A Bayesian Approach to Combining Information from a Census, a Coverage Measurement Survey and Demographic Analysis

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Abstract

Demographic analysis of data on births, deaths, and migration and coverage measurement surveys that use capture-recapture methods have both been used to assess US Census counts. These approaches have established that unadjusted Census counts are seriously flawed for groups such as young and middle-aged African-American men. There is considerable interest in methods that combine information from the Census, coverage measurement surveys and demographic information to improve Census estimates of the population. This article describes a number of models that have been proposed to accomplish this synthesis when the demographic information is in the form of sex ratios stratified by age and race. A key difficulty is that methods for combining information require modeling assumptions that are difficult to assess based on fit to the data. We propose some general principles for aiding the choice among alternative models. We then pick a particular model based on these principles, and imbed it within a more comprehensive Bayesian model for counts in poststrata of the population. Our Bayesian approach provides a principled solution to the existence of negative estimated counts in some subpopulations; provides for smoothing of estimates across poststrata, reducing the problem of isolated outlying adjustments; allows a test of whether negative cell counts are due to sampling variability or more egregious problems such as bias in Census or coverage measurement survey counts; and can be easily extended to provide estimates of precision that incorporate uncertainty in the estimates from demographic analysis and other sources. The model is applied to data for African-Americans aged 30-49 from the 1990 Census, and results compared with those from existing methods.

Key Words: Post-enumeration survey; model selection; Gibbs sampling; posterior predictive distribution.

1 Introduction

Capture-recapture methods (Seber 1982) and demographic analysis (DA) of data on births, deaths, and migration have been used to estimate the undercount in the US Census (Robinson et al. 1993).

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These approaches have established that unadjusted Census counts are seriously flawed for groups such as young and middle-aged African-American men. Demographic analysis indicates a 1990 male-female ratio among African-Americans 30-49 of 0.91, whereas the published 1990 Census counts indicate a ratio of 0.86; with imputations and erroneous enumerations removed, the 1990 Census identified only 0.78 males for every female in this age-race category (Bell et al. 1996). Considerable effort has been devoted during the past decade to develop methods that combine demographic information and capture-recapture analysis to improve Census estimates of the population (Isaki and Schultz 1986; Fay, Passel and Robinson 1988; Choi, Steel, and Skinner 1988; Das Gupta and Robinson 1990; Wolter 1990; Bell 1993; Bell et al. 1996). Two key difficulties arise: DA typically provides estimates only for national levels of aggregation, and capture-recapture methods require modeling assumptions that are difficult to assess based on fit to the data. In the remainder of this section we overview existing research on these problems. Section 2 suggests principles for choosing among capture-recapture models, and Section 3 imbeds these methods within a unified Bayesian model. Section 4 applies the results of Sections 2 and 3 to the US Census data for African-Americans 30-49.

The goal of the methods described here is to yield adjusted population counts in strata of the population, together with measures of uncertainty. Since the issue of combining information from macro data sources such as administrative records and from micro data sources such as surveys arises in many disparate fields (Raftery, Givens and Zeh 1995), the methodology described here has potentially broader application than the US Census.

1.1 Coverage Measurement Surveys

Since 1970, the Census has been supplemented by a coverage measurement survey (CMS), a detailed independent enumeration of households in a probability sample of Census blocks conducted immediately after the actual Census. In 1990, this CMS, termed the Post Enumeration Survey or PES, was conducted in July-September 1990, following the April-June 1990 data collection for the Census. To combine the Census and CMS data, individuals in the Census who were imputed or for whom insufficient information existed to match with the CMS ("imputations") were removed from the Census total. In addition, persons in the Census listing who did not appear in the CMS were re-checked for "erroneously enumeration". Examples of erroneously enumerations included persons whose primary residence was a dormitory, persons who were enumerated at a vacation home that was not their primary residence, or any other persons not resident at the enumerated dwelling unit as of April 1st. The Census count in the sampled blocks minus the imputations and an estimate of erroneous enumerations became the adjusted E-sample. Similarly, persons in the CMS – the P-sample – were cross-checked against Census records within the sample Census blocks and a set of surrounding blocks and assigned to either an In-Census or Out-Census category. After inflating the counts in the sampled blocks by the inverse of their probability of selection, a 2×2 table (In-Out Census; In-Out CMS) was formed for males and females ($S = M, F$) in each of K post-strata (typically defined by geographic area and owner vs. renter status) within age-by-race groupings (see Table 1). For more information about the details of the P- and E-samples in the 1990 PES, see Hogan (1992).

Table 1 also gives the "true" but unknown population counts in the kth poststratum for gender S that would be obtained if the CMS had itself been a census and if those missed in both the Census and CMS were known. These population quantities $\psi_k = \{\psi_{kij}^S : i = 0, 1; j = 0, 1; S = M, F\}$, where i is the Census enumeration status and j the CMS enumeration status in the kth poststratum (1 if included, 0 otherwise), are considered unknown parameters, and are estimated by the statistical methods to be described.

Table 1: Observed data and associated underlying population parameters from Census and Coverage Measurement Survey: y^S_{k11} and y^S_{k01} are estimated counts of individuals in and out of the Census on the basis of the CMS follow-up; $z^S_{k1.}$ is the Census count, minus imputations and an estimate of erroneous enumerations; ψ^S_{kij} is the population that would reside in the ijth cell if the CMS had been a complete census. (stratum=k; gender=S).

		Observed Data			Underlying Parameters		
		CMS			CMS		
		In	Out		In	Out	
Census	In	y^S_{k11}		$z^S_{k1.}$	ψ^S_{k11}	ψ^S_{k10}	$\psi^S_{k1.}$
	Out	y^S_{k01}			ψ^S_{k01}	ψ^S_{k00}	$\psi^S_{k0.}$
		$y^S_{k.1}$			$\psi^S_{k.1}$	$\psi^S_{k.0}$	$\psi^S_{k..}$

Table 2 displays the counts $\{y^S_{k11}, y^S_{k01}, z^S_{k1.}\}$ and associated estimated sampling errors from the 1990 Census for African-Americans aged 30-49, stratified into 12 poststrata. Poststrata 1 through 6 include those residing in owner-occupied dwelling units in urban area 250,000/+ in the Northeast (1), South (2), Midwest (3), and West (4); owners in urban areas under 250,000 (5); and owners in non-urban areas (6). Poststrata 7 through 12 include those residing in non-owner (rental) dwelling units in the corresponding geographic areas. The analysis is conducted only on this one age-race "super-stratum" because of time and length constraints. A more complete analysis will of course apply the methods to the entire United States population.

1.2 Model constraints

A fundamental problem (Bell 1993) is that Table 1 provides only 3 data elements to estimate the 4 parameters. Thus constraints must be placed on the parameters to obtain unique parameter estimates. One such constraint is to assume independence of capture and recapture (ICR), that is that the odds ratios for enumeration in the Census and the CMS for sex S in poststratum k, $\theta^S_k = \frac{\psi^S_{k11}/\psi^S_{k10}}{\psi^S_{k01}/\psi^S_{k00}}$, are all equal to 1 (Sekar and Deming 1949).

$$\text{ICR: } \theta^S_k = 1 \text{ for all } k, S. \tag{1}$$

As Sekar and Deming point out, the ICR assumption can be violated when either the probabilities of capture and recapture are unequal or when they differ across individuals (unobserved heterogeneity), leading to "correlation bias". Correlation bias tends to lead to an underestimate the undercount, since if it is due to unobserved heterogeneity then $\theta_k > 1$, leading to an underestimation of ψ^S_{k00}. This form of bias is indicated by a pattern of implausibly low values of the male-female sex ratio, a value considered highly reliable by demographers (Robinson 1995), when the 1990 Census estimates are adjusted using the ICR model. For example, under the ICR assumption, the estimated male-female ratio among African-Americans aged 30-49 is 0.84, substantially below the 0.91 DA estimate.

To correct for this bias, several models have been suggested that attribute the low observed male-female ratio to an undercount of males rather than an overcount of females. The estimated number of males in the population is increased to match the overall male-female ratio ($\rho = \psi^M_{...}/\psi^F_{...}$) from DA. The additional males are distributed over the post-strata using a method based on the assumptions of the model. In practice ρ is estimated from DA within an age-race group, so the models are applied separately to each age-race grouping. In particular, Wolter (1990) assumed that the odds of appearing in the follow-up survey given that one was enumerated in the Census relative to the odds of appearing in the follow-up survey given that one was not enumerated in the Census

Table 2: Observed In-CMS-In Census cell (y^S_{k11}), In CMS-Out Census cell (y^S_{k01}), and Census count minus imputations and erroneous enumerations ($z^S_{k1.}$), together with estimated standard errors in parentheses. African-Americans aged 30-49, 1990 US Census. S=Sex PS=Poststratum

Males	y^M_{k11}	y^M_{k01}	$z^M_{k1.}$
PS 1	141567(16461)	17672(3100)	205692(4262)
PS 2	379139(37049)	40835(6257)	460849(4856)
PS 3	210725(25075)	23433(3579)	260288(2794)
PS 4	178356(38354)	27737(7639)	124959(3907)
PS 5	314080(49222)	33038(6777)	315599(5602)
PS 6	246492(51920)	38469(12365)	278798(5100)
PS 7	216082(31396)	86040(15883)	300904(8555)
PS 8	279818(36308)	84300(12284)	429687(7476)
PS 9	136885(16663)	48690(6988)	215578(5497)
PS 10	111513(22167)	44969(12379)	169313(6255)
PS 11	313894(56391)	67312(9911)	297656(5216)
PS 12	52785(11822)	11778(3017)	80882(4856)

Females	y^F_{k11}	y^F_{k01}	$z^F_{k1.}$
PS 1	173719(20288)	24451(4556)	248862(4086)
PS 2	502134(53051)	35945(5698)	547036(4450)
PS 3	271782(32200)	22838(3855)	308192(3278)
PS 4	223704(57827)	42301(17562)	134557(3032)
PS 5	371117(58356)	25382(7092)	361009(3664)
PS 6	274158(52666)	39471(12800)	318979(6390)
PS 7	328988(54152)	95224(16530)	450241(7635)
PS 8	395789(46484)	59477(8247)	575657(13836)
PS 9	333752(47017)	51880(7168)	345949(4110)
PS 10	177245(36334)	23922(5913)	203188(5301)
PS 11	482488(87229)	50058(9590)	415852(4846)
PS 12	77267(21163)	8457(2772)	105417(3103)

are arbitrary for males and equal to 1 for females in a one-stratum design. We call this the fixed odds-ratio (FOR) model (here and elsewhere labels for the models are ours):

$$\text{FOR: } \theta^M = \theta \text{ and } \theta^F = 1. \tag{2}$$

Bell (1993) extended Wolter's approach by considering multiple post-strata and alternative behavior models. The fixed relative-risk (FRR) model assumes a constant relative risk for enumeration in the Census and CMS for males and independence for females:

$$\text{FRR: } \gamma_k^M = \frac{\psi_{k11}^M/\psi_{k1.}^M}{\psi_{k01}^M/\psi_{k0.}^M} = \gamma^M \text{ and } \theta_k^F = 1 \text{ for all } k. \tag{3}$$

The fixed sex-ratio (FSR) model assumes that the sex ratio in the out CMS - out Census cell is constant across strata:

$$\text{FSR: } \omega_k = \frac{\psi_{k00}^M}{\psi_{k00}^F} = \omega \text{ and } \theta_k^F = 1 \text{ for all } k. \tag{4}$$

The generalized behavioral response model (GBR) assumes the probability of being in the coverage measurement survey given that one was not enumerated in the Census divided by the probability of being in the Census is constant across the poststrata for males:

$$\text{GBR: } \lambda_k^M = \frac{\psi_{k01}^M/\psi_{k0.}^M}{\psi_{k1.}^M/\psi_{k..}^M} = \lambda^M \text{ and } \theta_k^F = 1 \text{ for all } k. \tag{5}$$

Das Gupta (Bell et al. 1996) proposed yet another model, which we term the fixed dual-coverage rate (FDCR) model. It assumes that the ratio of the Census-CMS combined coverage rate for males to the Census-CMS coverage rate or females is constant over all post-strata:

$$\text{FDCR: } \chi_k = \frac{1 - \psi_{k00}^M/\psi_{k..}^M}{1 - \psi_{k00}^F/\psi_{k..}^F} = \chi \text{ and } \theta_k^F = 1 \text{ for all } k. \tag{6}$$

All models adjust the fitted counts so that their sum across the poststrata matches the sex ratio ρ from DA. In practice ρ is estimated from DA within an age-race group, so the models are applied separately to each age-race grouping.

The models (2) through (6) add a single parameter (θ^M, γ^M, ω, λ^M, or χ) to the ICR model; all are "saturated" and provide an equally good fit to the data. Thus it is difficult to choose among alternative models, although they can yield adjustments with non-trivial differences. This is viewed as a major obstacle to combining the data from the CMS and DA (Bell 1993). Another problem is the existence of *negative* estimates of persons included in the Census but missed in the CMS, obtained by subtracting those estimated to have been in both the CMS and Census from the Census total ($\widehat{\psi}_{k10}^S = z_{k1.}^S - y_{k11}^S$). Bell (1993) determines the maximum likelihood estimates of $\psi_{k..}^S$ under the assumptions (1)-(5), with the constraint that $\widehat{\psi}_{k10}^S = max(0, z_{k1.}^S - y_{k11}^S)$, and that $\widehat{\psi}_{k11}^S = y_{k11}^S$ and $\widehat{\psi}_{k10}^S = y_{k10}^S$ if $\widehat{\psi}_{k10}^S > 0$ and $\widehat{\psi}_{k11}^S = z_{k1.}^S$ and $\widehat{\psi}_{k10}^S = y_{k10}^S z_{k1.}^S / y_{k11}^S$ if $\widehat{\psi}_{k10}^S = 0$. That is, strata with negative estimates of the In CMS-Out Census cell have their negative estimates set to zero and their In CMS column cells multiplied by $\frac{z_{k1.}^S}{y_{k11}^S}$ to maintain the marginal Census counts and, as it turns out, the ICR-based poststratum estimates. Formal justification for this approach is lacking. In addition, it is unclear how to account for uncertainty in the DA sex ratios. This article builds on previous research in the following respects:

1. Section 2 proposes principles by which one might choose among the saturated models (2)-(6), or others.

2. Section 3 provides a unified Bayesian foundation for model fitting that

   (a) addresses the negative cell problem in a principled manner;

   (b) provides for smoothing of estimates across poststrata, reducing the problem of isolated outlying adjustments;

   (c) allows a test of whether negative cell counts are due to sampling variability or more egregious problems such as bias in erroneous enumerations or imputations in Census counts or in assigning CMS subjects to In/Out Census categories;

   (d) includes a parameter to control variability of sex ratios across poststrata; and

   (e) can be extended to provide estimates of precision that incorporate uncertainty in the DA sex ratio estimates.

2 Principles for Choosing Between Models

Five alternative models (2) through (6) for combining CMS and DA data on sex ratios are proposed by Bell (1993) and Das Gupta (Bell et al. 1996), and many others might be envisaged. To simplify and reduce the scope of the model selection problem, we propose 6 principles for guiding the selection of a model for combining CMS and DA information:

1. PLAUSIBILITY: The model should imply a plausible description of Census behavior.

- Assessment of this issue requires expert opinion and careful exposition of the model assumptions.

2. FIT: The model should minimize contradiction with available data.

- The "no adjustment" model (that is, doing nothing) clearly fails this test, since it ignores the sizable body of evidence of differential undercount across demographic groups. A number of alternative models (including those considered by Bell and Das Gupta) provide better fits and hence should be preferred under this principle.

3. PREDICTION: The model should provide plausible predictions of key unobserved quantities, e.g. undercount rates should be within limits deemed reasonable.

- Models may yield implausible outlying predictors for certain cells. While a consistent pattern of unlikely predictions is evidence that the model is not appropriate, in isolated cases modifications that control the extent of adjustments might be considered. These may be achieved informally by ad-hoc adjustments, or more formally by a Bayesian analysis based on prior distributions that limit the size of the adjustments. Section 3 includes an example of the Bayesian approach. Since the Bayesian approach is no panacea – if non-informative prior distributions give unreasonable model predictions, constraining them via informative priors is only hiding the problem – we consider model fit via posterior predictive distributions in Section 4.2.

4. ICR INCLUSION: The model should include the ICR model, which assumes zero correlation bias within poststrata, as a possibility.

- It is harder to defend this principle as necessary on scientific grounds, but given the widespread adoption of the ICR model for CMS problems, it seems reasonable to restrict attention to the class of models that include that model for a particular choice of parameters. It is also in keeping with statistical parsimony: without evidence of correlation bias we would accept the independence model.

5. STABILITY: If alternative competing models are not distinguished on the basis of 1-4, models that yield more stable estimates of key estimands should be favored over models that yield less stable estimates.

- If little can be concluded about the relative biases of competing models, then a model that yields estimates with fewer potential outliers is to be preferred.

6. CONSERVATISM: If alternative competing models are not distinguished on the basis of 1-5, then models that are more conservative with respect to undercount adjustment should be favored over models that are less conservative.

- The principle of "one adjusts to the minimum extent necessary to be consistent with the data" is pragmatic rather than scientific: given the sentiment against any type of adjustment in some quarters, the goal of adjusting the Census counts to the minimal extent needed for consistency with DA and CMS data seems appropriate.

Of the six models discussed, only two – the FOR and FRR models – are both saturated and satisfy the ICR inclusion rule. Both models have simple interpretations and appear to satisfy both the plausibility and prediction principle. Comparing the maximum likelihood estimates of the undercount rates under the various models, Bell (1993) determines that the FRR model yields somewhat more stable and conservative results than the FOR model. Hence we highlight this model in the remainder of our paper.

3 A Fixed Relative-Risk Model Incorporating CMS and DA Data

This section outlines a comprehensive model for the underlying $8K$ population counts in the CMS tables that incorporates information about sex ratios from DA and eases prior specifications. For the FRR model (3), we reparameterize the eight population counts in poststratum k, $\psi_k = \{\psi_{kij}^S : i = 0, 1; j = 0, 1; S = M, F\}$, as $\psi_k^* = (\psi_{k..}, \rho_k, \delta_k^M, \delta_k^F, \phi_k^M, \phi_k^F, \gamma_k^M, \gamma_k^F)$ where:

1. $\psi_{k..}$ is the total population count in poststratum k.
2. $\rho_k = (\psi_{k..}^M)/(\psi_{k..}^F)$, the sex ratio ($\psi_{k..}^M + \psi_{k..}^F = \psi_{k..}$).
3. $\delta_k^S = (\psi_{k1.}^S)/(\psi_{k..}^S)$, the Census undercount proportion for sex S.
4. $\phi_k^S = (\psi_{k11}^S)/(\psi_{k1.}^S)$, the proportion of Census cases enumerated in the CMS for sex S.
5. $\gamma_k^S = \frac{\psi_{k11}^S/\psi_{k1.}^S}{\psi_{k01}^S/\psi_{k0.}^S}$, the relative proportion of Census and non-Census cases enumerated in the CMS for sex S.

The above parameterization is particularly useful for the FRR model; other choices of parameterizations are more natural for other models. For example, under the FOR assumption (2) we would replace the γ_k^S with $\theta_k^S = \frac{\psi_{k11}^S/\psi_{k10}^S}{\psi_{k01}^S/\psi_{k00}^S}$, while under the ICR assumption (1) we would force $\gamma_k^S = \theta_k^S = 1$ for all S and k. Priors can be chosen that avoid negative cell count estimates, borrow strength across post-strata to reduce outlying predictions, and moderate extreme undercount adjustments: for example, a proper prior for γ_k^S can prevent the risk ratio of enumeration from reaching unreasonable extremes. In our analysis of the FRR model we select the following independent priors for each parameter:

- $p(\psi_{k..}) \propto 1$, a flat prior corresponding to our lack of knowledge about the total population counts in each poststratum.
- $\rho_k \sim N(\rho, \sigma^2)$ subject to the constraint that $\sum_k w_k \rho_k = \rho$ where $w_k = (\psi_{k..})/(\sum_k \psi_{k..})$ and ρ is the DA-estimated nationwide sex ratio. Variation in the sex ratios across poststrata is modeled via the parameter σ^2. The normal distribution is chosen for computational convenience.
- $\delta_k^S \sim BETA(a^S, b^S)$ and $\phi_k^S \sim BETA(c^S, d^S)$, beta priors that smooth the proportion of Census undercounts and the proportion of Census cases enumerated in the CMS across poststrata independently for each sex and provide a support of [0,1] for these proportions.
- $\gamma_k^M = \gamma^M \sim GAMMA(\alpha, \beta)$ for all k; $\gamma_k^F = 1$ for all k. These priors assume that the relative proportion of Census and non-Census cases enumerated in the CMS for each poststratum is a constant across poststrata (likely greater than 1) for all males and is constant and known to be equal to 1 (under the independence assumption) for females.

In addition, we assume that

- $y_{k11}^S \mid \psi_{k11}^S \sim N(\psi_{k11}^S, (\tau_{k11}^S)^2)$
- $y_{k01}^S \mid \psi_{k01}^S \sim N(\psi_{k01}^S, (\tau_{k01}^S)^2)$
- $z_{k1.}^S \mid \psi_{k1.}^S \sim N(\psi_{k1.}^S, (u_{k1.}^S)^2)$

where y_{k11}^S, y_{k01}^S, and $z_{k1.}^S$ are all independent. The marginal covariance between y_{k11}^S and y_{k01}^S is expected to be very small unless the sampling fraction is quite large (see proof in Appendix C). The marginal covariance between y_{k11}^S and $z_{k1.}^S$ and y_{k01}^S and $z_{k1.}^S$ is more problematic – since the erroneous enumerations subtracted from the Census counts and the Census status of the persons sampled in the CMS are being estimated from the same sample, there may be non-trivial correlations regardless of the sample size. Because information on this covariance was not available, we assume independence as a simplifying assumption.

3.1 Estimating the Posterior Distribution

The mode of the posterior distribution of $\{\psi_k^*\}$ might be computed by a numerical optimizing algorithm. However, given the large number of parameters relative to the data and presence of peaks near the boundary of support for ϕ_k^S in cells where $z_{k1.}^S < y_{k11}^S$, Newton-Raphson and Fischer scoring algorithm performed poorly. Hence we used Gibbs sampling (Gelfand and Smith 1990; Gelman and Rubin 1992; Smith and Roberts 1993) to draw estimates of the population parameters from their joint posterior distribution. In brief, Gibbs sampling obtains draws from a joint distribution of $p(\theta \mid y)$ for $\theta = \{\theta_1, \ldots, \theta_n\}$ by initializing θ to some reasonable $\theta^{(0)}$ and drawing

$\theta_1^{(1)} \sim p(\theta_1 \mid \theta_2^{(0)}, \ldots, \theta_n^{(0)}, y)$, $\theta_2^{(1)} \sim p(\theta_2 \mid \theta_1^{(1)}, \theta_3^{(0)}, \ldots, \theta_n^{(0)}, y)$, and so forth. As $t \to \infty$, $\theta^{(t)} \dot{\sim} p(\theta_1, \ldots, \theta_n \mid y)$. The Gibbs sampling approach estimates the entire posterior distribution and thus allows for greater flexibility in terms of point estimation and inference.

Summarizing the conditional distributions:

- $\psi_{k..} \mid \text{Rest} \propto N(\eta_k^{\psi}, (\xi_k^{\psi})^2)$
- Draw $M_k = \frac{\rho_k}{1+\rho_k}$ (the fraction of males): $M_k \mid \text{Rest} \propto N(\eta_k^M, (\xi_k^M)^2)$. Draws made from $N_K(\eta^M, (\xi^M)^2)$ conditional on $\overline{M} = \rho/(1+\rho)$ where $\eta^M = (\eta_1^M, \ldots, \eta_K^M)'$, $\xi = \text{diag}((\xi_1^M)^2, \ldots, (\xi_K^M)^2)$ via the SWEEP operator (Goodnight 1979; Little and Rubin 1987).
- $\delta_k^S \mid \text{Rest} \propto (\delta_k^S)^{a^S-1}(1-\delta_k^S)^{b^S-1} exp(\eta_k^{\delta})$
- $\phi_k^S \mid \text{Rest} \propto (\phi_k^S)^{c^S-1}(1-\phi_k^S)^{d^S-1} exp(\eta_k^{\phi})$
- $\gamma^M \mid \text{Rest} \propto (\gamma^M)^{\alpha-1} exp\left\{-(\gamma^M/\beta) + \sum_k \eta_k^{\gamma}\right\}$

where $\eta_k^{\theta}, \xi_k^{\theta}$ are linear functions of ψ_k^* and data.

Draws from the non-standard distributions were made via the inverse CDF method by determining the conditional PDF up to a constant. The constant of proportionality was obtained by numerical integration using 32-point Gauss-Hermite quadrature on each of 1,000 subintervals of the support or line segment where the posterior is observed to be non-trivially greater than 0. For a complete description of the conditional distributions, see Appendix A.

3.2 Estimating the Hyperparameters

The variances $(\tau_{k11}^S)^2$, $(\tau_{k01}^S)^2$, and $(u_{k1.}^S)^2$ were treated as known. The beta hyperparameters a^S, b^S, c^S, d^S were estimated using Gibbs sampling assuming a uniform hyperprior distribution. Since little information is available to estimate the gamma hyperparameters α and β, we chose the "flattest" prior for which $P(\gamma^M \in [0.5, 2.0]) = 0.95$. Specially, α and β were chosen from a grid of values to maximize $\alpha\beta^2$ under the constraint that $P(0.5 < X < 2.0) = 0.95$ for $X \sim GAMMA(\alpha, \beta)$ (Casella and Berger 1992, p. 186-187); this yielded $\alpha = 9$ and $\beta = 0.1306$. Rather than attempting to find an empirical Bayes estimate of σ^2, we set $\sigma = 1$, which essentially allows the poststratum sex ratios to vary freely, subject to the constraint that they yield the DA estimate when aggregated.

4 Application to US Census Data

4.1 Population and Undercount Estimation

We now apply the methods described above to the 1990 Census data in Table 2. The poststratified results under the ICR model are given in Table 3 (reproduced from Table B-1 from Bell et al. 1996). Table 4 gives the maximum likelihood estimates under the FRR model using Bell's (1993) approach described in Section 1.2.

Initial parameter estimates for the Gibbs sampler for the FRR model of Section 3 are described in Appendix B, together with methods for assessing convergence of the Gibbs chain. Five chains of the Gibbs sampler, each containing 1,000 draws, were run from different starting points,

Table 3: Estimated Counts of Inclusion/Exclusion in Census and CMS (in thousands): ICR (Independence) Model. PS = Poststratum

Census:	In	In	Out	Out	
CMS:	In	Out	In	Out	Total
Males PS 1	142	64	18	8	231
PS 2	379	82	41	9	510
PS 3	211	50	23	6	289
PS 4	178	-53	28	-8	144
PS 5	314	2	33	0	349
PS 6	246	32	38	5	322
PS 7	216	85	86	34	421
PS 8	280	150	84	45	559
PS 9	137	79	49	28	292
PS 10	111	58	45	23	238
PS 11	314	-16	67	-4	361
PS 12	53	28	12	6	77
Total	2581	559	524	152	3817
Females PS 1	174	75	24	11	284
PS 2	502	45	36	3	586
PS 3	272	36	23	3	334
PS 4	224	-89	42	-17	160
PS 5	371	-10	25	-1	386
PS 6	274	45	39	6	365
PS 7	329	121	95	35	581
PS 8	396	180	59	27	662
PS 9	334	12	52	2	400
PS 10	177	26	24	4	231
PS 11	482	-66	50	-7	459
PS 12	99	28	8	3	117
Total	3612	403	479	69	4564

Table 4: Estimated Counts of Inclusion/Exclusion in Census and CMS (in thousands): Maximum Likelihood Estimates Based on Bell's FRR Model. In CMS-Out Census negative cell counts are set to 0 with Bell's marginal adjustment. PS = Poststratum

Census:	In	In	Out	Out	
CMS:	In	Out	In	Out	Total
Males PS 1	142	64	18	20	243
PS 2	379	82	41	32	534
PS 3	211	50	23	19	303
PS 4	125	0	19	9	154
PS 5	314	2	33	16	364
PS 6	246	32	38	26	343
PS 7	216	85	86	90	477
PS 8	280	150	84	106	620
PS 9	137	79	49	64	328
PS 10	112	58	45	56	270
PS 11	298	0	64	30	392
PS 12	53	28	12	15	107
Total	2512	628	512	483	4136
Females PS 1	174	75	24	11	284
PS 2	502	45	36	3	586
PS 3	272	36	23	3	334
PS 4	135	0	25	0	160
PS 5	361	0	25	0	386
PS 6	274	45	39	6	365
PS 7	329	121	95	35	581
PS 8	396	180	59	27	662
PS 9	334	12	52	2	400
PS 10	177	26	24	4	231
PS 11	416	0	43	0	459
PS 12	77	28	8	3	117
Total	3446	569	455	94	4564

with the first 200 discarded as in initial "burn-in". The estimated ratio of the marginal posterior variance for all sequences to the mean within-sequence posterior variance, $\widehat{R}$ was used to check of the degree of convergence for the parameters (Gelman, et al., p.331-332.). Examination of all parameters estimated under our model for the 24 African-American 30-49 strata shows that $1.00 < \sqrt{\widehat{R}} < 1.07$, well below $\sqrt{\widehat{R}} \approx 1.2$ which Gelman et al. indicates as acceptable convergence.

The posterior mean of ψ_{k11}^{M} is simulated as the average of of $\frac{\rho_k}{1+\rho_k}(\delta_k^S)^{(t_j)}(\phi_k^S)^{(t_j)}\psi_{k..}^{(t_j)}$, where subscript (t_j) denotes the tth cycle of the jth chain of the Gibbs algorithm. Similar transformations provide the posterior means of ψ_{k10}, ψ_{k01}, and ψ_{k00}. The resulting estimates are given in Table 5. Use of a beta prior for the In Census-In CMS to Total Census ratio has eliminated negative estimates for the In Census-In CMS and Out-Census-Out CMS cells; more generally, the assumption of the common prior distribution has reduced the larger positive values and increased the smaller positive values of this cell somewhat as well. The generally much larger estimates of the Out Census-Out CMS cell among men when compared with the ICR model is a consequence of the correlation bias estimated from the discrepancy between the ICR model-estimated and DA-estimated sex ratios. Note also that, in strata with negative cells, the In Census-In CMS cells are substantially reduced relative to their unadjusted values to compensate for the increase in the In Census-Out CMS cells; this is a result of the total Census counts having relatively smaller estimates of variability relative to the CMS counts, thus forcing the In CMS-In Census cells to "surrender" people to the Out CMS-In Census cells. In Census-In CMS counts increase somewhat in strata where all counts are positive to balance this loss.

Figure 1 and Table 7 indicate the differences between the population estimates in each poststratum for African-Americans 30-49 using (a) Census estimates (minus imputations and estimates of erroneous enumerations); (b) maximum likelihood estimates for the ICR model; (c) maximum likelihood estimates for the FRR model adjusted to DA sex ratios using Bell's (1993) approach; and (d) posterior means under the model of Section 3. Note that estimates for (b) and (c) are identical for females. Several key observations can be derived from Figure 1:

- The undercount appears to be greatest, as might be expected, in Poststrata 7 through 10 (renters residing in urban area of 250,000/+).
- The FRR models provide larger estimates than the ICR models because these models adjust for correlation bias by forcing the total male/female ratio to equal or approximate DA sex ratio estimates. (Recall that females are assumed to have zero correlation bias.) This bias appears to be associated with the undercount itself, appearing larger in the rental poststrata than in the owner poststrata.
- The estimates derived for males from the Bayesian approach of Section 3 under the FRR assumption generally fall between the maximum likelihood estimates of Bell (1993) for the FRR model and the MLE ICR estimates. A discrepancy between the MLE and Bayesian FRR model appears in Poststratum 10 for men (non-owners in Western urban areas). This can be explained in part by the fact that this poststratum (a) has the largest proportion of males as estimated by the post-CMS-adjusted data (SR=1.030 under maximum likelihood for the ICR model and SR=1.170 under maximum likelihood for the FRR model), and (b) has apparently poor Census coverage as estimated by the CMS: this postratum contained the smallest proportion of the CMS that were identified in the Census and third-smallest proportion of those in the Census who were estimated to have been in the CMS. However, this estimate of poor coverage is based on relatively unstable CMS estimates (the largest CV for y_{k01}^{M} and third-largest CV for y_{k11}^{M}). Thus the Bayesian approach identifies and "corrects" to some degree this potential outlier, increasing its estimated coverage toward the all-strata mean. Similiar discrepancies in Stratum 6 and 7 result in part from large CVs from the CMS

Table 5: Estimated Counts of Inclusion/Exclusion in Census and CMS: Posterior Means Based on Bayesian FRR Model.

Census:	In	In	Out	Out	
CMS:	In	Out	In	Out	Total
Males PS 1	145	60	17	20	242
PS 2	417	44	40	26	527
PS 3	236	24	23	15	298
PS 4	123	3	26	14	165
PS 5	309	7	32	17	364
PS 6	267	11	31	17	326
PS 7	239	60	82	74	456
PS 8	282	148	84	110	623
PS 9	139	76	48	64	327
PS 10	128	40	42	43	253
PS 11	292	6	66	34	398
PS 12	62	18	11	11	102
Totals	2638	496	501	445	4081
Females PS 1	179	66	26	10	281
PS 2	519	12	50	1	583
PS 3	290	10	29	1	331
PS 4	128	3	30	1	161
PS 5	343	6	34	1	384
PS 6	297	13	39	2	350
PS 7	358	59	120	22	559
PS 8	380	146	92	37	655
PS 9	307	6	83	2	398
PS 10	174	7	43	2	226
PS 11	383	6	75	1	465
PS 12	90	8	13	1	112
Totals	3450	343	632	81	4505

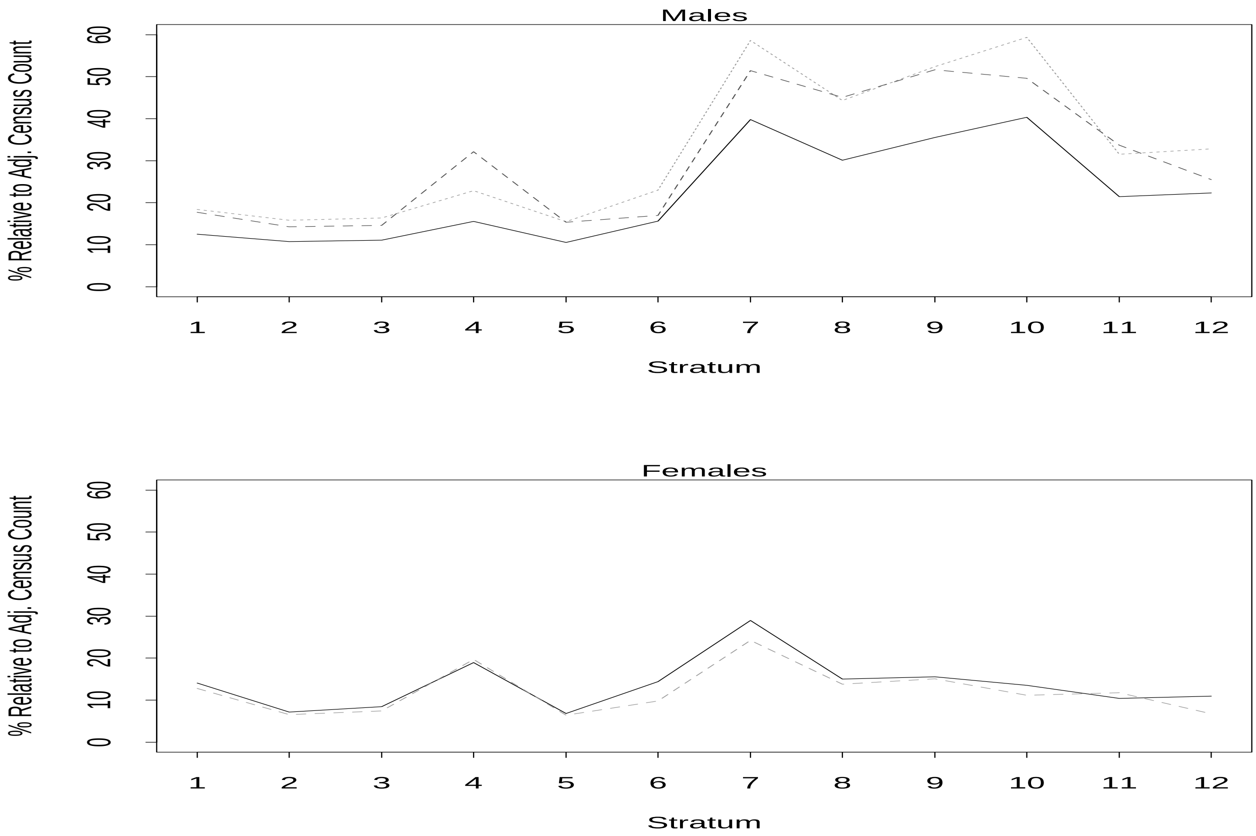

Figure 1: Percent of total population relative to Census counts minus imputations and erroneous enumerations ($z_{k1.}$) under the independence (ICR) model using ML estimates, under the fixed relative risk (FRR) model using ML estimates, and under the FRR Bayes model using posterior means, by gender and stratum.

estimates that allow the posterior results to be pulled toward the ICR estimates. The larger Bayes FRR estimate in Stratum 4 is a result of removing the large negative cell in the "raw" Out CMS- In Census cell and a similarly large CV in the CMS estimate.

- Female estimates under the Bayes FRR models are somewhat smaller on average than under the MLE FRR model, possibly a consequence of the smoothing of the sex ratios. Exceptions are Strata 4 and 11, where the removal of the large negative cells increases the estimate of the female population over the MLE estimates.

The total undercount for African-American women 30-49 when compared against the published Census estimate (US Census Bureau 1991) for African-Americans 30-49 is 1.7% under the ICR/MLE FRR model and 0.5% under the Bayesian FRR model. The undercount for African-American men 30-49 is estimated to be -1.1% under the ICR model, 6.7% under the FRR model using Bell's 1993 maximum likelihood approach, and 5.4% under the Bayes FRR model. The total undercount is 0.4% for the ICR model, 4.1% for FRR maximum likelihood, and 2.9% for FRR Bayes.

4.2 Model fit: Sex Ratios

Das Gupta (Bell et al. 1996) argues that the approach of Bell (1993) yields undue variation in estimated sex-ratio (SR) variation between strata (SRs under Bell's FRR model range between 0.822 and 1.170; SR's under Das Gupta's own model vary between 0.789 and 1.050). We introduce a prior for the poststratum sex ratios ρ_k which assumes that $\rho_k \sim N(\rho, \sigma^2)$, where $\rho = .9063$, the demographic estimate for this age-race group. This allows a degree of control over the variability of the sex ratios, ranging from forcing all sex ratios to equal the estimated nationwide sex ratio (which assumes $\sigma^2 = 0$) and estimating the poststratum sex ratio without restriction (which assumes $\sigma^2 = \infty$). Table 6 gives the male-to-female sex ratio for this model where $\sigma^2 = 1$, together with the adjusted Census estimates, the ICR model, and the FRR and FDCR maximum likelihood estimates. The choice of $\sigma^2 = 1$ allows the data from the likelihood to overwhelm the prior and thus essentially estimates sex ratios only under the constraint that $\sum_k w_k \rho_k = \rho$ where $w_k = (\psi_{k..})/(\sum_k \psi_{k..})$. Even when the sex ratio is essentially unconstrained by the prior variance, the variability across the poststrata estimates is less than for either the FRR and FDCR ML estimates.

However, the presence of sex ratios greater than 1 in Poststrata 4 and 10 seems unlikely, and suggests that the FRR model with this prior may be yielding overly-inflated estimates of sex ratio variability. Thus we considered a prior variance for ρ_k of $\sigma = .033$, which reduced $P(\rho_k > 1)$ to less than 0.002. The results are given in the far right column of Table 6. This moderately informative prior has greatly dampened the sex ratio variation, although owner strata still have slightly higher proportions of males, as would be expected. The extent to which σ^2 should be reduced to dampen the perhaps overly-variable within-strata sex ratio estimates is a matter that additional demographic expertise should address.

4.3 Model fit: Posterior Predictive Distributions

A key assumption in the above model is that negative differences $z^S_{k1.} - y^S_{k11}$ are attributable to sampling error *alone*, and not caused by systematic differences in the Census and CMS methods of enumeration. Overstatement of the number of matches from the CMS will bias upward the estimated In-Census In-CMS count y^S_{k11}, and overstatement of the rate of erroneous enumerations from the Census will bias downward the Census count $z^S_{k1.}$. This would clearly be the case if both $(\tau^S_{k11})^2$ and $(u^S_{k1.})^2$ were both near 0 – that is, we assumed that y^S_{k11} and $z^S_{k1.}$ were essentially exact estimates of the true population values ψ^S_{k11} and $\psi^S_{k1.}$. Hence the negative Out CMS-In Census counts potentially

Table 6: Estimates of the ratio of males to females within each poststratum: Census count minus imputations and estimate of erroneous enumerations; maximum likelihood estimate (MLE) from independence (ICR) model; MLE from fixed relative risk (FRR) model; posterior means from Bayes FRR model. PS=Poststratum

	Adj. Census Estimate	ICR MLE	FRR MLE	($\sigma = 1$) Bayes FRR	($\sigma = .033$) Bayes FRR
PS 1	0.826	0.815	0.858	0.863	0.890
PS 2	0.842	0.871	0.911	0.904	0.905
PS 3	0.845	0.866	0.907	0.901	0.902
PS 4	0.929	0.902	0.960	1.042	0.912
PS 5	0.874	0.904	0.945	0.948	0.931
PS 6	0.874	0.833	0.940	0.934	0.911
PS 7	0.668	0.725	0.822	0.817	0.900
PS 8	0.746	0.844	0.937	0.952	0.916
PS 9	0.623	0.731	0.822	0.822	0.890
PS 10	0.833	1.030	1.170	1.123	0.918
PS 11	0.716	0.788	0.853	0.857	0.893
PS 12	0.769	0.846	0.919	0.904	0.906
Total	0.782	0.836	0.906	0.906	0.906
$s(\times 10^{-2})$	9.18	8.24	9.20	8.49	1.23

provide evidence of lack of fit of the data to the model. To examine this possibility, we utilize posterior predictive distributions (PPD) (Gelman et al. 1995; Gelman, Meng, and Stern 1996).

Classic p-values represent the probability under the model (typically at some fixed parameter value θ) that the observed statistics $T(y)$ will be less than (or greater than) the values of the statistic that would be seen in repeated observations: $P(T(y) \leq T(y^{rep}) \mid \theta)$. The PPD p-value represents the probability that the observed statistic (which can be a function of both the data y and the parameter θ) is more extreme than replicated statistic, conditional on the observed data: $P(T(y, \theta) \leq T(y^{rep}, \theta) \mid y)$. PPD p-values can be obtained from the draws of θ generated by the Gibbs sampler; y^{rep} can be drawn from $f(y \mid \theta^{rep})$, and $T(y^{rep}, \theta^{rep})$ compared with $T(y, \theta^{rep})$. Specifically, we draw $(y^S_{k11})^{rep}$ from $N((\psi^S_{k11})^{rep}, (\tau^S_{k11})^2)$, where $(\psi^S_{k11})^{rep}$ is constructed from the draws of $\rho_k, \delta^S_k, \phi^S_k$, and $\psi_{k..}$ used to estimate $E(\psi^S_{k11} \mid y^S_{k11}, y^S_{k01}, z^S_{k1.})$ above. Similarly, we draw $(z^S_{k1.})^{rep}$ from $N((\psi^S_{k1.})^{rep}, (u^S_{k1.})^2)$. We then compare $(z^S_{k1.})^{rep} - (y^S_{k11})^{rep}$ with $z^S_{k1.} - y^S_{k11}$. Examining the histograms (not shown) shows that the only strata for which the observed data appear in the tail of the predictive distributions are poststratum 4 for both males and females: the PPD p-value for males is 0.070 and for females is 0.054. Thus the large negative Out Census-In CMS cells are not necessarily evidence of bias in the estimation of CMS capture or of erroneous enumeration and imputation.

Another PPD for diagnosing model fit is given by Figure 2, which plots the percentage of men in the draws of the total In-Census counts against in total In-Census count $[z^M_{k1.}/(z^M_{k1.} + z^F_{k1.})$ versus $(z^M_{k1.} + z^F_{k1.})]$. Figure 2 highlights two features of model fit. First, small estimates of the "true" In-Census count $(\psi^M_{k1.} + \psi^F_{k1.})$ are associated with higher percentages of men. This is because small In-Census estimates are associated with smaller estimates of follow-up enumeration $(\psi^S_{k11}/\psi^S_{k.1})$, which in turn are association with a greater degree of correlation bias, which leads to larger estimates of male population relative to the female population. Secondly, all of the observed In-Census sex ratios are well within the model predictions, although they tend to lie in the lower range of the predictions.

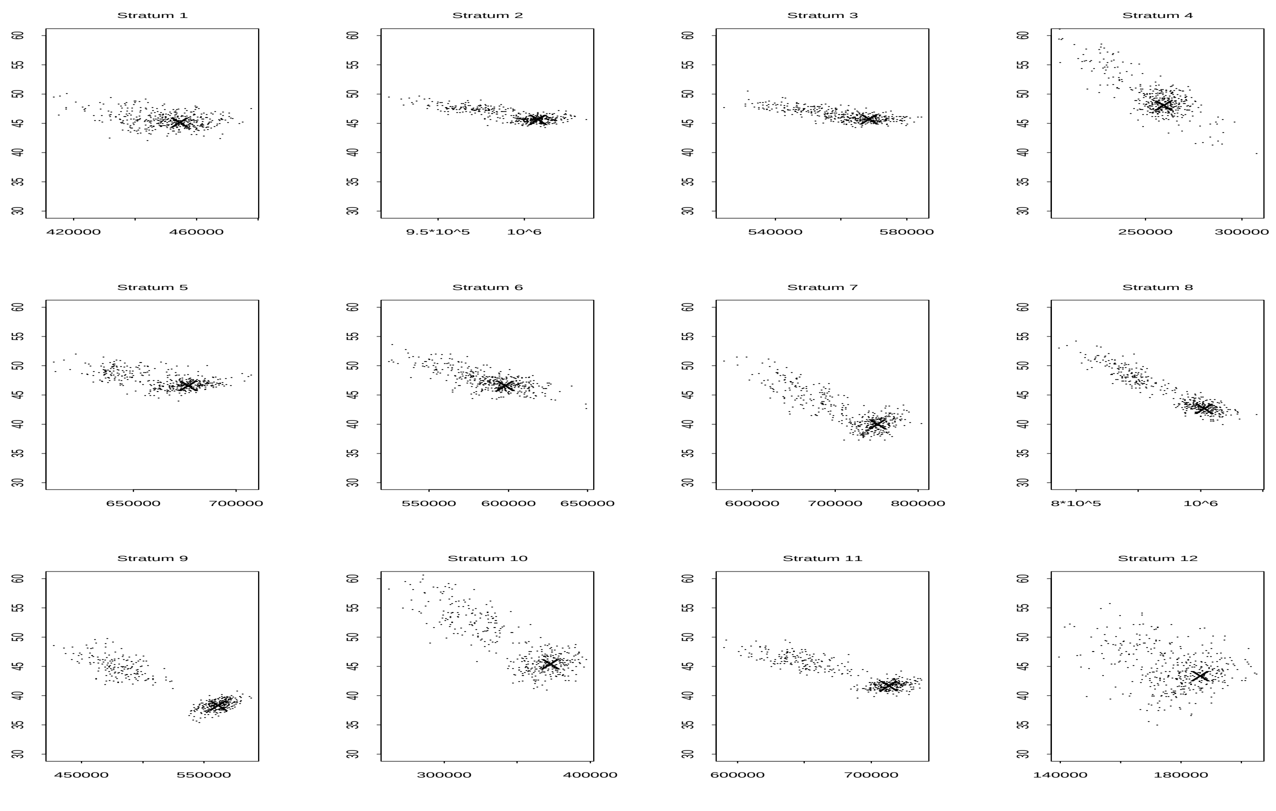

Figure 2: Posterior Predictive Draws of Adjusted Census Counts ($z_{k1.}$): Percent Male (y-axis) vs. Total (x-axis). Observed data given by "X".

Table 7: Estimates of total population within each poststratum (in thousands): Census count minus imputations and estimate of erroneous enumerations; maximum likelihood estimate (MLE) from independence (ICR) model; MLE from fixed relative risk (FRR) model; posterior means from Bayes FRR model, and 95% posterior probability interval (PPI) under Bayes FRR. PS = Poststratum

PS	Adj. Census Estimate	ICR MLE	FRR MLE	Bayes FRR	Bayes FRR 95% PPI
1	455	515	527	523	(498 - 550)
2	1008	1097	1120	1109	(1079 - 1142)
3	568	623	637	629	(610 - 650)
4	260	304	314	326	(278 - 384)
5	677	734	750	748	(716 - 781)
6	598	687	708	676	(614 - 739)
7	751	1001	1058	1015	(922 - 1128)
8	1005	1221	1282	1278	(1191 - 1381)
9	562	692	728	725	(682 - 774)
10	373	468	500	479	(419 - 555)
11	714	820	851	863	(821 - 909)
12	186	216	224	214	(192 - 242)

4.4 Inference about Populations

Inferences about the posterior distribution of parameters of interest can be easily obtained from the distribution of the Gibbs draws. For example, an estimate of the 95% posterior probability interval can be obtained by noting the 100th and 3900th smallest of the 4000 draws from the posterior distribution. Table 7 gives the mean and 95% posterior probability intervals (PPIs) for the total population in each of the 12 poststrata for African-Americans 30-49 under the Bayes FRR model, together with the adjusted Census estimate and Bell's (1993) maximum likelihood estimates under the ICR and FRR models. The 95% PPI for the undercount for females is (-1.8% - 2.6%); for males is (3.2% - 7.5%); and overall is (0.6%-5.2%).

Figure 3 plots draws of the total population against the proportion of the population that is male, and shows how easily other posterior predictive intervals, including multidimensional intervals, can be generated. Here we see that larger estimates of total population correspond with larger estimates of correlation bias and hence larger estimates of the proportion of the population that is male. ML estimates of the equivalent results for the ICR and FRR models are also presented, along with the adjusted Census counts.

5 Discussion

In this paper we summarize methods proposed for incorporating post-Census CMS and demographic data into estimates of Census subpopulation counts. All methods face the difficulty that the underlying cell counts in the 2 × 2 Census-CMS poststratification tables are unidentifiable unless a model is posited for the population. The simplifying assumption of independence – that the probabilities of capture and recapture are independent and homogeneous across the population – leads to ratios of males to females that are typically lower than estimates from demographic analysis. Numerous plausible models can be suggested that incorporate this sex ratio data, all providing perfect fits to the data. We have suggested six principles – PLAUSIBILITY, FIT, PREDICTION, INDEPENDENCE MODEL INCLUSION, STABILITY, and CONSERVATISM – to

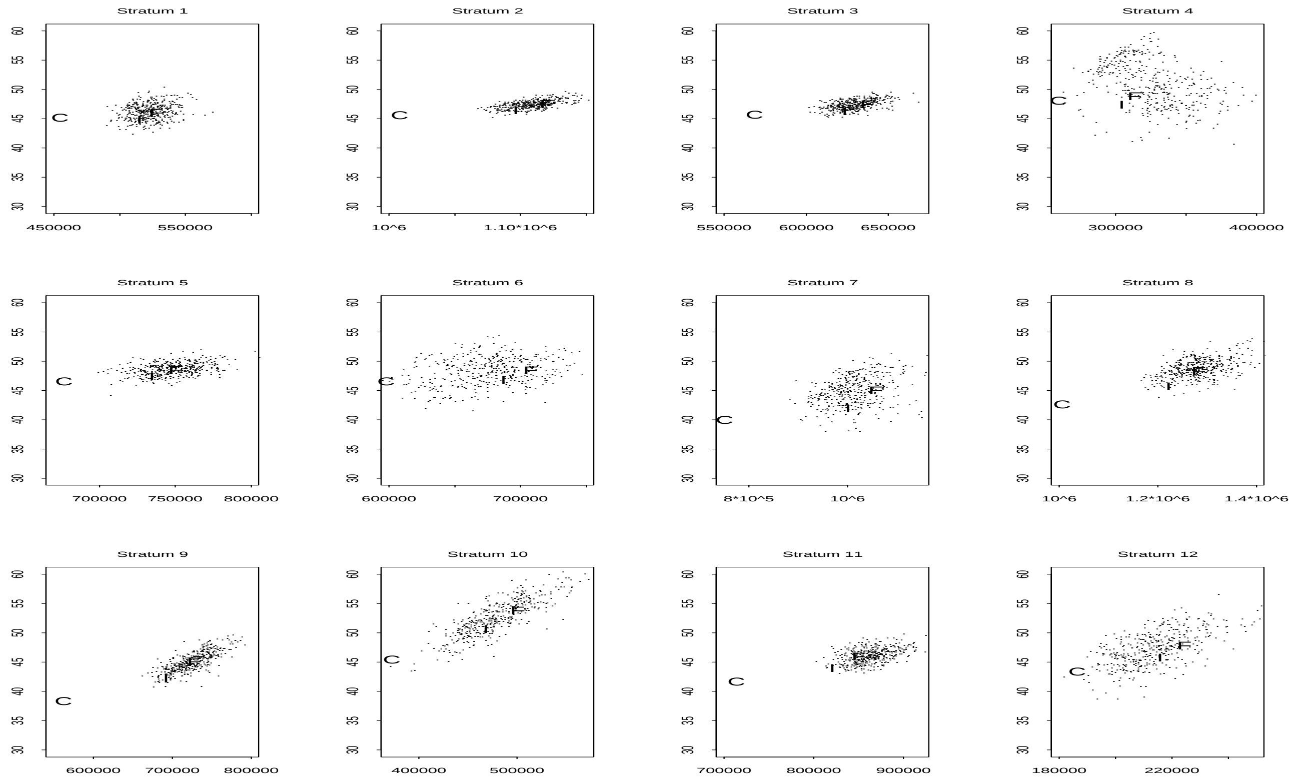

Figure 3: Gibbs Sampling Draws of the Total Population Estimates with each Poststratum: Percent Male (y-axis) vs. Total (x-axis). Adjusted Census counts and ML estimates for the ICR and FRR models given by "C", "I", and "F" respectively.

help choose among the models. (These principles are generally applicable to choosing among any statistical models whenever the FIT principle alone is deemed insufficient.) Use of these principles suggests a fixed relative-risk for enumeration in the CMS and Census (FRR) model [see (3)] as a leading candidate model for selection.

Beyond these qualitative discussions, we have described a more comprehensive statistical model that, through judicious choice of parameterization and prior distributions, eliminates negative cell estimates from the In Census-Out CMS cell of the poststratification tables and reduces outlying predictions of undercount rates. Applying this approach to the FRR model using 1990 Census data for African-Americans aged 30-49 yielded estimates of undercount of 0.5% for women, 5.4% for men, and 2.9% overall in this race-age category, with corresponding 95% posterior undercount intervals of (-1.8% - 2.6%), (3.2% - 7.5%), and (0.6%-5.2%). The estimates 1990 Census undercount among African-Americans aged 30-49 using the FRR MLEs (Bell 1993) are 1.7% for women, 6.7% for men, and 4.1% overall; no confidence intervals are easily available. Our approach identifies potential outliers in the poststrata tables and reduces their impact on post-CMS total population estimates. In addition, the model detects, through posterior predictive distributions, strata in which large negative raw In Census-Out CMS cells may be due to bias in, rather than variance in, CMS enumeration or Census estimates. Finally, our model allows direct control over the inter-strata sex ratio variation ρ_k through the variance parameter σ^2. To be conservative, we have utilized $\sigma^2 = 1$, which essentially flattens the prior distribution relative to the likelihood. As with the ML estimates, some of the resulting within-poststratum sex ratio estimates yielded implausibly large proportions of males. If prior estimates of σ are available – they were not to us – they might be utilized, in the process reducing poststrata sex ratio variability to more plausible values.

Many extensions of the methods and models described could be envisaged. One immediate extension would be to introduce a prior for ρ to account for known uncertainty in the DA-estimated nationwide sex ratio. Also, the estimates of variability in the CMS and Census data are treated as fixed; prior distributions could be assumed to estimate any uncertainty in their values. Additional demographic measures could be incorporated in our model through careful choice of parameterizations. Prior means for the poststrata cell data or sex ratios could be regressed on poststratum characteristics to further reduce the dimensionality of the model. In the future, covariates more closely associated with the probability of response (Slud 1998), such as number and type of contact attempts required, could be used to form the poststrata, reducing correlation bias.

Additionally, the sensitivity of the estimates under the various model assumptions (2)-(6) could be examined, as Bell (1993) did in the context of maximum likelihood estimation. A more complete extension would average the results across the various models by noting that for J models M_j

$$P(\psi \mid \text{data}) = \int P(\psi \mid \text{data}, M_j) P(M_j \mid \text{data}) dM_j$$

$$\propto \sum_{j=1}^{J} P(\psi \mid \text{data}, M_j) P(\text{data} \mid M_j) P(M_j)$$

Thus if all models are considered *a priori* likely, $P(M_1) = \cdots = P(M_J) = 1/J$, and if all are saturated so that $P(M_1 \mid \text{data}) = \cdots = P(M_J \mid \text{data})$, the marginal posterior expectation and variance of, say, the total population could be obtained as

$$\widehat{E}(\psi_{k..} \mid \text{data}) = (1/J) \sum_j \widehat{E}(\psi_{k..} \mid \text{data}, M_j) \tag{7}$$

$$\widehat{Var}(\psi_{k..} \mid \text{data}, M_j) = (1/J) \sum_j \widehat{Var}(\psi_{k..} \mid \text{data}, M_j) +$$

$$\frac{1+J^{-1}}{J-1}\sum_j(\widehat{E}(\psi_{k..} \mid \text{data}, M_j) - \widehat{E}(\psi_{k..} \mid \text{data}))^2).$$

One criticism of our proposed approach is that it is explicitly Bayesian, and hence incorporates subjective elements through the choices of model and prior. However, every method of modern Census enumeration aimed at getting counts of the full population requires subjective assumptions – including methods that leave raw Census counts unadjusted. The Bayesian framework makes these assumptions explicit and open to debate, rather than implicit in the estimation algorithm. A second criticism is that the computations are complex, and simple transparent methods that are relatively easy to explain to lay audiences are preferable. We too favor simplicity, but think a distinction needs to be made between the underlying assumptions of the model, which are not particularly complex and are capable of being transmitted in non-technical terms, and the algorithms used to simulate posterior distributions based on the model, which are very complex but need not be understood by non-statistical stakeholders. By analogy, least squares estimation for multiple linear regression is beyond the understanding of the large majority of the public who did not get beyond a single elementary statistics course, but that does not prevent its widespread use in real-world policy settings that involve analysis of data. More generally, complex micro-simulation and statistical models with subjective assumptions underlie the interpretations of much data that are used to inform public policy in the economic and health arenas.

A Conditional Distributions for Gibbs Sampling

The posterior distribution of $\{\psi_k^*\}$ given the DA estimate ρ and hyperparameters $a^S, b^S, c^S, d^S, \alpha, \beta, \tau_{k11}^S, \tau_{k01}^S$ and $u_{k1.}^S$ is

$$\begin{gathered}
p(\psi_k^* \mid y_{k11}^M, y_{k11}^F, y_{k01}^M, y_{k01}^F, z_{k1.}^M, z_{k1.}^F) \propto \\
p(\gamma^M)p(\rho_k)p(\delta_k^M)p(\delta_k^F)p(\phi_k^M)p(\phi_k^F)\times \\
p(y_{k11}^M \mid \psi_{k11}^M, (\tau_{k11}^M)^2)p(y_{k01}^M \mid \psi_{k01}^M, (\tau_{k01}^M)^2)p(z_{k1.}^M \mid \psi_{k1.}^M, (u_{k1.}^M)^2)\times \\
p(y_{k11}^F \mid \psi_{k11}^F, (\tau_{k11}^F)^2)p(y_{k01}^F \mid \psi_{k01}^F, (\tau_{k01}^F)^2)p(z_{k1.}^F \mid \psi_{k1.}^F, (u_{k1.}^F)^2) \propto \\
(\gamma^M)^{\alpha-1}e^{\gamma^M/\beta}\prod_{k=1}^K exp(-1/2((\rho_k-\rho)/\sigma)^2)\times \\
\prod_{S=M,F}\prod_{k=1}^K(\frac{\Gamma(a^S+b^S)}{\Gamma(a^S)\Gamma(b^S)}(\delta_k^S)^{a^S-1}(1-\delta_k^S)^{b^S-1}\times \\
\frac{\Gamma(c^S+d^S)}{\Gamma(c^S)\Gamma(d^S)}(\phi_k^S)^{c^S-1}(1-\phi_k^S)^{d^S-1}\times \\
exp((\frac{\psi_{k11}^S - y_{k11}^S}{\tau_{k11}^S})^2 + (\frac{\psi_{k01}^S - y_{k01}^S}{\tau_{k01}^S})^2 + (\frac{\psi_{k1.}^S - z_{k1.}^S}{u_{k1.}^S})^2)).
\end{gathered} \tag{8}$$

We determine the conditional distributions of each scalar parameter θ_j conditional on all data (Y=$\{y_{k11}^S, y_{k01}^S, z_{k1.}^S\}$)) and other parameters $\theta_{-\theta_j}$ by examining the posterior likelihood given by (A.1). Where possible, draws are obtained from standard distributions; otherwise draws are made via the inverse CDF method by determining the conditional PDF up to a constant. The constant of proportionality is obtained by numerical integration using 32-point Gauss-Hermite quadrature on each of 1,000 subintervals $[t_{i-1}, t_i]$ of the support or line segment where the posterior is observed to be non-trivially greater than 0. A draw X from $UNI(0,1)$ is then obtained and the midpoint of the subinterval $[t_{j-1}, t_j]$ where $min_j(\sum_1^j A_{[t_{i-1},t_i]}) > X$ is then taken as a draw of $\theta_j \mid Y, \theta_{-\theta_j}$, where $A_{[t_{i-1},t_i]}$ is the area under the normalized PDF between points t_{i-1} and t_i. We now discuss the conditional distributions required for the Gibbs cycle:

A.1 δ_k^S

$$p(\delta_k^M \mid Y, \theta^{(t)}_{-\delta_k^M}) \propto$$
$$(\delta_k^M)^{a^M-1}(1-\delta_k^M)^{b^M-1}exp\{\frac{-1}{2\sigma_1^2}(l\delta_k^M - x)^2 +$$
$$\frac{-1}{2\sigma_2^2}(m - m\delta_k^M - y)^2 + \frac{-1}{2\sigma_3^2}(n\delta_k^M - z)^2\} \propto$$
$$(\delta_k^M)^{a^M-1}(1-\delta_k^M)^{b^M-1}exp\{\frac{-(l^2/\sigma_1^2 + m^2/\sigma_2^2 + n^2/\sigma_3^2)}{2}\times$$
$$[\delta_k^M - \frac{(lx/\sigma_1^2 + (m-y)m/\sigma_2^2 + nz/\sigma_3^2)}{(l^2/\sigma_1^2 + m^2/\sigma_2^2 + n^2/\sigma_3^2)}]^2\}$$

where

$$l = \phi_k^M \frac{\rho_k}{1+\rho_k}\psi_{k..} \qquad m = \frac{\phi_k^M}{\gamma^M}\frac{\rho_k}{1+\rho_k}\psi_{k..} \qquad n = \frac{\rho_k}{1+\rho_k}\psi_{k..}$$

and

$$x = y_{k11}^M \quad y = y_{k01}^M \quad z = z_{k1.}^M$$
$$\sigma_1^2 = (\tau_{k11}^M)^2 \quad \sigma_2^2 = (\tau_{k01}^M)^2 \quad \sigma_3^2 = (u_{k1.}^M)^2$$

Draws from this distribution are made by the inverse CDF method.

Similar results hold for δ_k^F, with $\frac{\rho_k}{1+\rho_k}$ replaced with $\frac{1}{1+\rho_k}$

A.2 ϕ_k^S

$$p(\phi_k^M \mid Y, \theta^{(t)}_{-\phi_k^M}) \propto (\phi_k^M)^{c^M-1}(1-\phi_k^M)^{d^M-1}\times$$
$$exp\{\frac{-1}{2\sigma_1^2}(l\phi_k^M - x)^2 + \frac{-1}{2\sigma_2^2}(m\phi_k^M - y)^2\} \propto$$
$$(\phi_k^M)^{c^M-1}(1-\phi_k^M)^{d^M-1}exp\{\frac{-(l^2/\sigma_1^2 + m^2/\sigma_2^2)}{2}\times$$
$$[\phi_k^M - \frac{(lx/\sigma_1^2 + my/\sigma_2^2)}{(l^2/\sigma_1^2 + m^2/\sigma_2^2)}]^2\}$$

where

$$l = \delta_k^M \frac{\rho_k}{1+\rho_k}\psi_{k..} \quad m = \frac{(1-\delta_k^M)}{\gamma^M}\frac{\rho_k}{1+\rho_k}\psi_{k..}$$
$$x = y_{k11}^M \quad y = y_{k01}^M \quad \sigma_1^2 = (\tau_{k11}^M)^2 \quad \sigma_2^2 = (\tau_{k11}^M)^2$$

As with δ_k^F, similar results for ϕ_K^F hold replacing $\frac{\rho_k}{1+\rho_k}$ with $\frac{1}{1+\rho_k}$; draws are made by the inverse CDF method.

A.3 $\psi_{k..}$

$$p(\psi_{k..} \mid Y, \theta^{(t)}_{-\psi_{k..}}) \propto$$

$$exp\{\sum_{S=M,F}[\frac{-1}{2(\sigma_1^S)^2}(l^S\psi_{k..} - x^S)^2 + \frac{-1}{2(\sigma_2^S)^2}(m^S\psi_{k..} - y^S)^2 +$$

$$\frac{-1}{2(\sigma_3^S)^2}(n^S\psi_{k..} - z^S)^2]\}$$

$$\Rightarrow \psi_{k..} \mid Y, \theta^{(t)}_{-\psi_{k..}} \sim N(\eta_k, \xi_k^2)$$

with

$$\eta_k = \frac{\sum_{S=M,F}(l^S x^S/(\sigma_1^S)^2 + m^S y^S/(\sigma_2^S)^2 + n^S z^S/(\sigma_3^S)^2)}{((l^S)^2/(\sigma_1^S)^2 + (m^S)^2/(\sigma_2^S)^2 + (n^S)^2/(\sigma_3^S)^2)}$$

$$\xi_k^2 = \left(\sum_{S=M,F}\left[(l^S)^2/(\sigma_1^S)^2 + (m^S)^2/(\sigma_2^S)^2 + (n^S)^2/(\sigma_3^S)^2\right]\right)^{-1}$$

where

$$l^S = \delta_k^S \phi_k^S \frac{\rho_k^{I(S=M)}}{1+\rho_k} \qquad m^S = \frac{(1-\delta_k^S)\phi_k^S}{\gamma^S}\frac{\rho_k^{I(S=M)}}{1+\rho_k}$$

$$n^S = \delta_k^S \frac{\rho_k^{I(S=M)}}{1+\rho_k}$$

and

$$x^S = y_{k11}^S \quad y^S = y_{k01}^S \quad z^S = z_{k1.}^S$$

$$(\sigma_1^S)^2 = (\tau_{k11}^S)^2 \quad (\sigma_2^S)^2 = (\tau_{k11}^S)^2 \quad (\sigma_3^S)^2 = (u_{k1.}^S)^2$$

Draws are made directly from the normal distribution.

A.4 γ^M

$$p(\gamma^M \mid Y, \theta^{(t)}_{-\gamma^M}) \propto$$

$$(\gamma^M)^{\alpha-1} exp\left\{-\gamma^M/\beta) + \sum_k \left[\frac{-1}{2(\tau_{k01}^M)^2}(l_k/\gamma^M - y_{k01}^M)^2\right]\right\}$$

where

$$l_k = \phi_k^M(1-\delta_k^M)\frac{\rho_k}{1+\rho_k}\phi_{k..}$$

Draws are made via the inverse CDF method.

A.5 ρ_k

Rather than draw ρ_k directly, we draw $M_k = \frac{\rho_k}{1+\rho_k}$ (the fraction of males), where $p(\rho_k) \sim N(\rho, \sigma^2) \Rightarrow p(M_k) \dot{\sim} N(\frac{\rho}{1+\rho}, \frac{\sigma^2}{(1+\rho)^4})$.

$$p(M_k \mid Y, \theta^{(t)}_{-M_k}) \propto$$

$$exp\{\frac{-1}{2(\sigma_1^F)^2}(l^F(1-M_k)-x^F)^2 + \frac{-1}{2(\sigma_2^F)^2}(m^F(1-M_k)-y^F)^2 +$$

$$\frac{-1}{2(\sigma_3^F)^2}(n^F(1-M_k)-z^F)^2 + \frac{-1}{2(\sigma_1^M)^2}(l^M M_k - x^M)^2 +$$

$$\frac{-1}{2(\sigma_2^M)^2}(m^M M_k - y^M)^2 + \frac{-1}{2(\sigma_3^M)^2}(n^M M_k - z^M)^2 +$$

$$\frac{-1}{2\sigma^2/(1+\rho)^4}(M_k - \frac{\rho}{1+\rho})^2\}$$

$$\Rightarrow M_k \mid Y, \theta^{(t)}_{-M_k} \sim N(\eta, \xi_k^2)$$

with

$$\eta_k = (l^F(l^F - x^F)/(\sigma_1^F)^2 + m^F(m^F - y^F)/(\sigma_2^F)^2 + n^F(n^F - z^F)/(\sigma_3^F)^2 +$$

$$l^M x^M/(\sigma_1^M)^2 + m^M y^M/(\sigma_2^M)^2 + n^M z^M/(\sigma_3^M)^2 + \frac{(1+\rho)^4}{\sigma^2})/$$

$$(\sum_{S=M,F} [(l^S)^2/(\sigma_1^S)^2 + (m^S)^2/(\sigma_2^S)^2 + (n^S)^2/(\sigma_3^S)^2] + \frac{\rho(1+\rho)^3}{\sigma^2})$$

$$\xi_k^2 =$$

$$\left(\sum_{S=M,F} [(l^S)^2/(\sigma_1^S)^2 + (m^S)^2/(\sigma_2^S)^2 + (n^S)^2/(\sigma_3^S)^2] + \frac{\rho(1+\rho)^3}{\sigma^2} \right)^{-1}$$

where

$$l^S = \delta_k^S \phi_k^S \psi_{k..} \qquad m^S = \frac{(1-\delta_k^S)\phi_k^S \psi_{k..}}{\gamma^S} \qquad n^S = \delta_k^S \psi_{k..}$$

and

$$x^S = y^S_{k11} \quad y^S = y^S_{k01} \quad z^S = z^S_{k1.}$$

$$(\sigma_1^S)^2 = (\tau^S_{k11})^2 \quad (\sigma_2^S)^2 = (\tau^S_{k11})^2 \quad (\sigma_3^S)^2 = (u^S_{k1.})^2$$

Since the constraint $\sum_k \psi^M_{k..}/\sum_k \psi^F_{k..} = \rho$ implies that the draws must be make subject to the constraint $\sum_k w_k M_k = \overline{M} = \rho/(1+\rho)$ where $w_k = \psi_{k..}/\sum_k \psi_{k..}$, draws are made from $N_K(\eta, \xi)$ conditional on $\overline{M} = \rho/(1+\rho)$ where $\eta = (\eta_1, \ldots, \eta_K)'$ and $\xi = \text{diag}(\xi_1, \ldots, \xi_K)$. This is accomplished via the SWEEP operator (Goodnight 1979; Little and Rubin 1987, p.112-119), which shows that draws from this conditional multivariate normal are equivalent to draws from $N_{K-1}(\eta^*, \xi^*)$, where $\eta^*_k = \eta_k - \frac{w_k \xi_k}{\sum_k w_k^2 \xi_k}\overline{\eta}$ and $\xi^*_{kj} = \xi_k I\{k=j\} - \frac{w_k \xi_k w_j \xi_j}{\sum_k w_k^2 \xi_k}$ where $I\{\cdot\}$ is the indicator function. After drawing $M_1, \ldots, M_{K-1}$ from $N_{K-1}(\eta^*, \xi^*)$, M_K is given by $(\overline{M} - \sum_1^{K-1} w_k M_k)/w_K$.

A.6 a^S, b^S, c^S, d^S

$$p(a^S, b^S \mid Y, \theta^{(t)}_{-a^S, b^S} \propto \frac{\Gamma(a^S + b^S)}{\Gamma(a^S) + \Gamma(b^S)} \prod_{k=1}^{K} (\delta_k^S)^{a^S - 1}(1 - \delta_k^S)^{b^S - 1} =$$

$$\frac{\Gamma(a^S + b^S)}{\Gamma(a^S) + \Gamma(b^S)} e^{(a^S-1)\sum_k log \delta_k^S} e^{(b^S-1)\sum_k log(1-\delta_k^S)}$$

Draws are made from the joint distribution of a^S, b^S by determining the joint distribution of a^S, b^S over a grid, summing across the grid to approximate the marginal distribution of a^S, drawing a value of a^S via the inverse CDF method, and then drawing a value of b^S via the inverse CDF method from the distribution of b^S conditional on the drawn value of a^S.

A similar approach is used to draw c^S and d^S.

B Initial estimates and convergence of the Gibbs chain

To obtain initial estimates, we fit an alternative and somewhat simpler model that assumes $y_{k11}^S \sim N(\psi_{k11}^S, (\tau_{k11}^S)^2)$, $z_{k1.}^S \sim N(\psi_{k1.}^S, (u_{k1.}^S)^2)$, and $\psi_{k11}^S = \phi_k^S \psi_{k1.}^S$, where $\phi_k^S \sim BETA(c^S, d^S)$. We assume $\psi_{k01}^S = y_{k01}^S$, that is that ψ_{k01}^S has a degenerate prior distribution where $\psi_{k01}^S = y_{k01}^S$ with probability 1. Then, to a first-order approximation, $r_k^S \mid \phi_k^S \sim N(\phi_k^S, w_k^S)$ where $\frac{y_{k11}^S}{z_{k1.}^S} = r_k^S$ and $w_k^S = \frac{(\tau_{k11}^S)^2}{{z_{k1.}^S}^2} + (u_{k1.}^S)^2 \left(\frac{(y_{k11}^S)^2}{(z_{k1.}^S)^4}\right)$, the ratio estimator of the variance of r_k^S. To obtain the posterior distribution of ϕ_k^Swe note that, up to a constant,

$$f(\phi_k^S \mid r_k^S) \propto f(\phi_k^S) f(r_k^S \mid \phi_k^S) =$$

$$(\phi_k^S)^{(c^S-1)}(1 - \phi_k^S)^{(d^S-1)} exp(-\frac{1}{2w_k^S}(r_k^S - \phi_k^S)^2)$$

Empirical Bayes estimates of c^S and d^S can be obtained by integrating over ϕ_k:

$$r_k \approx N(\frac{c^S}{c^S + d^S}, E(w_k) + \frac{c^S d^S}{(c^S + d^S)^2 (c^S + d^S + 1)})$$

Evaluating $E(w_k)$ is a non-trivial task; however, when $\psi_{k11} >> \tau_{k11}$ and $\psi_{k1.} >> u_{k1.}$, as is the case here, $E(w_k) \doteq w_k$. Algebra then gives the empirical Bayes estimators

$$\widehat{c^S} = \frac{\overline{(r^S)}^2 (1 - \overline{r^S})}{S^2(r^S) - \overline{w^S}} - \overline{r^S} \qquad \widehat{d^S} = \widehat{c^S}(\overline{(r^S)}^{-1} - 1)$$

The estimator of the posterior mean $\phi_k^S = E(\phi_k^S \mid r_k^S)$ can then be obtained through numerical integration. The "In Census-In CMS" cell (ψ_{k11}^S) is then estimated by $\widehat{\phi_k^S} z_{k1.}^S$, and the "In Census-Out CMS" cell (ψ_{k10}^S) by $(1 - \widehat{\phi_k^S}) z_{k1.}^S$. The "Out CMS-Out Census" (ψ_{k00}^S) cell is estimated under the ICR model (assumption of independence of capture-recapture) with no adjustment for sex ratio ρ. The estimates of ψ_{kij}^S are then transformed using (7) to provide initial estimates of δ_k^S, ϕ_k^S, γ^M; the initial estimate of $\psi_{k..}$ is then drawn conditional on the other parameters as described in Section A.3. Initial estimates of the hyperparameters a^S and b^S can be then made as for c^S and d^S

above. (In the application in Section 4, the empirical Bayes estimators of c^M and c^F were negative and lead to improper posterior distributions. Thus initial estimates of c^S and d^S were estimated by crude method-of-moment estimators $(\overline{\hat{\phi^S}} = \frac{c^S}{c^S+d^S}, \widehat{Var}(\hat{\phi^S}) = \frac{c^s d^S}{(c^S+d^S)^2(c^s+d^s+1)} \Rightarrow c^S = (\frac{\overline{\hat{\phi^S}}(1-\overline{\hat{\phi^S}})}{Var(\hat{\phi^S})} - 1)\overline{\hat{\phi^S}}, d^S = (\frac{\overline{\hat{\phi^S}}(1-\overline{\hat{\phi^S}})}{Var(\hat{\phi^S})} - 1)(1 - \overline{\hat{\phi^S}})).)$

To formally test that convergence has occurred, we use the method suggested in Gelman et al. (1995, p.331-332). A series of J different startpoints of the Gibbs sampler are utilized, and a subset of n iterations at which it is believed convergence has been achieved is selected and the between- and within-sequence posterior variances and for each (scalar) parameter calculated. The estimated ratio of the marginal posterior variance for all sequences to the mean within-sequence posterior variance, $\widehat{R}$, then provides a check of the degree of convergence for the parameters. In particular, Gelman et al. suggest that $\sqrt{\widehat{R}} \approx 1.2$ or less indicates acceptable convergence has been obtained for most purposes.

We used 5 startpoints for the parameters: those obtained through the simplified model described above, and those estimates with ϕ_k^S and γ^M changed to a 2×2 design where ϕ_k^S=0.75 or 0.99 for all S, k and γ^M=0.75 or 2.5. The 201st for 1000th iterations were selected. Examination of all parameters estimated under our model for the 12 African-American 30-49 strata shows that $1.00 < \sqrt{\widehat{R}} < 1.07$.

The autocorrelation of the Gibbs sequence is ignored in the above approximation, which is reasonable if n is large (Gelman, et al. 1995, p. 330). Although a moderate degree of autocorrelation is present in the data, our posterior estimates are made from 4,000 draws from five independent chains, diminishing this effect.

C Correlation between In-Census-In CMS and Out-Census-In CMS counts

We make the simplifying assumption that the total number of persons sampled in the CMS of a given age-race-gender category in stratum k conditional on the true age-race-gender population $\psi_{k..}^F$ is $w_{k.1}^S \sim BIN(\psi_{k..}^S, \pi_k^S)$ where π_k is the probability of selection into the CMS. Then the number of persons in the CMS sample who were also observed in the Census conditional on the CMS sample size is $w_{k11}^S \mid w_{k.1}^S \sim BIN(w_{k.1}^S, p_k^S)$. Making the further somewhat simplified assumption that $y_{k11}^S = w_{k11}^S/\pi_k^S$, we have

$$Cov(y_{k11}^S, y_{k01}^S) = (1/\pi_k^{S\,2})Cov(w_{k11}^S, w_{k01}^S) =$$

$$(1/(\pi_k^S)^2)(E[-w_{k.1}^S p_k^S(1-p_k^S)] + Cov[w_{k.1}^S p_k^S, w_{k.1}^S(1-p_k^S)]) =$$

$$-\psi_{k..}^S p_k^S(1-p_k^S)$$

Similarly, $Var(y_{k11}) = \frac{\psi_{k..}^S p_k^S}{\pi_k^S}[1 - p_k^S\pi_k^S]$ and $Var(y_{k01}) = \frac{\psi_{k..}^S(1-p_k^S)}{\pi_k^S}[1-(1-p_k^S)\pi_k^S]$. Thus $Corr(y_{k11}^S, y_{k01}^S) = -\pi_k^S/(1-p_k^S\pi_k^S)(1-(1-p_k^S)\pi_k^S)$, which implies $Corr(y_{k11}^S, y_{k01}^S) \to 0$ as $\pi_k^S \to 0$.

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