The Science of Music

Exercises for Chapter 9

9.1 A pendulum oscillates back and forth once per second. If you took the same pendulum to a planet where the gravity was twice as strong as Earth's how fast would it then oscillate?

9.2 The vibrating part of a guitar string is 64.8 cm long and the string is made of solid steel with density 7900 kg/m^3 and diameter 0.33 mm. If its tension is 123 newtons what note does the string play?

9.3 The highest string on a violin is tuned to the note E5 and the vibrating part of the string is 32.5 cm long. The string is made of solid steel with a density of 7900 kg/m^3 and diameter 0.25 mm. What tension does it have?

9.4 A piano string that plays the note C4 is 1 meter long, 1 mm in diameter, and made of steel with density 7900 kg/m^3.

a) What is the tension on the string?
b) A piano has about 230 strings in total. If they all have about the same tension, what is the total tension force on the piano frame from all of the strings together?

9.5 Suppose that instead of being circular a steel instrument string is made square in cross-section, with width d along each side of the square. What would be the equivalent of Mersenne's law, Eq. (9.16), for the frequency of vibration of such a string?

9.6 The guzheng is a traditional Chinese string instrument with plucked steel strings about 0.5 mm in diameter, strung over a wooden frame. There is no fingerboard—each string plays its own note, as on a harp.

a) Given that the density of steel is 7900 kg/m^3, what is the tension of a 60 cm long guzheng string that plays the note C4?
b) The guzheng plays a pentatonic scale and has a range spanning about four octaves. About how many strings does it have?
c) Hence what is the total tension on the frame of a guzheng?

9.7 A piano string is made of solid steel, with length 50 cm and diameter 1 mm.

a) What is its mass m in grams?
b) The string plays the note C5. How many times is it vibrating back and forth per second?
c) Suppose it moves back and forth by 2 mm. What is the total distance it moves per second? This is its average velocity v.

d) Kinetic energy is equal to $\frac{1}{2}mv^2$. About how much energy does the string have in joules?
e) Suppose the note lasts 10 seconds before it dies away. Approximately what is the rate at which the string loses energy, in joules per second?

9.8 Suppose a guitar plays the note C3.

a) What is the frequency of vibration of the string?
b) If the soundboard moves back and forth by 0.001 mm as it vibrates, what is the total distance it travels per second? This is its average velocity u.
c) Using Eq. (1.2) estimate the sound pressure produced right next to the soundboard.

9.9 There are equivalents of Mersenne's law, Eq. (9.16), for vibrating objects other than strings.

a) For a thin membrane like a drum head the equivalent equation is

$$f = \frac{0.7655}{L}\sqrt{\frac{T}{h\rho}},$$

where L is the diameter of the drum, h is the thickness of the membrane, ρ is its density, and T is the tension. If we double the diameter of the drum head, how will the musical pitch of the note change? How about if we double the thickness?

b) For a solid bar like the bar of a xylophone, with length L and thickness h, the frequency is

$$f = \frac{3\sqrt{3}\pi h}{16L^2}\sqrt{\frac{E}{\rho}},$$

where ρ is again the density and E is a quantity called the Young's modulus, which measures the hardness of the material. If we double the length of such a bar, how will the musical pitch of the note change? How about if we double the thickness?