# Anticipatory Random Walks: A Dynamic Model of Crowdfunding

Saeed Alaei* Azarakhsh Malekian† Mohamed Mostagir‡

**Abstract**

We provide a model of all-or-nothing crowdfunding in which consumers arrive sequentially and make decisions about whether to pledge or not. Pledging is not costless, and hence consumers would prefer not to pledge if they think the campaign will not succeed. This uncertainty can lead to cascades where a campaign fails to raise the required amount even though there are enough consumers who want the product. The central contribution of the paper is the introduction of a novel stochastic process — *anticipatory random walks*— to analyze this collective action problem. We use this process to prove a series of inequalities that show that the success probability of crowdfunding is bimodal: the outcomes concentrate around succeeding with high probability or failing with certainty. In addition to crowdfunding, this random walk and its analysis can find wider applications in other contexts and dynamic problems where decisions made in the present are not only based on history but also stochastically depend on future outcomes.

## 1 Introduction

Crowdfunding is an approach that allows businesses and entrepreneurs to decentralize the funding process by directly appealing to the end consumer as a funding source. In a crowdfunding environment, a seller solicits financial contributions from the crowd, usually in the form of consumers buying a still-unrealized product, and commits to producing the product if the total amount pledged is above a certain threshold. The threshold is usually chosen to cover the amount of funds required to start production, and the product is not funded if the campaign is unable to reach that threshold within

*Google Research, Mountain View CA. email: alaei@google.com
† Rotman School of Management, University of Toronto. email:Azarakhsh.Malekian@rotman.utoronto.ca
‡ Ross School of Business, University of Michigan. email: mosta@umich.edu

an allotted amount of time. This funding paradigm can rectify some of the common inefficiencies found in the traditional supply and demand process, as a seller does not commit to production until demand is observed and necessary capital is raised, while consumers who are interested in products that might be considered too risky or too specialized get a chance to help bring these products to market. Crowdfunding platforms like Kickstarter and Indiegogo connect sellers with potential consumers, as well as regulate these campaigns by a) vetting the seller and the proposed project, and b) ensuring that consumers who make a pledge to buy follow through with their purchase if the campaign is successful.

This paper presents a dynamic model of crowdfunding where consumers arrive sequentially and make decisions on whether to back a product or not. The focus is on all-or-nothing funding schemes similar to the one used exclusively on Kickstarter. Consumers have their own valuations for the product and make their pledging decision after observing the following: a) the price of the product, b) the aggregate contribution made by previous backers, c) the campaign funding target, and d) the duration of the campaign. Pledging is not costless: after making a pledge and until the campaign is over, a backer may have to pass on opportunities to use their money in anticipation of using it to pay for their pledge if the campaign is successful. However, if the campaign fails, then not only does the backer not get the product, but she has also forfeited the option to use the funds in the interim. Because of this, consumers need to estimate the chances that a campaign will succeed before they decide to make a pledge. This can lead to cascades where the absence of earlier pledges makes those who arrive later pessimistic about the chances of campaign success, and therefore discourages them from pledging, leading to a vicious cycle. Conversely, a pattern may emerge where consumers estimate the success probability to be high and create a virtuous cycle through pledging. The goal of this paper is to understand how and when do these different dynamics arise.

**Contribution** The primary contribution of this paper is methodological. We analyze the behavior of consumers through introducing a novel stochastic process that we call an *anticipatory random walk*, which has the property that the transition probability at every step of the walk depends on the probability of the walk eventually reaching a particular success state in the future. This random walk is complicated to analyze because it anticipates the success probability at every time step, and this anticipation takes into consideration future anticipations as well, leading to a recursive structure. We

| % funding received | 0% | $1-20\%$ | $21-40\%$ | $41-60\%$ | $61-80\%$ | $81-99\%$ | $\geq 100\%$ |
|---|---|---|---|---|---|---|---|
| Number of campaigns | 55,847 | 205,635 | 32,268 | 13,562 | 5,605 | 4,210 | 199,581 |

Table 1: Total number of campaigns and the percentage of funding goal they received since the inception of Kickstarter. Data collected on 4/9/2021. More on `https://www.kickstarter.com/help/stats`

circumvent this difficulty by introducing a related dynamic process that is *completely deterministic*, and whose analysis allows us to obtain concentration results for the anticipatory random walk and the associated crowdfunding problem. In particular, we obtain bounds in terms of the problem's parameters that characterize the regions in which campaigns fail with certainty or succeed with high probability, and show the existence of a phase transition between these two regions. We believe that this random walk and its analysis can find wider applications in other contexts and dynamic problems where the decisions made by current actors are not only based on history but also stochastically depend on how they will affect the decisions of future actors. We mention some of these applications in Section 5.

On the modeling side, the paper suggests that uncertainty arising from the collective action problem can be one of the salient features that affect pledging behavior, and embeds this feature in a dynamic framework. We note that the predictions of our model agree with a sharp pattern observed in the data on crowdfunding campaigns, namely, the bimodal distribution of campaign outcomes. Campaigns either succeed, or fail to generate any substantial fraction of their funding goal, i.e. it is unlikely that a campaign would fail by raising, for example, 85% of its target funding. The possible outcomes are overwhelmingly tilted towards either success or dismal failure. Table 1 shows that out of all projects on Kickstarter, 50% failed to generate more than 20% of their goal while ≈40% were successfully funded. Our analysis of anticipatory random walks lead to similar concentration results, suggesting that the model might be capturing some element of the underlying dynamics of crowdfunding. It is important to point out that the paper does not claim to provide a singular explanation for all crowdfunding outcomes or what makes a campaign successful. Instead, the goal of the paper is to isolate what we believe is an important factor —uncertainty arising from the collective action problem— and examine its effect on campaign outcomes.

**Related Literature** The potential for using crowds to improve the decisions and operations of firms has attracted a lot of recent attention. Araman and Caldentey (2016) and Marinesi and Girotra (2013) explore the idea of using a crowd voting mechanism to gauge demand and interest in a product before committing to production. Voting is costless and does not come with a commitment to purchase, and hence their focus is on how the firm can use this information to update its estimates of demand and adjust its funding target. Lobel et al. (2017) study how crowd referrals can be used as a marketing tool for the firm. On the empirical front, Mollick (2014) provides a detailed analysis of data from $\sim$ 48,500 Kickstarter projects. Similar to Table 1, he finds that projects either succeed, or they fail by large margins, with failed campaigns raising only 8% of their funding target on average. Kuppuswamy and Bayus (2013) use data to try and empirically understand the dynamics of crowdfunding, and find that the propensity of backers to contribute is influenced by how much has already been pledged. Similar effects are found by Wu et al. (2014) in the context of group buying. This is consistent with the empirical and experimental literature on public goods. In a randomized experiment, List and Lucking-Reiley (2002) find that donors give more when they are told that the required funding for a project is near its goal. Similarly, Vesterlund (2003) finds that announcing contributions generates more contributions. This self-reinforcing behavior is also observed in our dynamic model, where backers' decisions are influenced by the current contribution level. Bagnoli and Lipman (1989) and Varian (1994) study if public goods can be curated through collecting private donations in an all-or-nothing mechanism. In contrast to their work, the product in our model is a private good produced through collective action, and no free-riding can happen since consumers only get the good if they pay for it. Additionally, prices and the number of people required for success are set by the campaign designer instead of letting donors decide their own contribution levels.

On the theory side, Belleflamme et al. (2014) develop a static model to explore when the campaign creator should offer a pre-ordering scheme (like the one explored in this paper) or an equity-based scheme where backers become investors. Anand and Aron (2003) also use a static model to examine when group-buying is preferred to a posted-price mechanism and the timing of pricing and production decisions when there is uncertainty about the size of the market. Jing and Xie (2011) show that social interactions between informed and uninformed consumers can improve the efficiency of group buying. In more recent work, Hu et al. (2015) adopt a product-line design approach to the crowdfunding problem. They show that offering slightly-differentiated products and an accompanying menu of prices is optimal, whereas our main interest is in understanding how consumers in a dynamic setting respond to the choices of product price and consequently the number of pledges required to reach an exogenous goal. Hu et al. (2013) compare sequential and simultaneous group-buying mechanisms. More recent work in Strausz (2017) and Belavina et al. (2020) examines the design of platform rules in the presence of moral hazard (for example, sellers running away with the backers' money), while Babich et al. (2020) investigate when should entrepreneurs select crowdfunding vs. other methods of funding. Chakraborty and Swinney (2020) show how a seller can use prices to communicate information to consumers about the quality of the unrealized product.

This paper is also related to the social and observational learning literature, e.g., Lobel and Sadler (2015); Acemoglu et al. (2011); Bikhchandani et al. (1992) and the survey in Acemoglu and Ozdaglar (2011), where agents arrive sequentially and observe other agents' choices, and then update their beliefs about an underlying state of the world. The important difference is that the environment we consider in this paper adds an element of collective action that is absent in that literature. Agents do not act just to maximize their current utility (for example, by choosing a better retailer or a better restaurant), but also because of the consequence of their choices on the actions of those agents who follow, whose own choices affect the agents' utility through the dependence of everyone's payoff on campaign success. Because of this, the underlying dynamics of these two environments are quite different.

Finally, global games is a standard methodology for modeling uncertainty arising from collective action (see, Carlsson and Van Damme (1993)). This framework captures similar situations to the one we study in this paper where a collection of agents need to coordinate on an action (for example, whether to show up for a protest, as in Ali (2011) or whether to go on a bank run Diamond and Dybvig (1983)) and one of the challenges of that framework is the multiplicity of equilibria (see Angeletos and Werning (2006)), which, as Dahleh et al. (2016) show, depends on the structure of the information exchanged between agents. We remark that a static version of our problem can perhaps be modeled following Dahleh et al. (2016), where information about the desirability of the product in a network can be summarized by signals that agents utilize and compare against a threshold in order to coordinate over whether to pledge or not. The setup employed in our paper however proposes that consumers arrive sequentially in order to capture the temporal aspect of these campaigns, which cannot be modeled directly by global games, in which agents move simultaneously.

The paper is organized as follows. We introduce our benchmark model in Section 2. Section 3 is the main section of the paper: it defines anticipatory random walks and analyzes a related deterministic process to obtain bounds on their success probability. Section 4 extends the model in several directions, some related to the analysis of the stochastic process itself and some related to extending the modeling choices of the benchmark model. Section 5 discusses model limitations and concludes the paper.

discusses the limitations of our model and the associated difficulties with generalizing some of the results, and Section 5 concludes the paper.

## 2 Benchmark Model

Our benchmark model has a seller who offers a (still-unrealized) product to consumers through an all-or-nothing crowdfunding platform. The seller is interested in raising an exogenous minimum target fund $G$ to cover the various production costs, and chooses a pledge amount, or price $p$, for the product being sold. The campaign has duration $N$ and consumers arrive sequentially, with one consumer arriving in each time period. We refer to the consumer arriving in period $i = 1, ..., N$, as consumer $i$. Consumers can pledge $p$ to the project upon their arrival or exit without pledging. If by the deadline $N$ the total amount raised is at least equal to the goal $G$ (equivalently, if the total number of pledges made is at least equal to $k = \left\lceil \frac{G}{p} \right\rceil$), then the campaign is successful: backers are charged $p$ each, funds are allocated to the seller, and production commences. Conversely, if the campaign falls short of its goal, no one is charged. We assume that the seller has a prior on consumer valuations. In particular, valuations are independent and consumer $i, i = 1, .., N$ has valuation $v_i$, where

$$v_i = \begin{cases} v^H & \text{with probability } \pi_i \\ v^L & \text{with probability } 1 - \pi_i \end{cases}$$

where $v^H$ is the consumer's high valuation, and $v^L < v^H$ is the consumer's low valuation. Different consumers need not have the same probability of having a high valuation, which can be used to model different situations; for example, high-valuation consumers being more likely to arrive earlier in the campaign and so on. If consumer $i$ decides to pledge, she gets utility equal to $v_i - p$ if the campaign is successfully funded. As discussed in the introduction, consumers who pledge anticipate paying the pledge amount at the conclusion of the campaign and therefore forfeit the opportunity to use the funds in the interim. If the campaign fails, then backers would have tied up their money and got nothing in return. This scenario is captured by having an opportunity (uncertainty) cost $c > 0$ that a backer incurs in case of campaign failure.

The utility of backer $i$ can thus be written as

$$u_i = \begin{cases} v_i - p & \text{if campaign successfully funded ,} \\ -c & \text{if not funded.} \end{cases} \tag{1}$$

A consumer who does not pledge gets utility zero. Based on this discussion, a consumer's decision to pledge depends on the valuation for the product as well as the price, but also crucially hinges on their belief about whether the campaign will succeed in reaching its goal $G$ or not. This belief changes from one backer to the next depending on the information set $I_i$ of backer $i$, where $I_i = \{m_i, N - i + 1, \boldsymbol{\pi}, v^H, c, p\}$. This information set summarizes the current state of the campaign, with $m_i$ being the total number of pledges made up to (and including) period $i - 1$, $N - i + 1$ indicating how much time is left in the campaign (including the current period), and the remaining components indicating the primitives of the problem, with $\boldsymbol{\pi}$ being the vector of probabilities of high valuations. Let $\alpha(I_i)$ be the campaign success probability that consumer $i$ estimates using her information set, which we will denote by $\alpha_i$ for short, then using (1) to write the expected utility $E[u_i]$, we get

$$E[u_i] = \alpha_i(v_i - p) - (1 - \alpha_i)c,$$

which means that backer $i$ will pledge only if her valuation $v_i$ satisfies

$$v_i \geq \left( \frac{(1 - \alpha_i)c}{\alpha_i} + p \right)$$

or equivalently, if the estimated success probability satisfies

$$\alpha_i \geq \frac{c}{v_i - p + c} = \beta_i$$

The previous inequality indicates that a consumer will pledge if they estimate the probability of success to be at least equal to a threshold $\beta_i$. For backers to make a decision then, they need to be able to estimate the success probability $\alpha_i$. The sequential nature of the problem can be described by a recursion that we introduce and analyze in the next section.

## 3 Backer Behavior and Anticipatory Random Walks

We start this section by observing that if the price satisfies $p \leq v^L$, then everyone pledges as long as $Np \geq G$ and no one pledges otherwise, i.e. there is no uncertainty about the outcome. If on the other hand the price satisfies $v^L < p \leq v^H$, then low valuation consumers will not pledge, which is the more involved case and the one we consider in the rest of the paper. We can then assume without loss of generality that $v^L$ is equal to zero. As discussed in the previous section, consumers estimate success probabilities in order to decide whether to pledge or not. We can rewrite the inequality above so that high-valuation consumers pledge only if the success probability they estimate satisfies

$$\alpha_i \geq \frac{c}{v_i - p + c} = \frac{c}{v^H - p + c} = \beta \tag{2}$$

Note that the consumer's problem is *time* and *path*-dependent: it matters not only how many consumers have high valuations, but also the times at which these consumers arrive and what happened up to that time.

The following recursion computes the probability of success at each time step. Let $s_i^j$ be the success probability that consumer $i$ estimates given that $j$ pledges have been made up to and including her own pledge. Recall that $\pi_i$ is the probability that consumer $i$ has high valuation and that a campaign succeeds if at least $k$ pledges are made by time $N$. The probabilities $s_i^j$ can be written recursively as follows:

$$s_i^j = \begin{cases} 0 & i = N,\, j < k \\ 1 & i = N,\, j \geq k \\ s_{i+1}^j & i < N,\, s_{i+1}^{j+1} < \beta \\ \pi_{i+1} s_{i+1}^{j+1} + (1 - \pi_{i+1}) s_{i+1}^j & j < N,\, s_{i+1}^{j+1} \geq \beta \end{cases}. \tag{3}$$

We are interested in the campaign success probability $s_0^0$. The next subsection defines a stochastic process that corresponds to the above recursion and proves a few concentration inequalities that can be used to characterize the likelihood of different campaign outcomes.

### 3.1 Anticipatory Random Walks

An anticipatory random walk is a stochastic process that is characterized by having current transitions depend on the probability that future transitions satisfy a certain event. Formally

**Definition 1.** Given a probability $\beta \in (0, 1)$, a target $k \in \mathbb{N}$, and a sequence of independent Bernoulli random variables $X_1, \ldots, X_n$ with known means $\pi_1, \ldots, \pi_n \in (0, 1]$, an *anticipatory random walk* is given by a sequence of positions on the real line denoted by $Y_0, Y_1, \ldots, Y_n$ defined as follows:

$$Y_i \mid Y_{i-1}, X_i = \begin{cases} Y_{i-1} + X_i & \text{if } \mathbf{Pr}[Y_n \geq k \mid Y_i = Y_{i-1} + X_i] \geq \beta \\ Y_{i-1} & \text{if } \mathbf{Pr}[Y_n \geq k \mid Y_i = Y_{i-1} + X_i] < \beta \end{cases}$$

$$Y_0 = 0$$

A realization of an anticipatory random walk is called *successful* if it advances at least a total of $k$ units by the end of time $n$, i.e. if $Y_n \geq k$.

Informally, an anticipatory random walk advances one unit at time $i$ if $X_i = 1$ and if the probability of eventual success conditioned on current position at time $i$ is at least equal to $\beta$. We are interested in computing the probability of success at time $0$. We note that the term *anticipatory* in our setting is used to indicate thinking about, rather than actually seeing into, the future: transitions depend on probabilities of future outcomes and not on the knowledge of the actual outcomes themselves. This departs from how the term is sometimes used in the stochastic processes literature. In the parlance of that literature, the anticipatory random walk as defined above is an adapted process (see Grimmett and Stirzaker (2001)).

**Connection to crowdfunding** Consider a scenario with $n$ backers where the parameters of the problem are such that a consumer with high value pledges only if she believes that the campaign will be funded with probability at least $\beta$ (c.f. Equation (2)). This can be modeled as an anticipatory random walk as follows:

- The target parameter $k$ of the anticipatory random walk is the same as the minimum number of pledges required for the success of the crowdfunding campaign.
- The parameter $\beta$ from the anticipatory random walk is equal to the pledging threshold $\frac{c}{v^H - p + c}$ of a backer in the crowdfunding problem.
- Each Bernoulli random variable $X_i$ in the anticipatory random walk corresponds to the arrival (or not) of one backer in the crowdfunding problem.
- $\mathbf{Pr}[X_i = 1] = \mathbf{Pr}[v_i = v^H] = \pi_i$, i.e. the probability that the Bernoulli random variable $X_i = 1$ is the same as the probability that backer $i$ has high valuation.

Thus, the success probability $s_i^j$ of the crowdfunding campaign when $j$ pledges are made by $i$ backers is equal to $\mathbf{Pr}[Y_n \geq k | Y_i = j]$, the success probability of an anticipatory random walk at time $i$ given that it advanced $j$ steps. In particular, the ex-ante success probability of the anticipatory random walk by the end of $n$ time steps, i.e., $\mathbf{Pr}[Y_n \geq k]$, is exactly equal to the ex-ante success probability $s_0^0$ of the corresponding crowdfunding campaign.

The main result of our paper is the following concentration theorem:

**Theorem 1.** *An anticipatory random walk as given in Definition 1 satisfies the following two properties:*

- *if* $\sum_i \pi_i \geq k(\ln(1/(1-\beta)) + 1)$, *then* $\mathbf{Pr}[Y_n \geq k] \geq \beta$,
- *if* $\sum_i \log(\frac{1}{1-\pi_i}) \leq k \ln(1/(1-\beta))$, *then* $\mathbf{Pr}[Y_n \geq k] < \beta$.

The next section is devoted to proving this theorem. One novel aspect of our analysis is that, while the process in Definition 1 is stochastic, the process we introduce and analyze in the next section is completely deterministic.

### 3.2 A Flow Process

This section analyzes a dynamic flow process that enables us to obtain bounds on $s_i^j$ and in particular, on $s_0^0$, the probability of success of the anticipatory random walk and the corresponding crowdfunding campaign. The process is deterministic and involves a fluid material flowing between nodes according to set rules. We note some differences between this process and a traditional network flow process: in the process we analyze, nodes are not merely conduits for flow, but also function as "containers" and *store* fluid themselves, up to unit capacity. Unlike classical flow problems, the focus is not on the maximum or minimum amount of flow in a network of arbitrary topology, but rather on the amount of fluid stored in these nodes as a function of how much time had elapsed.

**Definition 2** (Dynamic Flow Process)**.** Given a sequence $q_1, \ldots, q_n \in (0,1)$ and an infinite sequence of initially empty nodes indexed by integer numbers, the amount of fluid in node $j \in \mathbb{N} \cup \{0\}$ at the end of time step $t \in \{0, \ldots, n\}$ is given by $b_j^t$, defined as follows:

$$b_j^t = \begin{cases} q_t b_{j-1}^{t-1} + (1-q_t) b_j^{t-1} & b_{j-1}^{t-1} \geq \beta \\ 0 & b_{j-1}^{t-1} < \beta \\ 0 & t = 0, j \geq 1 \\ 1 & j = 0 \end{cases} \tag{4}$$

This process can be visualized by seeing that the following takes place at every time step $t \in \{1, \ldots, n\}$:

- Simultaneously for all nodes $j \in \mathbb{N}$: if the amount of fluid in node $j$ is at least $\beta$, a fraction $q_t$ of the fluid in that node is moved to node $j+1$, otherwise a fraction $q_t$ is discarded.
- $q_t$ units of fluid are added to node $1$.

| Parameter | Crowdfunding | Anticipatory Random Walk | Flow Process |
|---|---|---|---|
| Probability (Fraction of Fluid) | $\mathbf{Pr}[v_i = v^H] = \pi_i$ | $\mathbf{Pr}[X_i = 1] = \pi_i$ | $q_{n-i+1} = \pi_i$ |
| Success Probability (Amount of Fluid) | $s_i^j$ | $\mathbf{Pr}[Y_n \geq k \mid Y_i = j]$ | $b_{k-j}^{n-i}$ |

Table 2: Notation for equivalent quantities across the different Models

**Connection to anticipatory random walks** An anticipatory random walk with parameters $\beta$, $k$, and $\pi_1, \ldots, \pi_n$ is related to a flow process with parameters $q_1, \ldots, q_n$ as follows:

- The target parameter $k$ of the anticipatory random walk is the index of the $k^{th}$ node in the flow process.
- For all nodes $j$, the threshold $\beta$ from the anticipatory random walk is equal to the amount of fluid in node $j$ beyond which a fraction of fluid moves from node $j$ to node $j+1$ (and below which a fraction of fluid in $j$ is discarded).
- $\mathbf{Pr}[X_i = 1] = \pi_i = q_{n-i+1}$, i.e., the probability that the Bernoulli random variable $X_i = 1$ is equal to the fraction of fluid moved out of each node at time $n-i+1$.

Thus, the success probability of an anticipatory random walk at time $i$ and given that it advanced $j$ steps, i.e., $\mathbf{Pr}[Y_n \geq k \mid Y_i = j]$, is the same as the amount of fluid in node $k-j$ at time $n-i$. Then from the equivalence of $\mathbf{Pr}[Y_n \geq k \mid Y_i = j]$ and $s_i^j$, we have

$$s_i^j = b_{k-j}^{n-i}. \tag{5}$$

Table 2 provides the equivalent notation across the three processes. We are interested in computing bounds on the success probability $s_0^0$, which from Equation (5), is the same as the amount of fluid stored in node $k$ at time $n$, $b_k^n$. Thus bounding this quantity enables us to get our desired result. To this end, we give a few definitions and auxiliary results that help us derive the main theorem at the end of this section.

**Definition 3** (Critical Node)**.** For each $t \in \{1, \ldots, n\}$, the *critical node* at the end of time $t$ is the node whose index $\lambda^t$ is given by

$$\lambda^t = \max\left\{j \mid b_j^t > 0, j \in \mathbb{N}\right\} \qquad \forall t \in \{1, \ldots, n\}. \tag{6}$$

We also define $B^t$ to denote the total amount of fluid in all nodes (except node $0$) at time $t$, i.e.

$$B^t = \sum_{j \in \mathbb{N}} b_j^t \qquad \forall t \in \{0, \ldots, n\}. \tag{7}$$

**Definition 4** (Critical Onset Time)**.** For each $j \in \mathbb{N}$, the critical onset time of node $j$, denoted by $\tau_j$, is the first time at which node $j$ becomes the critical node:

$$\tau_j = \min\left(\left\{t \mid \lambda^t = j\right\}, \infty\right) = \min\left(\left\{t \mid b_j^t > 0\right\}, \infty\right) \tag{8}$$

Observe that node $j$ is the critical node only within the times $t \in \{\tau_j, \ldots, \tau_{j+1} - 1\}$, assuming $\tau_{j+1} < \infty$.

We are now ready to present our formal results about the above flow process. These results are used to prove Theorem 1. The proofs of all results can be found in the appendix.

The following lemma states that the amount of fluid in a node increases over time and that nodes are filled left-to-right, with any node having at most an amount of fluid that is equal to the amount of fluid in a node with a lower index.

**Lemma 1.** *The value of $b_j^t$ is (weakly) decreasing in $j$ and (weakly) increasing in $t$. Furthermore, $b_{j'}^{t'} \geq \beta$ for all $j' \leq j$ and all $t' \geq \tau_{j+1} - 1$, and $b_{j'}^{t'} < \beta$ for all $j' \geq j$ and all $t' < \tau_{j+1} - 1$.*

Lemma 1 together with Definition 2 imply that in each iteration, the only discarded fluid comes from the critical node. We use this property to characterize the total increase in fluid between two consecutive time steps.

**Lemma 2.** *The increase in the total amount of fluid in all nodes at time step $t \in \{1, \ldots, n\}$ is*

$$B^t - B^{t-1} = q_t \cdot (1 - b_{\lambda^t}^{t-1}) \tag{9}$$

We next calculate an upper bound on the amount of fluid in a node at any time $t$ after that node becomes critical. As we show shortly, combining that upper bound with Lemma 2 provides a lower-bound on the total amount of fluid added in each step of the process.

**Lemma 3.** *The amount of fluid in node $j \in \mathbb{N}$ at time $t \geq \tau_j$ is at most*

$$b_j^t \leq 1 - \prod_{t'=\tau_j}^{t} (1 - q_{t'}). \tag{10}$$

The next result is the main lemma used to prove the theorem at the end of this section. Utilizing the shorthand notation $(x)^+$ to denote $\max(x, 0)$, we can write

**Lemma 4.** *The increase in the total amount of fluid in all nodes during the times that node $j \in \mathbb{N}$ is critical is at least*

$$B^{t^\dagger} - B^{\tau_j - 1} \geq \beta + (1 - \beta) \left( \sum_{t=\tau_j}^{t^\dagger} q_t - \ln(1/(1 - \beta)) \right)^+ \tag{11}$$

*where $t^\dagger = \tau_{j+1} - 1$, or $t^\dagger < \tau_{j+1} - 1$ and $\sum_{t=\tau_j}^{t^\dagger} q_t \geq \ln(1/(1 - \beta))$.*

We now give the main result of this section.

**Theorem 2.** *The flow process in Definition 2 satisfies the following two properties for any $k \in \mathbb{N}$:*

- *if $\sum_t q_t \geq k\left(\ln(1/(1 - \beta)) + 1\right)$, then $b_k^n \geq \beta$*
- *if $\sum_t \log(\frac{1}{1-q_t}) < k \ln(1/(1 - \beta))$, then $b_k^n < \beta$.*

The idea of the proof is to show that if the first condition in the theorem holds by the end of time $n$, then the total amount of fluid in all nodes indexed 1 and higher must be greater than $k$, but since each node has unit capacity, it must be that there are at least $k + 1$ non-empty nodes. But by Definition 2 and Equation (4), node $k + 1$ will only start filling up if node $k$ has more than $\beta$ amount of fluid in it, and so the result follows.

Having proved Theorem 2, we can show the analogous result for anticipatory random walks:

**Proof of Theorem 1** The proof follows immediately from Equation (5) and the fact that $\mathbf{Pr}[Y_n \geq k] = s_0^0 = b_k^n$. ∎

### 3.3 Phase Transition and Success with High Probability

We use the results from the previous section to show the existence of a *phase transition* in the success probability of the anticipatory random walk. In particular, the next proposition establishes the existence of a threshold above which the random walk fails with certainty.

**Proposition 1.** *(Phase Transition) An anticipatory random walk has a phase transition around $k^*$, for which the following properties hold:*

- *if $k \leq k^*$, then $\mathbf{Pr}[Y_n \geq k] \geq \beta\pi_1$,*
- *if $k > k^*$, then $\mathbf{Pr}[Y_n \geq k] = 0$.*

*Furthermore,*

$$\frac{\sum_{i=2}^{n} \pi_i}{\ln(1/(1-\beta)) + 1} \leq k^* - 1 \leq \frac{\sum_{i=2}^{n} \ln(\frac{1}{1-\pi_i})}{\ln(1/(1-\beta))}$$

In contrast to Proposition 1, Proposition 2 shows that a slight perturbation of the first condition in Theorem 1 leads to the anticipatory random walk succeeding with probability close to 1: for every $0 < \epsilon < 1$, there is a corresponding threshold value such that if $k$ is less than that value then the walk succeeds with probability $1 - \epsilon$.

**Proposition 2.** *(Success with High Probability) For any $\epsilon > 0$, there exists $\delta_0(\epsilon, n_0) > 0$ such that for all $\delta \geq \delta_0(\epsilon, n_0)$, if $\sum_{i=1}^{n} \pi_i \geq (k+1)(\ln \frac{1}{1-\beta} + 1)(1+\delta)$, then $\mathbf{Pr}[Y_n \geq k] \geq 1 - \epsilon$, where $n_0$ is such that $\sum_{i=n_0}^{n} \pi_i \geq k(\ln \frac{1}{1-\beta} + 1)$.*

Propositions 1 and 2 show that the outcomes of the anticipatory random walk concentrate around success and failure, similar to the underlying crowdfunding application. Together with Theorem 1, these results can apply to the problem of price selection. In particular, combining the fact that $p = G/k$ and the expression for $\beta$ from Equation (2), we can write the first condition of Theorem 1 as

$$\begin{aligned} G &\leq \sum_i \pi_i \frac{p}{\ln\left(1 + \frac{c}{v^H - p}\right) + 1} \\ &= \sum_i \pi_i h(p) \end{aligned}$$

where $h(p)$ is a concave function in $p$ and is maximized at $p^*$. This gives a quick bound on $G$ as any target goal with value less than $\sum_i \pi_i h(p^*)$ succeeds with probability at least $\beta$. Similarly, we can rewrite the condition from Proposition 2 to obtain a sufficient but more complicated expression for the relationship between the parameters of the problem that guarantees success with probability $1-\varepsilon$ for $\epsilon > 0$. We have

$$\begin{aligned}
\frac{1}{1+\ln\frac{1}{1-\beta}}\sum_{i=1}^{n}\pi_i &\geq (k+1)(1+\delta) \\
&= (G/p+1)\left(1+\frac{(\ln(1/\epsilon)\ln(n))^{1/2}}{n(k/n)(1+\ln\frac{1}{1-\beta})}\right) \\
&= (G/p+1)\left(1+\frac{(\ln(1/\epsilon)\ln(n))^{1/2}}{(G/p)(1+\ln\frac{1}{1-\beta})}\right)
\end{aligned}$$

where the first equality follows from using the lower bound for $\delta$ from the proof of Proposition 2 (see page 29). Noting that $1+\ln\frac{1}{1-\beta} > 1$, we see that given parameters $n, c, v^H, \pi_i$, and target goal $G$ and $\epsilon > 0$, the campaign succeeds with probability at least $1-\varepsilon$ if the price $p$ satisfies

$$\frac{1}{1+\ln\left(1+\frac{c}{v^H-p}\right)}\sum_{i=1}^{n}\pi_i \geq (G/p+1)\left(1+\frac{(\ln(1/\epsilon)\ln(n))^{1/2}}{G/p}\right)$$

# 4 Extensions

## 4.1 Infinite and Stochastic Number of Periods

The focus of our paper has been on examining the behavior of the anticipatory random walk when the number of periods is fixed. As is standard in the stochastic processes literature, we examine how our results change if a) the number of periods is infinite or b) the number of periods is stochastic. While not directly applicable to crowdfunding, these variations model situations like startup or venture funding within a single investment round. Such endeavors usually consist of a sequence of stages that should be completed, and it is possible that the process terminates at one of these stages before successfully concluding. Anticipating the probability of such termination changes the transition probabilities themselves.

The next result shows that the anticipatory random walk always succeeds when the number of periods goes to infinity.

**Proposition 3.** *For any fixed $k$, $\lim_{n\to\infty} \mathbf{Pr}\left[Y_n \geq k\right] \geq \beta$.*

We now consider what happens with uncertain ending times. Continuing from above, assume that the process can go on indefinitely, but that it can terminate every period with probability $1-r$. Denote the random time of termination by $\bar{n}$, then Definition 1 changes to:

$$Y_i \mid Y_{i-1}, X_i = \begin{cases} Y_{i-1} + X_i & \text{if } \mathbf{Pr}[Y_{\bar{n}} \geq k \mid Y_i = Y_{i-1} + X_i] \geq \beta \\ Y_{i-1} & \text{if } \mathbf{Pr}[Y_{\bar{n}} < k \mid Y_i = Y_{i-1} + X_i] < \beta \end{cases}$$

$$Y_0 = 0$$

The process above fails if $Y_{\bar{n}} < k$ and succeeds otherwise. In what follows, we let $\pi_i = \pi$ for all $i$. We note that without this assumption, the process is non-homogeneous and computing the absorption probabilities in the proof of the next result becomes more challenging.

**Proposition 4.** *An anticipatory random walk with target $k$, continuation probability $r$ at every step, and threshold $\beta^* = \left(\frac{r\pi}{1-(1-\pi)r}\right)^k$ satisfies the following for all $\beta \in (0,1)$:*

- *If $\beta \leq \beta^*$, then $\mathbf{Pr}[Y_{\bar{n}} \geq k] = \left(\frac{r\pi}{1-(1-\pi)r}\right)^{k+1}$*
- *If $\beta > \beta^*$, then $\mathbf{Pr}[Y_{\bar{n}} \geq k] = 0$.*

### 4.2 Heterogeneous Costs

The benchmark model assumes that all consumers have the same uncertainty cost $c$. We now consider the case where these costs are different from one backer to the next. This can account for the fact that backers have different outside options and opportunity costs for tying up their money. We consider the setup we have worked with so far (binary valuations) but let consumer $i$ have cost $c_i$. Recall that $\beta_{n-i+1} = \frac{c_i}{v^H - p + c_i}$, and hence this is equivalent to having thresholds $\beta_1, ..., \beta_n$, with the conditions in Equation (4) being $b_j^t > 0$ only if $b_{j-1}^{t-1} \geq \beta_{j-1}$ for all $j$.

In Appendix B, we verify that our results hold for the case of heterogeneous uncertainty costs, with slight modifications to our main theorem. To demonstrate the result, note that for example, customers who arrive earlier wait for longer until the campaign concludes, and hence also tie up their funds for longer and as a result might incur higher penalties in case of failure, which will be captured by having $c_1 \geq c_2 \geq ... \geq c_n$. Recall from Equation (5) that node indices in the flow process run in the opposite direction to consumer indices, so that $\beta_1 \leq ... \leq \beta_n$. This implies that node $i$ takes longer to fill up to its respective $\beta_i$ as $i$ increases (which corresponds to earlier arrivals requiring higher success probabilities to pledge), therefore reducing the quantity of fluid in $b_k^n$. Nevertheless, just like Theorem 2, Theorem 3 in Appendix B gives us a bound on $b_k^n$ in terms of the other parameters of the problem.

### 4.3 Managing Price

We have assumed that the price is fixed throughout the horizon. However, prices can be used as a lever throughout the campaign in order to deal with specific realizations of sample paths as well as compensate consumers differently for the level of risk they take on when they pledge. A consumer who arrives on the last day of the campaign or after the campaign is successful faces no uncertainty compared to someone who arrived before the funding target was met, and can therefore be offered a different price to account for this fact. We now examine what happens if the price can be updated as the campaign goes along. The primary difference between this case and the one we analyzed is that once the price is fixed, as in the latter, the success probability is only a function of future arrivals. In the former case however, the success probability is a function of both future arrivals and future prices. Denote by $g_i$ the amount of target goal left after period $i$, with $g_0 = G$, and let $\alpha_i(g_i)$ be the success probability that consumer $i$ estimates after she had made her pledging decision and the remaining amount is $g_i$.

**Proposition 5.** *Denote by $p_i^*(g_{i-1})$ the optimal price in period $i = 1, ..., N$ when there is $g_{i-1}$ target goal left, then $p_i^*(g_{i-1})$ satisfy*

$$p_i^*(g_{i-1}) = \begin{cases} v^H & g_{i-1} > (N-(i-1))v^H \\ \max\left\{0, v^H + c\left(1 - \frac{1}{\pi\alpha_{i+1}(g_{i-1}-p_i^*-p_{i+1}^*)+(1-\pi)\alpha_{i+1}(g_{i-1}-p_i^*)}\right)\right\} & g_{i-1} \leq (N-(i-1))v^H \end{cases}$$

The right hand side can be considered consumer $i$'s *effective* valuation, which combines her valu-

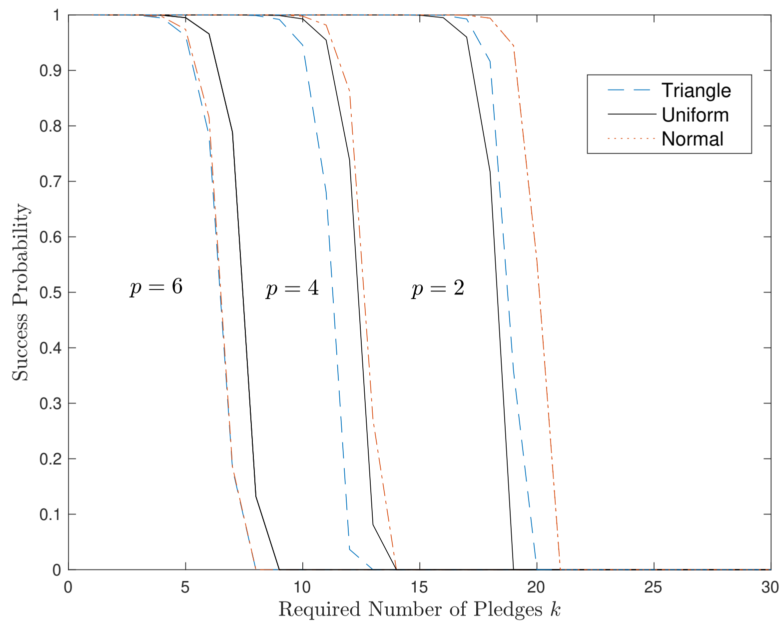


Figure 1: Success Probability for uniform, normal, and triangle distributions

ation for the product with the uncertainty she faces (assuming that future prices are optimally set). The price $p_i^*$ extracts the entire effective valuation. Note that the resulting prices can fluctuate up and down on any sample path, and do not necessarily have properties like earlier customers getting charged less. Nevertheless, we remark that in this setting these prices are "fair", in the sense that they have built-in discounts to compensate consumers for the risk they take on.

## 5 Discussion and Model Limitations

The success of a crowdfunding campaign hinges on understanding and accurately predicting the behavior of potential backers. A model that accounts for the myriad behavioral and other idiosyncratic factors that go into a backing decision is likely to be analytically intractable. In this paper, we isolate what we believe is a salient feature of these campaigns – backers' uncertainty about the outcome resulting from the collective action problem– and embed it in a dynamic framework whose analysis helps illustrate the prevalent success and failure pattern observed in crowdfunding outcomes.

The paper's central contribution is the introduction of anticipatory random walks. These stochastic processes have the property that their transitions at each time step probabilistically depend on the walk reaching a certain state in the future, which complicates their analysis. Our technical results show that outcomes of these walks concentrate around failure and success with high probability, and provide conditions under which each of these outcomes materialize. These walks and their analysis can be applied to a broad range of settings. One particular setup that resembles our framework is how donations are made to candidates running for office (e.g., Mutz (1995)). Donors anticipating that a particular candidate will not generate a lot of donation money are less inclined to put in their own money. Similarly, Andreoni (2006) discusses how a large donor might back a candidate in anticipation of how this backing can signal future potential donors to contribute as well. Another example is initial coin offerings (e.g., Gan et al. (2019)), which are quite similar to how crowdfunding operates except that valuations can also change over time with how the process evolves (which ties this setup to the correlated values discussion in the previous section). Single-round venture funding mentioned in Section 4 is another example where present actions are affected by the likelihood of future outcomes.

**Limitations** Our model makes some simplifying assumptions to focus on the uncertainty aspect and achieve tractability, but it can be extended on several fronts. The first of these would be to consider the model in a setting where the valuations and costs come from arbitrary distributions. We believe that the bimodal structure will still be maintained in this case, though the analysis becomes more challenging (we refer the interested reader to Appendix B for details). The model in this case is similar to a Markov jump system where the probabilities in the jump matrix $P_{ij}$ at time $t$ depend on the future jump probabilities. We are unaware of any work that addresses or even defines this problem.

Despite the difficulty of deriving analytical results for the general valuations case, we conjecture that the bimodal structure is preserved. Figure 1 provides simulation results for three valuation distributions that have the same mean (5) and standard deviation (2.88): uniform, normal, and triangle distributions. It can be seen that for different prices and number of required pledges, the structure is quite similar to the binary case we have analyzed in the paper.

Other venues for extension could be to consider correlated valuations, where not only the actions but also the valuations of consumers are influenced by past backing behavior (so that projects that attract early interest for example become more desirable). We also assume that consumers either pledge or exit, and do not have the option to wait before making a pledge. There is a wealth of literature on pricing for strategic and/or patient agents (e.g., Besbes and Lobel (2015); Lobel (2020), but no work that we are aware of that addresses the collective action problem of this paper, which will require consumers to solve correlated optimal stopping problems whose equilibria are difficult to pin down.

We are hopeful that the anticipatory random walk finds use beyond the setting developed in this paper. There are multiple directions to extend this work in terms of application and methodology. Beyond the suggestions highlighted above, continuous time analogs, for example, “anticipatory Brownian motion” are also a potential extension and constitute yet another interesting area of future research.

# Appendix

## A Proofs

**Proof of Lemma 1.** We first show that $b_j^t$ is decreasing in $j$, using induction on $t$. For $t = 0$, $b_0^0 = 1$ and $b_j^0 = 0$ for all $j > 0$. We next assume that for all $t' < t$, $b_j^{t'} \geq b_{j+1}^{t'}$, for all $j$, and we prove that $b_j^t \geq b_{j+1}^t$, for all $j < \lambda^{t-1}$. Using (4), we have

$$b_j^t = q_t b_{j-1}^{t-1} + (1 - q_t) b_j^{t-1} \geq q_t b_j^{t-1} + (1 - q_t) b_{j+1}^{t-1} = b_{j+1}^t,$$

where the first inequality holds by induction. Moreover, for $j = \lambda^{t-1}$, by definition $b_j^t > b_{j+1}^t = 0$, and for $j > \lambda^{t-1}$, $b_j^t = b_{j+1}^t = 0$, completing the inductive proof of the first part. Using this property, we next show that $b_j^t$ is weakly increasing in $t$. For all nodes with $b_j^{t-1} > 0$, using (4), we have

$$b_j^t = q_t b_{j-1}^{t-1} + (1 - q_t) b_j^{t-1} = b_j^{t-1} + q_t (b_{j-1}^{t-1} - b_j^{t-1}) \geq b_j^{t-1},$$

where the last inequality follows from $b_j^t$ being weakly decreasing in $j$.
Finally by definition, at $\tau_{j+1}$, $b_{j+1}^{\tau_{j+1}} > 0$ for the first time, which can only happen if $b_j^{\tau_{j+1}-1} \geq \beta$. Using the first two parts of this lemma, it implies that for all $j' \leq j$, and for all $t' \geq \tau_{j+1} - 1$, $b_{j'}^{t'} \geq \beta$. Similarly, at $t' < \tau_{j+1}$, $b_{j+1}^{t'} = 0$, which implies that $b_j^{t'-1} < \beta$. Combining this with the first two parts of the lemma, we have for all $t' < \tau_{j+1} - 1$, and for all $j' \geq j$, $b_{j'}^{t'} < \beta$, completing the proof.

■

**Proof of Lemma 2.** Using Lemma 1 at time $t - 1$, we have $b_j^{t-1} \geq \beta$ for all $j \leq \lambda^t - 1$, which implies that except for the first node (with index 1) and the critical node (with index $\lambda^t$), any movement of fluid is confined to be from one node to another, and thus the net change is equal to the amount of fluid that enters the first node, which is $q_t$, minus the amount of fluid that is discarded from the critical node, which implies that

$$B^t - B^{t-1} = q_t \cdot (1 - b_{\lambda^t}^{t-1}).$$

■

**Proof of Lemma 3.** The proof is by induction on $t$. The base case is $t = \tau_j$ for which the statement holds trivially because

$$\begin{aligned} b_j^{\tau_j} &= q_{\tau_j} b_{j-1}^{\tau_j - 1} + (1 - q_{\tau_j}) b_j^{\tau_j - 1} \\ &\leq q_{\tau_j} \end{aligned}$$

where the equality follows from (4) and the inequality from the facts that $b_{j-1}^{\tau_j-1} \leq 1$ and $b_j^{\tau_j-1} = 0$. To prove the induction step for $t > \tau_j$ observe that

$$\begin{aligned} b_j^t &= q_t b_{j-1}^{t-1} + (1 - q_t) b_j^{t-1} \\ &\leq q_t + (1 - q_t) \left( 1 - \prod_{t'=\tau_j}^{t-1} (1 - q_{t'}) \right) \\ &= 1 - \prod_{t'=\tau_j}^{t} (1 - q_{t'}). \end{aligned}$$

where the inequality follows from $b_{j-1}^{\tau_j-1} \leq 1$ and the induction hypothesis. ■

**Proof of Lemma 4.** First observe that if $\sum_{t=\tau_j}^{t^\dagger} q_t \leq \ln(1/(1-\beta)$ and $t^\dagger = \tau_{j+1} - 1$, the right hand side of eq. (11) is $\beta$, so the inequality holds trivially because

$$\begin{aligned} B^{t^\dagger} - B^{\tau_j - 1} &\geq \sum_{j'} b_{j'}^{t^\dagger} - b_{j'}^{\tau_j - 1} \\ &\geq b_j^{t^\dagger} - b_j^{\tau_j - 1} \\ &\geq \beta \end{aligned}$$

where the second inequality is by monotonicity of $b_{j'}^{t'}$ in $t'$ and the third inequality is because $b_j^{\tau_j-1} = 0$ and $b_j^{t^\dagger} \geq \beta$ by Lemma 1. For the rest of the proof we assume without loss of generality there exists a $t^* \in \{\tau_j, \ldots, t^\dagger\}$ and $\Delta \in [0, q_{t^*}]$ such that $\sum_{t=\tau_j}^{t^*-1} q_t + \Delta = \ln(1/(1-\beta)$. Observe that by definition of $\tau_j$ for all times $t \in \{\tau_j, \ldots, t^\dagger\}$, the critical node is $j$, that is $\lambda^t = j$. Therefore

$$B^{t^\dagger} - B^{\tau_j - 1} = \sum_{t=\tau_j}^{t^\dagger} B^t - B^{t-1}$$

$$
\begin{aligned}
&= \sum_{t=\tau_j}^{t^\dagger} q_t \cdot (1 - b_j^{t-1}) && \text{by Lemma 2 and } \lambda^t = j \\
&= \sum_{t=\tau_j}^{t^*-1} q_t \cdot (1 - b_j^{t-1}) + \Delta \cdot (1 - b_j^{t^*-1}) \\
&\quad + \overbrace{(q_{t^*} - \Delta)(1 - b_j^{t^*-1}) + \sum_{t=t^*+1}^{t^\dagger} q_t \cdot (1 - b_j^{t-1})}^{\Psi} \\
&\geq \sum_{t=\tau_j}^{t^*-1} q_t \cdot \prod_{t'=\tau_j}^{t-1} (1 - q_{t'}) + \Delta \cdot \prod_{t'=\tau_j}^{t^*-1} (1 - q_{t'}) + \Psi && \text{by Lemma 3} \\
&= \sum_{t=\tau_j}^{t^*-1} (1 - (1 - q_t)) \cdot \prod_{t'=\tau_j}^{t-1} (1 - q_{t'}) + \Delta \cdot \prod_{t'=\tau_j}^{t^*-1} (1 - q_{t'}) + \Psi \\
&= \sum_{t=\tau_j}^{t^*-1} \left( \prod_{t'=\tau_j}^{t-1} (1 - q_{t'}) - \prod_{t'=\tau_j}^{t} (1 - q_{t'}) \right) + \Delta \cdot \prod_{t'=\tau_j}^{t^*-1} (1 - q_{t'}) + \Psi \\
&= 1 - (1 - \Delta) \prod_{t'=\tau_j}^{t^*-1} (1 - q_{t'}) + \Psi && \text{by refactoring} \\
&\geq 1 - e^{-\Delta - \sum_{t'=\tau_j}^{t^*-1} q_{t'}} + \Psi && \text{by } e^{-x} \geq 1 - x \\
&= \beta + \Psi && \text{by definition of } t^* \text{ and } \Delta \\
&= \beta + (q_{t^*} - \Delta)(1 - b_j^{t^*-1}) + \sum_{t=t^*+1}^{t^\dagger} q_t \cdot (1 - b_j^{t-1}) && \text{by expanding } \Psi \\
&\geq \beta + (q_{t^*} - \Delta)(1 - \beta) + \sum_{t=t^*+1}^{t^\dagger} q_t \cdot (1 - \beta) && \text{by Lemma 1} \\
&= \beta + (1 - \beta) \left( \sum_{t=\tau_j}^{t^\dagger} q_t - \ln(1/(1 - \beta)) \right) && \text{by definition of } t^* \text{ and } \Delta
\end{aligned}
$$

■

**Proof of Theorem 2.** We start by proving the first statement. We show that the index of the critical node at time $n$ must be at least $k+1$ which then implies $b_k^n \geq \beta$. The proof is by contradiction. Suppose the index of the charging node at time $n$ is $j^* \leq k$ and first assume that $\sum_{t=\tau_{j^*-1}}^{n} q_t \geq \ln(1/(1-\beta)$ then

$$
\begin{aligned}
B^n &= \left(\sum_{j=1}^{j^*-1} B^{\tau_{j+1}-1} - B^{\tau_j-1}\right) + B^n - B^{\tau_{j^*}-1} \\
&\geq \left(\sum_{j=1}^{j^*-1} \beta + (1-\beta)\left(\sum_{t=\tau_j}^{\tau_{j+1}-1} q_t - \ln(1/(1-\beta))\right)^+\right) + B^n - B^{\tau_{j^*}-1} \quad \text{by Lemma 4} \\
&\geq j^* \cdot \beta + (1-\beta)\left(\sum_{t=1}^{n} q_t - j^* \ln(1/(1-\beta))\right)^+ \\
&> j^* \cdot \beta + (1-\beta)\left(k \cdot \ln(1/(1-\beta)) + k - j^* \cdot \ln(1/(1-\beta))\right) \quad \text{by theorem's hypothesis} \\
&\geq j^*,
\end{aligned}
$$

which is a contradiction, implying that $j^* > k$. Note that each node has at most $1$ unit of fluid and the critical node has at most $\beta$ amount of fluid. We next consider $\sum_{t=\tau_{j^*-1}}^{n} q_t \leq \ln(1/(1-\beta)$. Similar to the previous argument we have

$$
\begin{aligned}
B^n &= \left(\sum_{j=1}^{j^*-1} B^{\tau_{j+1}-1} - B^{\tau_j-1}\right) + B^n - B^{\tau_{j^*}-1} \\
&\geq \left(\sum_{j=1}^{j^*-1} \beta + (1-\beta)\left(\sum_{t=\tau_j}^{\tau_{j+1}-1} q_t - \ln(1/(1-\beta))\right)^+\right) + B^n - B^{\tau_{j^*}-1} \quad \text{by Lemma 4} \\
&\geq (j^*-1) \cdot \beta + (1-\beta)\left(\sum_{t=1}^{\tau_{j^*-1}} q_t - (j^*-1)\ln(1/(1-\beta))\right) + B^n - B^{\tau_{j^*}-1} \\
&\geq (j^*-1) \cdot \beta + (1-\beta)\left(\sum_{t=1}^{n} q_t - \sum_{t=\tau_{j^*-1}}^{n} q_t - (j^*-1)\ln(1/(1-\beta))\right)^+ + B^n - B^{\tau_{j^*}-1} \\
&\geq (j^*-1) \cdot \beta + (1-\beta)\left(\sum_{t=1}^{n} q_t - j^* \ln(1/(1-\beta))\right)^+ + B^n - B^{\tau_{j^*}-1} \\
&> (j^*-1) \cdot \beta + (1-\beta)\left(k \cdot \ln(1/(1-\beta)) + k - j^* \cdot \ln(1/(1-\beta))\right) + B^n - B^{\tau_{j^*}-1} \\
&> (j^*-1) + b^n_{j^*},
\end{aligned}
$$

which is a contradiction (where the next to last inequality follows from the theorem's hypothesis), again implying $j^* > k$.

We now prove the second statement. Suppose, $b_k^n \geq \beta$, then combining with Lemma 3, we have

$$\begin{aligned}(1-\beta)^k &\geq (1-b_k^n)\prod_{i=1}^{k-1}(1-b_i^{\tau_{i+1}-1}) \\ &\geq \left(1-(1-\prod_{t=\tau_k}^{n}(1-q_t))\right)\times\prod_{i=1}^{k-1}\left(1-(1-\prod_{t=\tau_i}^{\tau_{i+1}-1}(1-q_t))\right) \\ &= \prod_{t=1}^{n}(1-q_t) = e^{\sum_t \ln(1-q_t)} \\ &> e^{-k\ln(\frac{1}{1-\beta})} = (1-\beta)^k,\end{aligned}$$

which is a contradiction, implying $b_k^n < \beta$ and completing the proof.

■

**Remark 1:** Using a Taylor series expansion, the left-hand side of the second bullet in the statement of the theorem can be written as $\sum_i q_t - O(q_t^2)$. Assuming $q_t$ are sufficiently small, a first-order approximation of the sum can be rewritten as $\sum_t q_t < k\ln(1/(1-\beta)$.

**Remark 2:** Theorem 2 can be generalized to the case where the uncertainty costs are heterogeneous. Please see Appendix B for details.

**Proof of Proposition 1.** We first show the existence of the threshold $k^*$. Let

$$k^* = \max\{k : \mathbf{Pr}(Y_n \geq k|Y_1 = 1) \geq \beta\}$$

If $k > k^*$, we show that $Y_n = 0$ via induction. Using Definition 1, we have $Y_1 = Y_0 = 0$. We next show that $Y_i = 0$, assuming $Y_{i-1} = 0$. We have

$$\mathbf{Pr}\left[Y_n \geq k \mid Y_i = 1\right] < \mathbf{Pr}\left[Y_n \geq k \mid Y_1 = 1\right] < \beta,$$

which implies that $Y_i = Y_{i-1} = 0$ as desired. Now, if $k \leq k^*$ then we have

$$\mathbf{Pr}\left[Y_n \geq k\right] \geq \mathbf{Pr}\left[Y_n \geq k \mid Y_i = 1\right] \times \mathbf{Pr}\left[Y_i = 1\right] \geq \beta \times \pi_1,$$

which shows the first part of the proposition. The proof of the upper and lower bounds for $k^*$ follows from the equivalence in Equation (5) and noticing that $s_1^1 = b_{k-1}^{n-1}$. Using the first statement of Theorem 2, we have

$$k^* \geq k_1 = \frac{\sum_{t=2}^n q_t - q_1}{\ln(\frac{1}{1-\beta}) + 1} + 1 \geq \frac{\sum_t q_t}{\ln(\frac{1}{1-\beta}) + 1}$$

Also, using the second statement of Theorem 2, we have

$$k^* \leq k_2 = \frac{\sum_{t=2}^n \ln(\frac{1}{1-q_t})}{\ln(\frac{1}{1-\beta})} + 1,$$

completing the proof. ∎

**Proof of Proposition 2.** Define $\theta = \ln \frac{1}{1-\beta}$, and $\bar{\pi} = \frac{k}{n}$, i.e. $\bar{\pi}$ is the fraction of required pledges to the total number of arrivals (and is a constant). We break the proof into the following steps:

1. We first show that under the proposition assumption for all $1 \leq i \leq n_0$, $Y_i = Y_{i-1} + X_i$.

2. We then show that under the proposition assumption, for $t > n_0$, under the event that $\sum_{j=1}^i X_j \geq \sum_{j=1}^i \pi_j - \delta(1+\theta)(1+k)$, we have $Y_i = Y_{i-1} + X_i$, for all $i \leq t$.

3. Finally, we show that under the proposition assumption, the probability that $\sum_{j=1}^t X_j \geq \sum_{j=1}^t \pi_j - \delta(1+\theta)(1+k)$ for all $t \geq n_0$, is at least $1-\epsilon$.

4. Combining the preceding three steps implies that under the proposition assumption, with probability $1-\epsilon$, we have $Y_n = \sum_{i=1}^n X_i \geq k$, completing the proof.

To show the first step we should show that $Y_i = Y_{i-1} + X_i$ for all $0 \leq i \leq n_0$, i.e. a consumer will pledge if she has high valuation. By the definition of $n_0$, we have

$$\sum_{i=n_0}^n \pi_i \geq k(1+\theta).$$

Using Theorem 2, this would imply that at step $i \leq n_0$, even if no one before $i$ pledges, $\mathbf{Pr}[Y_n > k | Y_i = Y_{i-1} + X_i] \geq \beta$, therefore by Definition 1 we have $Y_i = Y_{i-1} + X_i$, i.e., $Y_{n_0} = \sum_{j=1}^{n_0} X_j$, completing the proof of the fist part.

We next show the proof of the second step.

We let $A_t$ denote the event that at time $t$, $\sum_{j=1}^{t} X_j \geq \sum_{j=1}^{t} \pi_j - \delta(1+\theta)(1+k)$. We next show that when the proposition condition holds, i.e., $\sum_{i=1}^{n} \pi_i \geq (k+1)(\theta+1)(1+\delta)$, and when the event $\bigcap_{i=n_0}^{t} A_i$ also holds, then $Y_i = Y_{i-1} + X_i$, for all $i \leq t$. As was shown in step (1), for all $i \leq n_0$, $Y_i = Y_{i-1} + X_i$, therefore, we only need to prove the statement for $i > n_0$. The proof of this step is via induction.

Assume by way of induction that for $t-1$, under the proposition assumption and when the event $\bigcap_{i=n_0}^{t-1} A_i$ holds, that $Y_i = Y_{i-1} + X_i$ for all $i \leq t-1$. We prove that at time $t$, if the event $\bigcap_{i=n_0}^{t} A_i$ holds, then $Y_i = Y_{i-1} + X_i$ for all $i \leq t$ as well. Using the induction hypothesis, and since $\bigcap_{i=n_0}^{t-1} A_i$ holds, for all $i \leq t-1$, we have $Y_i = Y_{i-1} + X_i = \sum_{j=1}^{i} X_j$ . Therefore, it suffices to show that $Y_t = Y_{t-1} + X_t$. Using Definition 1, $Y_t = Y_{t-1} + X_t$ if $\mathbf{Pr}[Y_n \geq k | Y_t = Y_{t-1} + X_t] \geq \beta$. We next show that under the proposition's assumption when event $\bigcap_{i=n_0}^{t} A_i$ holds, $\mathbf{Pr}[Y_n \geq k | Y_t = Y_{t-1} + X_t] \geq \beta$. Let $s$ denote the integer for which $s(1+\theta) \leq \sum_{i=t+1}^{n} \pi_i \leq (s+1)(1+\theta)$. This implies that $\sum_{i=1}^{t} \pi_i \geq (k+1)(1+\delta)(1+\theta) - (s+1)(1+\theta) = (k-s)(1+\theta) + \delta(1+\theta)(1+k)$. We then have

$$\begin{aligned}
\mathbf{Pr}\left[Y_n \geq k | Y_t = Y_{t-1} + X_t\right] &= \mathbf{Pr}\left[Y_n \geq k | Y_t = \sum_{j=1}^{t-1} X_j + X_t\right] \\
&= \mathbf{Pr}\left[Y_n \geq k | Y_t = \sum_{j=1}^{t} X_j\right] \\
&\geq \mathbf{Pr}\left[Y_n \geq k | Y_t \geq (k-s)(1+\theta)\right] \\
&\geq \mathbf{Pr}\left[Y_n - Y_t \geq s - (k-s)\theta\right],
\end{aligned}$$

where the first equality follows from the induction hypothesis and the next to last inequality holds because we assumed event $A_t$ holds. Moreover, we have $\sum_{j=t}^{n} \pi_j \geq s(1+\theta)$. Combining this with Theorem 2 implies that

$$\mathbf{Pr}\left[Y_n - Y_t \geq s - (k-s)\theta\right] \geq \mathbf{Pr}\left[Y_n - Y_t \geq s\right] \geq \beta,$$

completing the induction proof, and therefore the proof of the second step.

Finally, we prove that assuming $\sum_{i=1}^{n} \pi_i \geq (k+1)(1+\theta)(1+\delta)$, for $\delta \geq \delta_0(\epsilon, n_0) = \left(\frac{\ln(1/\epsilon)\ln(n-n_0)}{n\bar{\pi}^2(1+\theta)^2}\right)^{1/2}$, then the probability that $\bigcap_{i=n_0}^{n} A_i$ holds is at least $1-\epsilon$. We let $\bar{A}_i$ denote the complement of event $A_i$, i.e., $\bar{A}_i$ denote the event that at time $i$, $\sum_{j=1}^{i} X_j \le \sum_{j=1}^{i} \pi_j - \delta(1+\theta)(1+k)$. We show that for any $i \ge n_0$, $\mathbf{Pr}[\bar{A}_i] \le \epsilon$. Using Hoeffding's inequality we have,

$$\begin{aligned}
\mathbf{Pr}\left[\bar{A}_i\right] &= \mathbf{Pr}\left[\sum_{j=1}^{i} X_i \le \sum_{j=1}^{i} \pi_j - \delta(1+\theta)(1+k)\right] \\
&= \mathbf{Pr}\left[(E_i[X] - \frac{1}{i}\sum_{j=1}^{i} X_i) \ge \frac{\delta(1+\theta)(1+k)}{i}\right] \\
&\le e^{-2i(\frac{\delta(1+\theta)(1+k)}{i})^2}.
\end{aligned}$$

Using the bound on $\mathbf{Pr}[\bar{A}_i]$ we next bound the the probability of the event $\bigcap_{i=n_0}^{n} A_i$. In particular, we have

$$\begin{aligned}
\mathbf{Pr}\left[\bigcap_{i=n_0}^{n} A_i\right] = 1 - \mathbf{Pr}\left[\bigcup_{i=n_0}^{n} \bar{A}_i\right] &\ge 1 - \sum_{i=n_0}^{n} e^{-2i(\frac{\delta(1+\theta)(1+k)}{i})^2} \\
&\ge 1 - \sum_{i=n_0}^{n} e^{-2n(\delta(1+\theta)\bar{\pi})^2} \ge 1 - (n-n_0)e^{-2n(\delta(1+\theta)\bar{\pi})^2},
\end{aligned}$$

where the second inequality follows from the definition of $\bar{\pi}$. Therefore for $\delta \ge \delta_0(\epsilon, n_0) = \left(\frac{\ln(1/\epsilon)\ln(n-n_0)}{n\bar{\pi}^2(1+\theta)^2}\right)^{1/2}$, the anticipatory random walk succeeds with probability at least $1-\epsilon$.[1] ■

**Proof of Proposition 3.** This is immediate from the first condition in Theorem 1 and the fact that $\pi_i > 0$ for all $i$, as

$$\lim_{n\to\infty} \sum_{i=1}^{n} \pi_i = \infty > k(\ln(1/(1-\beta)) + 1$$

and therefore $b_k^n \ge \beta$. ■

**Proof of Proposition 4.** We construct the transitions in Figure 2 on top of the random walk, where the state $j$ denotes $0, 1, ..., k-1$, along with the two absorbing states $F$ and $S$ to denote the process failing or succeeding, respectively. Let $s_j = \mathbf{Pr}[Y_n = k \mid Y_i = j]$, i.e. the (non-anticipatory) probability of reaching state $S$ from state $i$, then from the transition probabilities in Figure 2, we get

$$s_j = \left(\frac{r\pi}{1-(1-\pi)r}\right)^{k-j+1}$$

[1] Note that $\delta_0(\epsilon) \to 0$ as $n$ grows larger.

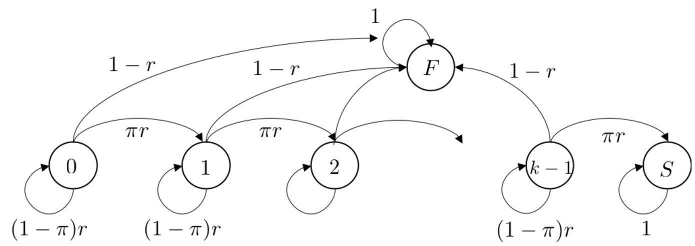


Figure 2: Transition probabilities for stochastic end times. State **S** denotes success while state **F** denotes Failure.

i.e. $s_j > s_{j-1}$. In particular, $s_1 = \left(\frac{r\pi}{1-(1-\pi)r}\right)^k = \beta^* \geq \beta \implies s_j \geq \beta$ for all $j > 1$. Thus if $s_1 < \beta$, $Y_n = 0$ for all $n$. In particular, $Y_{\bar{n}} = 0$ and the walk fails, as in the second statement of the proposition. Similarly, If $s_1 \geq \beta$, then the process succeeds with $\mathbf{Pr}[Y_n = k \mid Y_0 = 0] = s_0 = \left(\frac{r\pi}{1-(1-\pi)r}\right)^{k+1}$, proving the first statement of the proposition.

■

**Proof of Proposition 5.** First note that when the condition $g_{i-1} > (N-(i-1))v^H$ holds then there are not enough consumers to raise the required amount. The price in this case does not matter because no one pledges.

The recursion for the success probability $\alpha_i(g_i)$ is given by:

$$\alpha_i(g_i) = \begin{cases} 1 & g_i \leq 0 \\ 0 & g_i > (N-i)v^H \\ \pi\alpha_{i+1}(g - p^*_{i+1}) + (1-\pi)\alpha_{i+1}(g_i) & 0 < g_i \leq (N-i)v^H \end{cases}$$

Recall that if consumer $i$ estimates success probability $\alpha_i(g_i)$ then she pledges only if $\beta_i(p^*) \leq \alpha_i(g_i)$. Thus the maximum price $p^*_i$ that she can be charged satisfies

$$\begin{aligned} \beta_i(p^*_i) &= \alpha_i(g_i) \\ \frac{c}{v^H - p^*_i + c} &= \alpha_i(g_{i-1} - p^*_i) \\ &= \pi\alpha_{i+1}(g_{i-1} - p^*_i - p^*_{i+1}) + (1-\pi)\alpha_{i+1}(g_{i-1} - p^*_i) \end{aligned}$$

from which,

$$p_i^* = v^H + c\left(1 - \frac{1}{\pi\alpha_{i+1}(g_{i-1} - p_i^* - p_{i+1}^*) + (1-\pi)\alpha_{i+1}(g_{i-1} - p_i^*)}\right).$$

■

# B Additional Material

## B.1 Heterogeneous Costs

We provide the details of how our analysis changes when consumers have different heterogeneous costs. The first three lemmas immediately hold. However, Lemma 4 becomes:

**Lemma 5.** *The increase in the total amount of fluid in all nodes during the times that node $j \in \mathbb{N}$ is critical is at least*

$$B^{\tau_{j+1}-1} - B^{\tau_j - 1} \geq \beta_j + (1-\beta_j)\left(\sum_{t=\tau_j}^{\tau_{j+1}} q_t - \ln(1/(1-\beta_j))\right)^+$$

The analogous result for this case shows that we again have a concentration result with slightly different conditions that mirror the heterogeneity in $\beta_i$:

**Theorem 3** (Theorem 2 Revisited)**.** *The flow process with $\beta_1, ..., \beta_n$ and $\beta_{max} = \max\{\beta_1, ..., \beta_n\}$ satisfies the following two properties for any $k \in \mathbb{N}$:*

- *if $\sum q_t \geq \frac{\sum_{i=1}^{k}(1-\beta_i)}{1-\beta_{max}} + \sum_{j=1}^{k} \ln(1/(1-\beta_j))$, then $b_k^n \geq \beta_k$*
- *if $\sum_t \ln(\frac{1}{1-q_t}) < \sum_{i=1}^{k} \ln(1/(1-\beta_i))$, then $b_k^n < \beta_k$.*

The proofs of Lemma 5 and Theorem 3 resemble the proofs of the benchmark case with some slight modifications. They are presented below. We omit the $\Delta$ term found in the proof of Lemma 4 for simplicity (the presence of this term ensures integrality but is not crucial for the correctness of the argument).

**Proof of Lemma 5** Suppose $\sum_{t=\tau_j}^{\tau_{j+1}} q_t > A$ and let $t^*$ be such that $\sum_{t=\tau_j}^{t^*} q_t = A$, then

$$
\begin{aligned}
B^{\tau_{j+1}-1} - B^{\tau_j-1} &= \sum_{t=\tau_j}^{\tau_{j+1}-1} B^t - B^{t-1} \\
&= \sum_{t=\tau_j}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&= \sum_{t=\tau_j}^{t^*} q_t(1-b_j^{t-1}) + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&\geq \sum_{t=\tau_j}^{t^*} q_t \prod_{t'=\tau_j}^{t-1} (1-q_{t'}) + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&= \sum_{t=\tau_j}^{t^*} (1-(1-q_t)) \prod_{t'=\tau_j}^{t-1} (1-q_{t'}) + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&= \sum_{t=\tau_j}^{t^*} \left( \prod_{t'=\tau_j}^{t-1} (1-q_{t'}) - \prod_{t'=\tau_j}^{t-1} (1-q_{t'}) \right) + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&= 1 - \prod_{t'=\tau_j}^{t^*-1} (1-q_{t'}) + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&\geq 1 - e^{-\sum_{t'=\tau_j}^{t^*-1} q_{t'}} + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t(1-b_j^{t-1}) \\
&= \beta_j + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t \cdot (1-b_j^{t-1}) \\
&\geq \beta_j + \sum_{t=t^*+1}^{\tau_{j+1}-1} q_t \cdot (1-\beta_j) \\
&= \beta_j + (1-\beta_j) \left( \sum_{t=\tau_j}^{\tau_{j+1}-1} (q_t - A) \right) \\
&= \beta_j + (1-\beta_j) \left( \sum_{t=\tau_j}^{\tau_{j+1}-1} (q_t - \ln(1/1-\beta_j)) \right)
\end{aligned}
$$

■

**Proof of Theorem 3** Denote by $A = \frac{\sum_{i=1}^{k}(1-\beta_i)}{1-\beta_{max}} + \sum_{j=1}^{k} \ln(1/(1-\beta_j))$

$$
B^n = \sum_{j=1}^{j^*-1} B^{\tau_{j+1}-1} - B^{\tau_j-1} + B^n - B^{\tau_{j^*}-1}
$$

$$\geq \sum_{j=1}^{j^*-1} \beta_j + (1-\beta_j)\left(\sum_{t=\tau_j}^{\tau_{j+1}-1} q_t - \ln(1/(1-\beta_j))\right)^+ + B^n - B^{\tau_{j^*}-1}$$
$$\geq \sum_{j=1}^{j^*-1} \beta_j + (1-\beta_j)\left(\sum_{t=\tau_j}^{\tau_{j+1}-1} q_t - \ln(1/(1-\beta_j))\right)^+$$

Noting that $\sum_{t=\tau_j}^{\tau_{j+1}-1} q_t \geq A$, the last expression is minimized when it is equal to

$$= \sum_{j=1}^{k} \beta_j + (1-\beta_{max})(A - \sum_{j=1}^{k} \ln(1/(1-\beta_j)))$$
$$\geq \left(\sum_{j=1}^{k} \beta_j + (1-\beta_{max})(\frac{\sum_{i=1}^{k}(1-\beta_i)}{1-\beta_{max}} + \sum_{j=1}^{k} \ln(1/(1-\beta_j)) - \sum_{j=1}^{k} \ln(1/(1-\beta_j))\right)$$
$$= k$$

which proves the first part of the theorem as the index of the critical bucket is at least $k+1$ and hence $b_n^k \geq \beta_k$.

Similarly, suppose that the conditional statement in the second bullet of the theorem holds but that $b_k^n \geq \beta_k$, then combining with Lemma 5, we have

$$\prod_{i=1}^{k}(1-\beta_i) \geq (1-b_k^n)\prod_{i=1}^{k-1}(1-b_i^{\tau_{i+1}-1})$$
$$\geq \left(1-(1-\prod_{t=\tau_k}^{n}(1-q_t))\right) \times \prod_{i=1}^{k-1}\left(1-(1-\prod_{t=\tau_i}^{\tau_{i+1}-1}(1-q_t))\right)$$
$$= \prod_{t=1}^{n}(1-q_t) = e^{\sum_t \ln(1-q_t)}$$
$$> e^{-\sum_{i=}^{k} \ln(\frac{1}{1-\beta_i})} = \prod_{i=1}^{k}(1-\beta_i)$$

which is a contradiction, establishing that $b_k^n < \beta_k$. ■

## B.2 General Types

Suppose that each consumer has a two dimensional type $(c_i, v_i)$ where the costs $c_i$ and the valuations $v_i$ come from separate iid distributions over $(0, \infty)$. Fix a price $p$ and recall that a consumer pledges when she estimates a probability of success $\alpha_i > \frac{c_i}{v_i - p + c_i} = \beta_i$. Thus, under price $p$ consumer $i$ has an equivalent one-dimensional type $\beta_i(p)$. We will remove the dependence on $p$ in what follows but it should be implicit from that point on that everything is parameterized by $p$. The price $p$ together with the distributions on $c$ and $v$ induce a distribution $G(\beta)$ over $(-\infty, \infty)$. In particular, $v_i > p \iff \beta_i \in (0,1)$. Let $\pi = G(1) - G(0)$ and denote by $F(b) = Pr(\beta < b | 0 \leq \beta \leq 1)$, i.e $F(\beta)$ is the (conditional) distribution over those $\beta$ values that fall in the unit interval and represent thresholds of consumers who have an actual chance of pledging. This means that if consumer $i$ estimates success probability $\alpha_i$, then she pledges with probability $\pi F(\alpha_i)$.

We then write the success probability recursion in terms of the above parameters:

$$s_i^j = \begin{cases} \pi F(s_{i+1}^{j+1}) \cdot s_{i+1}^{j+1} + (1 - \pi F(s_{i+1}^{j+1})) \cdot s_{i+1}^{j} & i < N \\ 1 & i = N, j \geq k \\ 0 & i = N, j < k \end{cases}$$

The corresponding anticipatory random walk is given as follows, with the difference between this walk and the one we analyzed being the heterogeneous thresholds $\beta_i$:

$$Y_i | Y_{i-1}, X_i = \begin{cases} Y_{i-1} + X_i & \text{if } \mathbf{Pr}[Y_n \geq k | Y_i = Y_{i-1} + X_i] \geq \beta_i \\ Y_{i-1} & \text{if } \mathbf{Pr}[Y_n \geq k | Y_i = Y_{i-1} + X_i] < \beta_i \end{cases}$$

$$Y_0 = 0$$

Finally, the flow process can be written as:

$$b_j^t = \begin{cases} \pi F(b_{j-1}^{t-1}) b_{j-1}^{t-1} + (1 - \pi F(b_{j-1}^{t-1})) b_j^{t-1} & t \geq j, j \geq 1 \\ 0 & t < j \\ 1 & j = 0 \end{cases} \tag{12}$$

We can write $q_t^j = \pi F(b_{j-1}^{t-1})$ in the above recursion to facilitate comparison with the original recursion in Equation (4). In the latter, the probabilities $q_t$ are exogenous and independent of the indices of the nodes. By contrast, the probabilities $q_t^j$ in Equation (12) are endogenous to the process and index-dependent. This makes it significantly harder to analyze. In particular, the model resembles a Markov jump system where the probabilities in the jump matrix $P_{ij}$ at time $t$ depend on the future jump probabilities. This makes it difficult to prove an analog of Lemma 4, which is the key lemma that helps us bound the increase in flow in every time step in order to prove the main theorems in the paper. Nevertheless, based on the simulations in Figure 1, we conjecture that the properties we prove in the paper continue to hold in this case.

# References


Acemoglu, Daron, Munther A Dahleh, Ilan Lobel, and Asuman Ozdaglar (2011), "Bayesian learning in social networks." *The Review of Economic Studies*, 78, 1201–1236.

Acemoglu, Daron and Asuman Ozdaglar (2011), "Opinion dynamics and learning in social networks." *Dynamic Games and Applications*, 1, 3–49.

Ali, Amir Hatem (2011), "The power of social media in developing nations: New tools for closing the global digital divide and beyond." *Harv. Hum. Rts. J.*, 24, 185.

Anand, Krishnan S and Ravi Aron (2003), "Group buying on the web: A comparison of price-discovery mechanisms." *Management Science*, 49, 1546–1562.

Andreoni, James (2006), "Leadership giving in charitable fund-raising." *Journal of Public Economic Theory*, 8, 1–22.

Angeletos, George-Marios and Iván Werning (2006), "Crises and prices: Information aggregation, multiplicity, and volatility." *american economic review*, 96, 1720–1736.

Araman, Victor F and Rene Caldentey (2016), "Crowdvoting the timing of new product introduction." *Available at SSRN 2723515*.

Babich, Volodymyr, Simone Marinesi, and Gerry Tsoukalas (2020), "Does crowdfunding benefit entrepreneurs and venture capital investors?" *Manufacturing & Service Operations Management*.

Bagnoli, Mark and Barton L Lipman (1989), "Provision of public goods: Fully implementing the core through private contributions." *The Review of Economic Studies*, 56, 583–601.

Belavina, Elena, Simone Marinesi, and Gerry Tsoukalas (2020), "Rethinking crowdfunding platform design: mechanisms to deter misconduct and improve efficiency." *Management Science*.

Belleflamme, Paul, Thomas Lambert, and Armin Schwienbacher (2014), "Crowdfunding: Tapping the right crowd." *Journal of business venturing*, 29, 585–609.

Besbes, Omar and Ilan Lobel (2015), "Intertemporal price discrimination: Structure and computation of optimal policies." *Management Science*, 61, 92–110.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch (1992), "A theory of fads, fashion, custom, and cultural change as informational cascades." *Journal of Political Economy*, 992–1026.

Carlsson, Hans and Eric Van Damme (1993), "Global games and equilibrium selection." *Econometrica: Journal of the Econometric Society*, 989–1018.

Chakraborty, Soudipta and Robert Swinney (2020), "Signaling to the crowd: Private quality information and rewards-based crowdfunding." *Manufacturing & Service Operations Management*.

Dahleh, Munther A, Alireza Tahbaz-Salehi, John N Tsitsiklis, and Spyros I Zoumpoulis (2016), "Coordination with local information." *Operations Research*, 64, 622–637.

Diamond, Douglas W and Philip H Dybvig (1983), "Bank runs, deposit insurance, and liquidity." *Journal of Political Economy*, 91, 401–419.

Gan, Jingxing Rowena, Gerry Tsoukalas, and Serguei Netessine (2019), "Inventory, speculators and initial coin offerings." *The Wharton School Research Paper*.

Grimmett, Geoffrey and David Stirzaker (2001), *Probability and random processes*. Oxford University Press.

Hu, Ming, Xi Li, and Mengze Shi (2015), "Product and pricing decisions in crowdfunding." *Marketing Science*, 34, 331–345.

Hu, Ming, Mengze Shi, and Jiahua Wu (2013), "Simultaneous vs. sequential group-buying mechanisms." *Management Science*, 59, 2805–2822.

Jing, Xiaoqing and Jinhong Xie (2011), "Group buying: A new mechanism for selling through social interactions." *Management Science*, 57, 1354–1372.

Kuppuswamy, Venkat and Barry L Bayus (2013), "Crowdfunding creative ideas: the dynamics of projects backers in kickstarter." *SSRN Electronic Journal*.

List, John A and David Lucking-Reiley (2002), "The effects of seed money and refunds on charitable giving: Experimental evidence from a university capital campaign." *Journal of Political Economy*, 110, 215–233.

Lobel, Ilan (2020), "Dynamic pricing with heterogeneous patience levels." *Operations Research*.

Lobel, Ilan and Evan Sadler (2015), "Preferences, homophily, and social learning." *Operations Research*, 64, 564–584.

Lobel, Ilan, Evan Sadler, and Lav R Varshney (2017), "Customer referral incentives and social media." *Management Science*, 63, 3514–3529.

Marinesi, Simone and Karan Girotra (2013), "Information acquisition through customer voting systems."

Mollick, Ethan (2014), "The dynamics of crowdfunding: An exploratory study." *Journal of Business Venturing*, 29, 1–16.

Mutz, Diana C (1995), "Effects of horse-race coverage on campaign coffers: Strategic contributing in presidential primaries." *The Journal of Politics*, 57, 1015–1042.

Strausz, Roland (2017), "A theory of crowdfunding: A mechanism design approach with demand uncertainty and moral hazard." *American Economic Review*, 107, 1430–76.

Varian, Hal R (1994), "Sequential contributions to public goods." *Journal of Public Economics*, 53, 165–186.

Vesterlund, Lise (2003), "The informational value of sequential fundraising." *Journal of Public Economics*, 87, 627–657.

Wu, Jiahua, Mengze Shi, and Ming Hu (2014), "Threshold effects in online group buying." *Management Science*, 61, 2025–2040.