Submission to the *Journal of Answers*
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Problem suggested in class: For what conditions does the following hold?

$$E\tilde{x}u'(s+\tilde{x}) = 0 \Rightarrow Eu'(s+\tilde{x}) \leq u'(s)$$

A figure helps to establish the forbidden region (see figure 1 on next page).
Applying the diffidence theorem,

$$Eu'(s+\tilde{x}) - u'(s) \leq mE\tilde{x}u'(s+\tilde{x})$$

If true for all $\tilde{x}$, then our necessary and sufficent condition is

$$u'(s+x) - u'(s) \leq mxu'(s+x)$$

Taking derivatives with respect to x yields

$$\begin{aligned} u''(s+x)|_{x=0} &= m[u'(s+x) + xu''(s+x)]|_{x=0} \\ u''(s) &= m[u'(s)] \\ m &= \frac{u''(s)}{u'(s)} \leq 0 \end{aligned}$$

Now take the third derivative:

$$\begin{aligned} u'''(s+x)|_{x=0} &\leq m[2u''(s+x) + xu'''(s+x)]|_{x=0} \\ u'''(s) &\leq m * 2u''(s) \\ u'''(s) &\leq 2u''(s)\frac{u''(s)}{u'(s)} \\ \frac{u'''(s)}{u''(s)} &\geq \frac{2u''(s)}{u'(s)} \end{aligned}$$

Which gives our local necessary condition. Plugging our solution to m back into the necessary and sufficient condition:

$$\begin{aligned} u'(s+x) - u'(s) &\leq xu'(s+x)\frac{u''(s)}{u'(s)} \\ \frac{1}{u'(s)} - \frac{1}{u'(s+x)} &\leq x\frac{u''(s)}{[u'(s)]^2} \\ \frac{1}{u'(s)} - x\frac{u''(s)}{[u'(s)]^2} &\leq \frac{1}{u'(s+x)} \end{aligned}$$

As seen from the above equation, $\frac{1}{u'(s+x)}$ must be locally convex. See figure 2 on the next page for a graphical representation. Thus for the initial statement to be true for all s, $\frac{1}{u'(s+x)}$ must be globally convex.

Figure 1

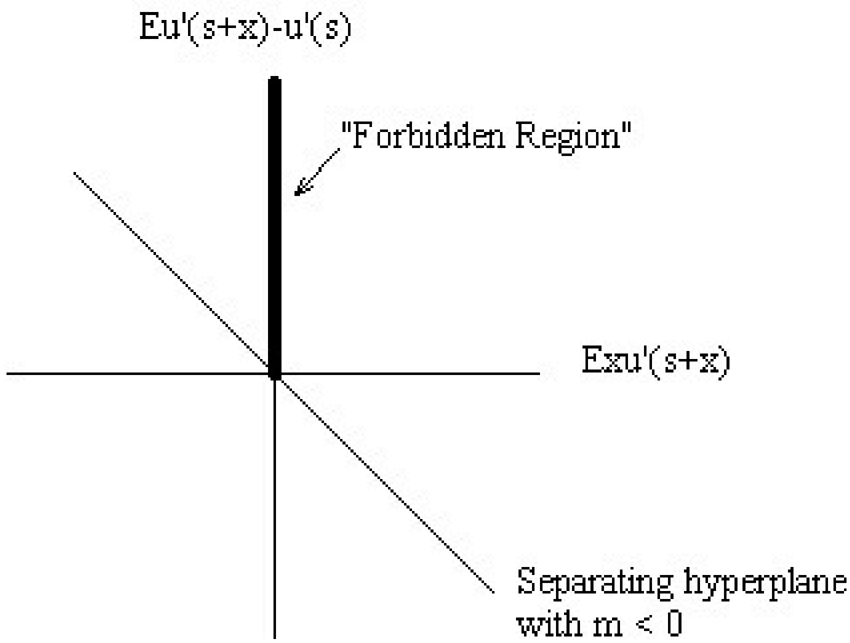

Figure 2