periodic waveforms correspond to sounds with a clear pitch, like a note on a piano. Non-periodic ones have no clear pitch, like the sound of footsteps.
- Three properties of waveforms are particularly important for music: the **frequency** (which determines musical pitch), the **amplitude** (which determines loudness), and the **shape** of the waveform (which determines the quality or timbre of the sound).

Exercises

1.1 A rectangular room is 5 meters long, 4 meters wide, and has a 3 meter high ceiling.

a) What is the volume of the room?
b) How much does the air in the room weigh, in kilograms?
c) What is the area of the ceiling?
d) What is the total force on the ceiling (in newtons) from the air pressure in the room?
e) About how much is this in tons (or tonnes)?
f) In what direction is this force pushing?

1.2 A playing field is 100 meters long and 50 meters wide.

a) What is the area of the field?
b) What is the total force exerted by air pressure downward on the surface of the field in units of newtons?
c) What is the equivalent force in units of metric tonnes?

1.3 How far, in either kilometers or miles, would sound travel in a minute?

1.4 Lightning strikes one kilometer away. How long after the strike will you hear the thunderclap if the temperature outside is (a) 20°C and (b) 30°C?

1.5 A marching band is playing on a football field.

a) If the band is spread out across the entire field, how long will it take from when a player at one end of the field plays a note until a player at the end other hears that note? You can assume that the field is 100 meters long, which is close enough.

b) Based on your calculation, why is it important that the marching band has a conductor?

1.6 The diaphragm or "cone" of a loudspeaker vibrates 1000 times a second and moves back and forth a distance of 10^{-7} m. Roughly what is the sound pressure in pascals the loudspeaker produces, measured right next to the cone?

1.7 A person walks across a wooden floor and the sound of their footsteps creates a sound pressure of around 0.1 Pa close to the floor.

a) Explain why footsteps make a sound.
b) About how fast is the floor moving when the person's feet strike it?