

ED 781-001 [31745]
The Study of Mathematics Instruction

Instructor: Magdalene Lampert
4113 School of Education Building
647-2447

Why this course came to be called "The *Study of Mathematics Instruction*:"

The American Heritage® Dictionary of the English Language, Third Edition defines the word "study" as follows:

study v. 1. To apply the mind in acquiring knowledge of: to *study* physics. 2. To examine; search into: to *study* a problem. 3. To look at attentively; scrutinize: the *study* one's reflection in a mirror. 4. To endeavor to memorize, as a part in a play. 5. To give thought and attention to, as something to be done or devised. 6. To apply the mind in acquiring knowledge. 7. To follow a regular course of instruction; be a student. 8. To muse, meditate.

Studying includes, but is not limited to, doing research on something. We will not be doing any memorizing of parts in this course, but we will hopefully engage in all of the other modes of studying. Note especially that the course involves *thinking* and *musings*.

As asserted by David K. Cohen, Stephen W. Raudenbush, and Deborah Loewenberg Ball in their essay, "Resources, Instruction, and Research,"

Instruction consists of interactions among teachers and students around content, in environments. The interactions occur in distance learning, small groups in classrooms, informal groups, tutorials, and large lectures. "Interaction" refers to no particular form of discourse but to teachers' and students' connected work, extending through, days, weeks, and months. Instruction evolves as tasks develop and lead to others, as students' engagement and understanding waxes and wanes, and organization changes (Lampert, 2001). Instruction is a stream, not an event, and it flows in and draws on environments -- including other teachers and students, school leaders, parents, professions, local districts, state agencies, and test and text publishers. This view of instruction has roots in early 19th century ideas, yet many researchers and practitioners still refer to teaching as though it was something done by teachers to learners.
(Educational Evaluation and Policy Analysis, Summer, 2003, Vol 25, no.2, p.122)

Cohen, Raudenbush, and Ball illustrate this phenomenon as follows:

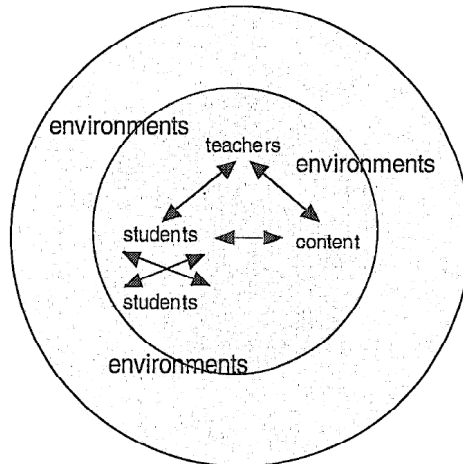


FIGURE 1. *Instruction as interaction.*

Although we will consider what teachers do as the practice of teaching and what students do as the practice of learning, it is assumed that in school, these are "relational practices," that is, the work of both teachers and students includes making productive relationships, between teacher and students, individually and in groups, among students in classrooms, between teacher and content and between students and content, individually and in groups. We will read and analyze together some of the "classic" works in the field of mathematics instruction that cover a broad swath of the terrain, and we will examine specific research on elements of instruction. We will also study videos representing mathematics instruction. As a group, we will focus our investigations of instruction on one "content" theme and one "process" theme as they play out in a particular environment, namely, the school classroom. (In terms of the definition of instruction above, "process" can be thought of as a means of structuring interaction between teacher and students in an instructional environment.) The content theme will be "graphing." This could be called a mathematical "topic" and it is included in the curriculum at many levels. It includes theoretical notions like the Cartesian plane and applied notions like box-and-whisker-plots. The process theme will be "classroom discourse." Within this theme we will investigate cognitive notions like what talking has to do with thinking and learning, and social issues like establishing classroom culture and learning to disagree.

Required assignments:

1. By October 5, choose a fundamental mathematical topic (e.g. multiplication, decimal numbers, operating on rational numbers) and a pedagogical process (e.g. use of representations, task design, inclusion of culturally diverse students) through which you will do an independent investigation of the relevant research on mathematics teaching and learning in school. For example, you might investigate *the use of multiple representations in the teaching and learning of negative numbers* or *the use of alternative counting systems to teach and study place value*. Or you might look at *instruction in small groups to teach and study mathematical definitions*. As a class, we will brainstorm possible combinations of topics and processes from which you can choose. You may work on this assignment individually or in pairs, and partners may choose to investigate one topic and two processes or two topics and one process. Alternatively, partners may choose the same topic and process, and investigate research on instruction at different levels of schooling.

In December, we will have one or two "poster sessions" at which you will present your findings to members of the class and others in the field of mathematics education. Each student will also submit a final paper, making use of collaborative work, but independently written. The paper will take the form of a "lit review" but it should be organized around an argument about mathematics instruction and not be simply a report on what you have read.

Some guiding questions for developing an argument:

For the topic:

What would it mean to "profoundly understand" this piece of mathematics?

What might teachers do to teach and students do to learn this topic?

What do learners have trouble understanding in this domain?

For the process:

What is the role of the teacher?

What is the role of the student?

What are some arguments in the field about the efficacy of this process?

2. Keep a progressive journal in which you enter on a weekly basis:
 - a. a list of new terms that you come across in your reading and definitions of each term in your own words. Where possible describe in writing an example of what you think the term means.
 - b. your developing thoughts about the following three questions as informed by what we talk about in class each week, and your reading:
 - i. What is learning?
 - ii. What is teaching?
 - iii. What is mathematics?
 - c. Your choice of one reference from the bibliography of the assigned reading that interests you. Look it up in the library and write a few sentences about why it was interesting to you and what you learned from reading it. These readings may follow a theme, if you like, or not.

This work should be submitted electronically on the following dates:

1. September 28
2. October 26
3. November 16
4. December 14

In order to receive credit for your submission, the file must be labeled with the course number, your initials, and the date in the following format, for example, if I were submitting a journal on October 26, I would name it

ED781ML10-26-04

The journal is called "progressive" because each time you submit your journal, I will make comments and raise questions, and return it to you. Add the next journal entry to the one on which I have written comments.

Expectations and grading policy:

Your grade will be assigned on the basis of the following four components:

1. Attendance and active participation in all class discussions based on thoughtful reading of assigned books and articles and thoughtful viewing and analysis of video representations of instruction. Appropriate participation includes a focused analysis of the readings as well as attention to the contributions of other class members and a poster presentation/discussion in class of your investigation of a topic/process of instruction (may be done with a partner).
2. A research paper on a topic/process of instruction
3. Your progressive journal with running records of
 - terms and your definitions from the readings
 - reflections on the meaning of mathematics, teaching and learning
 - annotated bibliography of independently identified resources

A letter grade (A, B, etc.) will be assigned on the basis of a composite judgment that weighs approximately equally the three major expectations described above. All forms of student work and participation will be factored into the final evaluation.

RESOURCE MATERIALS

Required books:

Chapin, Susan H.; O'Connor, Catherine; Anderson, Nancy. Classroom Discussions: Using Math Talk to Help Students Learn, Grades 1-6. (Sausalito, CA: Math Solutions Publications, 2003). ISBN 0-941355-53-5 (paper)

Chazan, Dan. Beyond Formulas in Mathematics and Teaching: Dynamics of the High School Algebra Class (NY: Teachers College Press, 2000). ISBN 0-8077-3918-9 (paper)

Hiebert, James; Carpenter, Thomas P.; Fenemma, Elizabeth; Fuson, Karen C.; Wearne, Diana; Murray, Hanlie; Oliver, Alwyn; and Human, Piet. Making Sense: Teaching and Learning Mathematics with Understanding. (Portsmouth, NH: Heinemann, 1997). ISBN 0435071327

Hiebert, James, et al. Teaching Mathematics in Seven Countries: Results from the TIMMS 1999 Video Study (Washington, DC: National Center for Educational Statistics, March 2003) (with CD-Rom) for ordering info go to <http://nces.ed.gov/timss>

Lampert, Magdalene. Teaching Problems and the Problems of Teaching. (New Haven, CT: Yale University Press, 2001). ISBN 0-300-09947-9 (paper)

Ma, Liping. Knowing and Teaching Elementary Mathematics. (Mahwah, NJ: Erlbaum, 1999). ISBN 0-8058-2909-1 (paper)

Sfaard, Anna & McClain, Kay. Analyzing Tools: Perspectives on the Role of Designed Artifacts in Mathematics Learning. Special Issue of The Journal of the Learning Sciences. Vol. 11, Nos. 2 & 3, 2002. The text of this book is available on line at <http://www.leaonline.com/toc/jls/11/2-3>

The complete journal, with CD-Rom including electronic text linked to video examples can be ordered from the publisher or through the instructor at a reduced rate.

Articles and chapters for possible use in class and for assignments (available electronically or on reserve):

Atweh, Bill; Bleicher, Robert E.; Cooper, Tom J. "The Construction of the Social Context of Mathematics Classrooms: A Sociolinguistic Analysis," Journal for Research in Mathematics Education, 1998, Vol. 29, No. 1, 63-82.

Ball, D. L. (2000). "Bridging practices: Intertwining content and pedagogy in teaching and learning to teach." Journal of Teacher Education, 51, 241-247. (D. Ball website)

Nicolas Balacheff "Contract and Custom: Two Registers of Didactical Interactions" (Translated and edited by Patricio Herbst) The Mathematics Educator Volume 9, Number 2 can be found at <http://jwilson.coe.uga.edu/DEPT/TME/Issues/v09n2/1balacheff.html>

Balacheff, Nicolas. "Towards a *problématique* for research on mathematics teaching." Journal for Research in Mathematics Education. . (Proquest full text)

Ball, D. L. , & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics*, (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics. (D. Ball website) Pair with Chazen, Brousseau, Herbst.

Boaler, Jo. "Open and Closed Mathematics: Student Experiences and Understandings," *Journal for Research in Mathematics Education*, 1998, Vol. 29, No. 1, 42-62. [PDF: Wilson Select Plus](#)

Brousseau, Guy. *Theory of Didactical Situations in Mathematics (Didactique des Mathématiques, (1970-1990))*. Edited and translated by Nicolas Balacheff, Martin cooper, Rosamund Sutherland, and Virginia Warfield. Editors' preface, Biography of Guy Brousseau, Introduction: Setting the Scene with an Example: The Race to 20, Chapter 1: Foundations and Methods of *Didactique* (pp. 1-75), and Appendix: The Center for Observation: The Ecole Jules Michelet at Talence (pp. 275-293).

Carpenter, Thomas P.; Franke, Megan; Jacobs, Victoria; Fennema, Elizabeth; Empson, Susan B. "A Longitudinal Study of Invention and Understanding in Children's Multidigit Addition and Subtraction," *Journal for Research in Mathematics Education*, 1997, Vol 29, No. 1, 3-20. (Jstor)

Chazan, D. & Ball, D. L. (1999). "Beyond being told not to tell." *For the Learning of Mathematics*, 9, 2-10. (D.Ball website)

Cohen, D., Raudenbush, S., & Ball, D. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, 25 (2), 1-24. (Proquest full text or D.Ball website)

Friel, Susan N.; Curcio, Frances R.; Bright, George W. "Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications," *Journal for Research in Mathematics Education*, 2001, Vol 32, No. 2, 124-158. (Proquest full text)

Gravemaijer, Koeno. *Developing Realistic Mathematics Education (Ontwikken van Realistisch Reken/Wiskundeonderwijs)*. Chapter 3; Mediating Between Concrete and Abstract (pp. 77-106).

Herbst, Patricio. "Using Novel Tasks in Teaching Mathematics: Three Tensions Affecting the Work of the Teacher," *American Educational Research Journal*, Spring 2003, Vol.40, No. 1, pp. 197-238.

Leinhardt, Gaea; Zaslavsky, Orit; and Stein, Mary Kay. "Functions, Graphs, and Graphing: Tasks, Learning, and Teaching," *Review of Educational Research*, Vol. 60, No. 1. (Spring, 1990), pp. 1-64. (Jstor)

Stable URL: <http://links.jstor.org/sici?sici=0034-6543%28199021%2960%3A1%3C1%3AFGAGTL%3E2.0.CO%3B2-Y>

Mack, Nancy. "Building Informal Knowledge Through Instruction in a Complex Content Domain: Partitioning, Units, and Understanding Multiplication of Fractions," *Journal for Research in Mathematics Education*, 2001, Vol. 32, No. 3, 267-295. . (Proquest full text)

McClain, Kay & Cobb, Paul. "An Analysis of Development of Sociomathematical Norms in One First-Grade Classroom," Journal for Research in Mathematics Education, 2001, Vol 32, No. 3, 236-266. (Proquest full text)

Moschkovich, Judit. "Supporting the Participation of English Language Learners in Mathematical Discussions," For the Learning of Mathematics, Vol. 19, No. 1 (March 1999).

Roth, Wolff-Michael & Bowen, G. Michael. "Professionals Read Graphs: A Semiotic Analysis," Journal for Research in Mathematics Education, 2001, Vol 32, No. 2, 159-194. Proquest full text (Pair with JLS, Leinhardt, and Friel)

Sfard, Anna. "Steering (Dis)Course between Metaphors and Rigor: Using Focal Analysis to Investigate an Emergence of Mathematical Objects," Journal for Research in Mathematics Education, 2000, Vol. 31, No. 3, 296-327. Proquest full text (Pair with O'Connor, McClain and Cobb)

Sfard, Anna. (1998). "On two metaphors for learning and the dangers of choosing just one." Educational Researcher, 27, 4-13.

Sfard, Anna. "Learning Mathematics as Developing a Discourse," Paper delivered at the 21st Annual Conference of Psychology of Mathematics Instruction. Available electronically at [www-leibniz.imag.fr/Didatech/ Seminaires2002/Sfard/PME-NA-21.pdf](http://www-leibniz.imag.fr/Didatech/Seminaires2002/Sfard/PME-NA-21.pdf)

Simon, Martin. "Reconstructing Mathematics Pedagogy from a Constructivist Perspective," Journal for Research in Mathematics Education, 1995, Vol. 26, No.2, 114-145. Jstor (pair with Balachef)

Steiner, Hans-Georg, "Philosophical and Epistemological Aspects of mathematics and their Interaction with Theory and Practice in Mathemaitcs Education," For the Learning of Mathematics, Vol. 7 No. 1 (February, 1987)

DVD representations of mathematics instruction:

Carpenter, Thomas and Romberg, Thomas. Powerful Practices in Matheamtics and Scinece: Research Based Practices for Teaching and Learning, National Center for Improving Student Learning and Achievement in Mathematics and Science, University of Wisconsin, Madison.

Carpenter, Thomas, Fennema, Elizabeth, Franke, Megan, Levi, Linda; and Empson, Susan B. Children's Mathematics: Classroom Episodes Portsmouth, NH: Heinemann.

Hiebert, James, et al. Teaching Mathematics in Seven Countries: Results TIMMS 1999 Video Study Sample Lessons

Possibly useful websites

Freudenthal Institute

<http://www.fi.uu.nl/en/welcome.html>

see Mathematics education in The Netherlands, a guided tour by Marja van den Heuvel-Panhuizen

Highlights from the TIMSS Videotape Classroom Study

<http://nces.ed.gov/pubs2000/2000094.pdf>

Math Forum <http://forum.swarthmore.edu/library/ed>

Proof: International Newsletter in the Teaching and Learning of Mathematical Proof

<http://www.lettredelapreuve.it/> Newsletter

Tentative semester schedule Fall 2004

Sept 7

Introduction to course structure and syllabus

Analysis of content of Journal of Research in Mathematics Education for studies of teaching

Sept 14

Conceptions of that which is to be taught and learned: the example of "graphs"

Leinhardt, Gaea; Zaslavsky, Orit; and Stein, Mary Kay. "Functions, Graphs, and Graphing: Tasks, Learning, and Teaching," Review of Educational Research, Vol. 60, No. 1. (Spring, 1990), pp. 1-64. ([Jstor](#))

Stable URL: <http://links.jstor.org/sici?sici=0034-6543%281990%21%2960%3A1%3C1%3AFGAGTL%3E2.0.CO%3B2-Y>

Roth, Wolff-Michael & Bowen, G. Michael. "Professionals Read Graphs: A Semiotic Analysis," Journal for Research in Mathematics Education, 2001, Vol 32, No. 2, 159-194. ([jstor](#))

Friel, Susan N.; Curcio, Frances R.; Bright, George W. "Making Sense of Graphs: Critical Factors Influencing Comprehension and Instructional Implications," Journal for Research in Mathematics Education, 2001, Vol 32, No. 2, 124-158. ([jstor](#))

Sept 28 (journals due)

What it means to say teaching is "complex"

The idea of "zooming": Lessons, interactions, big sets of connected topics, small notions, year, month

Read:

Lampert, Magdalene. Teaching Problems and the Problems of Teaching: Chapters 1, 2, 3, 4, 5, and 8 (pp. 1-119, 179-211)

Oct 5 (have a topic chosen for your project)

Zooming in: the use of tools and inscriptions in instruction

Read:

last revised September, 2004

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Sfaard and McClain, Analyzing Tools: Perspectives on the Role of Designed Artifacts in Mathematics Learning. pp.153 to 274 and 365-368; NB especially how each author addresses the idea of how classroom culture is established and supports the mathematics students can work on

Oct 12

Zooming out: When instruction is situated in an algebra course in a working class high school: the problem of motivation for mathematics

Read:

Chazan, Dan. Beyond Formulas in Mathematics and Teaching: Dynamics of the High School Algebra Class (all)

Oct 26 (After fall recess: journals due)

Zooming in: the instructional mechanics of classroom talk, and how teachers and students interact with content in doing it

Read:

Chapin, Susan H.; O'Connor, Catherine; Anderson, Nancy. Classroom Discussions: Using Math Talk to Help Students Learn, Grades 1-6. (all)

O'Connor, M.C. & Michaels, S. (1993). Aligning Academic Task and Participation Status through Revoicing: Analysis of a Classroom Discourse Strategy. Anthropology and Education Quarterly, Vol. 24, No. 4, 318-335.

Lampert, Teaching Problems and the Problems of Teaching chapters 6 and 7

Nov 2 (Election Day: If you are registered, please vote!)

Zooming out: Teaching all participants through involvement in classroom talk

Read:

Moschkovich, Judit. "Supporting the Participation of English Language Learners in Mathematical Discussions," For the Learning of Mathematics, Vol. 19, No. 1 (March 1999). [to be made available in class]

Boaler, Jo. "Open and Closed Mathematics: Student Experiences and Understandings," Journal for Research in Mathematics Education, 1998, Vol. 29, No. 1, 42-62. PDF: Wilson Select Plus

Gutierrez, Rochelle (2003). Beyond Essentialism: The Complexity of Language in Teaching Mathematics to Latina/Latino Students. American Educational Research Journal, Vol. 39, No. 4, pp. 1047-1088.

Lampert, Teaching Problems and the Problems of Teaching, chapters 10, 11, 12

Nov 9

What when and why something is taught and studied in math classes

Covering the curriculum

Scope and sequence

Read:

Lampert, Teaching Problems and the Problems of Teaching, chapter 9, 13

Hiebert, James, et al. Teaching Mathematics in Seven Countries: Results from the TIMMS 1999 Video Study (Washington, DC: National Center for Educational Statistics, March 2003) (with CD-Rom) for ordering info go to <http://nces.ed.gov/timss>

Steiner, Hans-Georg, "Philosophical and Epistemological Aspects of mathematics and their Interaction with Theory and Practice in Mathematics Education," For the Learning of Mathematics, Vol. 7 No. 1 (February, 1987)

Nov 16 (journals due)

Understanding and making sense: revisiting PUFM

(be prepared to show a poster that captures a representation of PUFM for the topic of your project)

Read:

Ma, Knowing and Teaching Elementary Mathematics, chapters 3,4 and 5

Mack, Nancy. "Building Informal Knowledge Through Instruction in a Complex Content Domain: Partitioning, Units, and Understanding Multiplication of Fractions," Journal for Research in Mathematics Education, 2001, Vol. 32, No. 3, 267-295. . (Proquest full text)

Carpenter, Thomas P.; Franke, Megan; Jacobs, Victoria; Fennema, Elizabeth; Empson, Susan B. "A Longitudinal Study of Invention and Understanding in Children's Multidigit Addition and Subtraction," Journal for Research in Mathematics Education, 1997, Vol 29, No. 1, 3-20. (Jstor)

Nov 23 (just before Thanksgiving) (Gloriana)

Can sense be made in the classroom?

Read:

Herbst, Patricio. "Using Novel Tasks in Teaching Mathematics: Three Tensions Affecting the Work of the Teacher," American Educational Research Journal, Spring 2003, Vol.40, No. 1, pp. 197-238.

Nicolas Balacheff "Contract and Custom: Two Registers of Didactical Interactions"

(Translated and edited by Patricio Herbst) The Mathematics Educator

Volume 9, Number 2 can be found at

<http://jwilson.coe.uqa.edu/DEPT/TME/Issues/v09n2/1balacheff.html>

Balacheff, Nicolas. "Towards a *problématique* for research on mathematics teaching." Journal for Research in Mathematics Education. (Proquest full text)

Brousseau, Guy. Theory of Didactical Situations in Mathematics (Didactique des Mathématiques, (1970-1990)). Edited and translated by Nicolas Balacheff, Martin cooper, Rosamund Sutherland, and Virginia Warfield. Editors' preface, Biography of Guy Brousseau, Introduction: Setting the Scene with an Example: The Race to 20, Chapter 1: Foundations and Methods of *Didactique* (pp. 1-75), and Appendix; The Center for Observation: The Ecole Jules Michelet at Talence (pp. 275-293).

Nov 30

How can sense be made in the classroom?

(submit readings on your topic for next week that can be found by your classmates on the web)

Read:

Ball, D. L. , & Bass, H. (2003). Making mathematics reasonable in school. In J. Kilpatrick, W. G. Martin, and D. Schifter (Eds.), *A Research Companion to Principles and Standards for School Mathematics*, (pp. 27-44). Reston, VA: National Council of Teachers of Mathematics. (D. Ball website)

van den Heuvel-Panhuizen, Marja "Mathematics Education in The Netherlands, a Guided Tour " at <http://www.fi.uu.nl/en/welcome.html> the website of the Freudenthal Institute

Hiebert, James; Carpenter, Thomas P.; Fenemma, Elizabeth; Fuson, Karen C.; Wearne, Diana; Murray, Hanlie; Oliver, Alwyn; and Human, Piet. Making Sense: Teaching and Learning Mathematics with Understanding (all)

Yackel, E. & Cobb, P. (1996). Sociomathematical Norms, Argumentation, and Autonomy in Mathematics, Journal for Research in Mathematics Education, vol. 27, No. 4, 458-477.

(Proquest full text)

Dec 7 (Gloriana)

How can sense be made in the classroom, part 2

Looking at the instructional processes that you have investigated

Read articles chosen by classmates, to be announced

Dec 14 (journals due)

Revisiting the concept of "instruction": what have we studied and how does it all fit together?

Read:

Cohen, D., Raudenbush, S., & Ball, D. (2003). Resources, instruction, and research. Educational Evaluation and Policy Analysis, 25 (2), 1-24. (Proquest full text or D.Ball website)

Lampert, Teaching Problems and the Problems of Teaching, chapter 14

Dec 17

Write up of project due

Please note: extensions will be granted in only very extreme circumstances.