Abstracts Week 3

Dori Bejleri

Title: The cotangent complex for pairs

Abstract: I will describe a new approach to the deformation theory of pairs (X, Z) using derived algebraic geometry with applications toward moduli spaces of higher dimensional log pairs. This is joint work in progress with G. Inchiostro and E. Elmanto.

Harold Blum

Title: Moduli of Fano varieties with complement

Abstract: While the theories of KSBA stability and K-stability have been successful in constructing compact moduli spaces of canonically polarized varieties and Fano varieties, respectively, the case of K-trivial varieties remains less well understood. I will discuss a new approach to this problem in the case of pairs (X, D), where X is a Fano variety and D is an anticanonical $\mathbf{Q}$-divisor, in which we consider all semi-log-canonical degenerations. One challenge of this approach is that the moduli stack can be unbounded. Nevertheless, if we consider pairs (X, D), where X is degeneration of $\mathbf{P}^2$, we show that there exists a projective moduli space. This is joint work with K. Ascher, D. Bejleri, K. DeVleming, G. Inchiostro, Y. Liu, and X. Wang.

Rankeya Datta

Title: Mittag-Leffler modules and Frobenius

Abstract: A central conjecture in prime characteristic commutative algebra is that test elements exist for the class of excellent domains. The conjecture holds for F-finite noetherian domains, domains that are essentially of finite type over excellent local rings and excellent domains with universally injective Frobenius. However, the problem remains elusive in general, even in interesting geometric settings. For example, due to recent constructions of Murayama and myself, we do not know if the rigid analytic analog of the coordinate ring of an affine variety admits test elements over an arbitrary non-Archimedean field of prime characteristic. Sharp proposed an approach to tackle the conjecture for rings of this latter type via a technical notion called Frobenius intersection flatness. This notion was first introduced by Hochster and Huneke in their efforts to show the existence of test elements. In unrelated seminal work on the descent of projectivity, Raynaud and Gruson introduced the notion of a Mittag-Leffler module. In this talk we will connect the intersection flatness condition to the Mittag-Leffler condition. Consequently, we will show how the existence of test elements in the rigid analytic and related settings reduces to a simple question about the openness of universally injective loci for

certain maps of modules. This talk is based on joint work with Neil Epstein and Kevin Tucker.

Tommaso de Fernex

Title: Geometry of arc spaces

Abstract: The arc space of a variety is an infinite dimensional scheme whose geometric structure captures, in a way that is yet to be fully understood, certain features of the singularities of the variety. Focusing on its local rings and invariants of these rings such as embedding dimension and codimension, we explore the local structure of arc spaces. Arc spaces are defined as the inverse limit of jet schemes and are often studied via their truncation maps; here we will present an alternative approach using projections to infinite dimensional affine spaces and finiteness properties of arc fibers. The talk is based on joint work with Christopher Chiu and Roi Docampo.

Mattias Jonsson

Title: Divisorial stability

Abstract: Let (X, B) be a complex projective klt pair. We define a notion of divisorial stability for an ample numerical class on X, defined in terms of an invariant on convex combinations of divisiorial valuations. For the class of an ample line bundle, divisorial stability coincides with K-stability for filtrations, as studied by C. Li, and is thus a stronger notion than (uniform) K-stability, and a sufficient condition for the existence of an cscK metric when X is smooth. For a Fano variety, divisorial stability is the same as uniform K-stability. We prove that divisorial stability is an open condition on the numerical class. This is joint work with S. Boucksom.

Joaquin Moraga

Title: Fundamental groups of algebraic singularities

Abstract: In this talk, we will review the history of fundamental groups of algebraic singularities and explain some new developments. We start by reviewing classic results regarding normal, quotient, rational, and Cohen-Macaulay singularities. Then, we will discuss more modern classes of singularities: log terminal and log canonical. We will explain some new results regarding fundamental groups of lc singularities. For instance, in dimension 3, no free group of rank at least 2 is the fundamental group of an lc singularity. On the other hand, in dimension 4, every free group is the fundamental group of an lc singularity.

Yusuke Nakamura

Title: Minimal log discrepancies of quotient singularities

Abstract: The minimal log discrepancy (MLD) is an invariant of singularity defined in the context of the minimal model program. In this talk, we will discuss the minimal log discrepancies of quotient singularities. I will explain that the PIA (precise inversion of adjunction) conjecture and Shokurov's index conjecture hold for quotient singularities. For the PIA conjecture, the theory of the arc space of a quotient singularity established by Denef and Loeser is an essential tool. For the index conjecture, Jordan's theorem on finite linear groups is essentially used. This is joint work with Kohsuke Shibata.

Zsolt Patakfalvi

Title: Varieties with nef anti-canonical have surjective Albanese

Abstract: I will present a joint work with Sho Ejiri showing that smooth projective varieties with nef anti-canonical divisor have surjective Albanese morphism. The statement was conjectured in the Kähler setting by Demailly-Peternell-Schneider in 1993, and it was shown in characteristic zero by Zhang in 1996. Our contribution is that it also holds in positive characteristic, and hence over any field. This is the first arbitrary dimensional positive characteristic result on varieties with nef anti-canonical divisor that is not sensitive to wild behavior: wild action of Frobenius on cohomology, wild singularities of the general fibers over the Albanese image, etc. I will also mention a few corollaries and generalizations.

Kenta Sato

Title: General hyperplane section of log canonical threefolds in positive characteristic

Abstract: In this talk, we prove that if a 3-dimensional quasi-projective variety X over an algebraically closed field of characteristic $p > 3$ has only log canonical singularities, then so does a general hyperplane section H of X. In the course of the proof, we give a sufficient condition for log canonical surface singularities over a field to be geometrically log canonical.

Teppei Takamatsu

Title: Fedder-type criteria for quasi-F-splitting

Abstract: In algebraic geometry in positive characteristic, singularities defined by the Frobenius map, including the notion of F-splitting, have played a crucial role. Moreover, there are powerful criteria, known as Fedder's criteria, for confirming such

properties. On the other hand, Yobuko introduced the notions of quasi-Frobenius-splitting and F-split heights, which generalize and quantify the notion of F-splitting. In this talk, I will present a generalization of Fedder's criteria to quasi-F-splittings and explain some applications of such criteria. This talk is based on a joint paper with Tatsuro Kawakami and Shou Yoshikawa.

Joe Waldron

Title: The log minimal model program for excellent threefolds

Abstract: The log minimal model program has recently been completed for klt threefolds over regular excellent base schemes of residue characteristic $p > 5$. In this talk I will survey the known results, together with some motivations and applications for working in this more general setup.

Jakub Witaszek

Title: Singularities in mixed characteristic via the Riemann-Hilbert correspondence

Abstract: In my talk, I will start by reviewing how various properties of characteristic zero singularities can be understood topologically by ways of the Riemann-Hilbert correspondence. After that, I will explain how similar ideas can be applied in the study of mixed characteristic singularities. This is based on a joint work (in progress) with Bhargav Bhatt, Linquan Ma, Zsolt Patakfalvi, Karl Schwede, Kevin Tucker, and Joe Waldron.

Ziquan Zhuang

Title: Boundedness of singularities and minimal log discrepancies of Kollár components

Abstract: The local volume of a klt singularity is an interesting invariant that comes up in the local study of K-stability. It is conjectured that K-semistable Fano cone singularities form a bounded family if their local volumes are bounded away from zero. I'll discuss some recent work on this conjecture through the minimal log discrepancies of Kollár components.