13.1

Computer project problems

Katta G. Murty Lecture slides

1. A Product mix problem: A company has machines of types 1 and 2. Using them and two raw materials R_1, R_2, the company makes and sells products P_1, P_2, P_3. Relevent data given below. In the table M_1, M_2 are type 1, 2 machine times (in minutes).

Resource	Input units/ton of product			Daily availability
	P_1	P_2	P_3	
M_1	1	2	1	430
M_2	3	0	2	460
R_1(tons)	1	4	0	420
R_2(tons)	1	1	1	300
Min daily demand		70		
Max daily demand			240	
Net profit (\$/ton)	300	200	500	

Formulate problem of maximizing total daily net profit of company, subject to all the constraints as an LP.

Solve the LP using AMPL (will get you a higher grade if you use this software), or any other software. Write a report explaining the output, and all the useful planning information that can be derived from it.

2. Problem with Local content constraint: A US company is planning to set up a plant in a foreign country to manufacture a hi-tech product to take advantage of the cheaper labor in that country. The product needs 10 components, some in multiple copies. For $i = 1$ to 10, let

$m_i =$ no. copies of component i used to produce 1 unit of final product

$c_{ih} =$ delivered unit purchase cost (\$) of component i from cheapest supplier in host country

$c_{ig} =$ delivered unit purchase cost (\$) of component i from cheapest global supplier .

Data provided in following table.

Component i	m_i	c_{ig}	c_{ih}
1	20	0.25	0.30
2	2	2.20	4.00
3	4	5.30	6.20
4	1	80.00	95.00
5	1	45.50	52.00
6	2	7.25	7.25
7	12	2.40	3.20
8	4	3.00	4.50
9	4	12.00	19.25
10	1	50.00	100.00

The local supplier is the cheapest supplier globally for component 6, but all other components have a foreign source as the cheapest supplier.

Assume that no supplier has capacity constraints.

The host country has a domestic content policy which stipulates that a specified fraction α or higher of the total value of all the components in the manufactured product must come from domestic suppliers. For $i = 1$ to 10, define the decision variables:

$x_i =$ fraction of component i sourced from cheapest local supplier

$1 - x_i = y_i =$ fraction of component i sourced from cheapest global supplier.

Assume that the annual production target is $T = 10^6$ units of the product.

Using the above decision variables, formulate the problem of minimizing the total component cost, subject to the domestic content policy, as an LP.

Solve the model for several values of α, say, 0, 0.25, 0.50, and 1. Discuss how the total component cost per unit product varies as α increases.

3. Cooking oil blending: A small company makes a very fine cooking oil used in used in high-quality restaurants for salad oil. It is a blend of 5 different vegetable oils.

Following table gives the posted prices of raw vegetable oils over a 6 month period of next year. These prices can be locked in now by means of futures contracts.

Raw oil prices ($/100 liter vat)

Month	Price of oil				
	Corn	Peanut	Saflower	Sesame	Sunflower
Jan	179	172	203	188	172
Feb	180	140	172	203	203
Mar	149	156	203	219	172
Apr	195	187	187	172	187
May	164	172	234	188	156
Jun	211	125	218	156	141
Hardness	5.0	4.2	2.0	6.1	8.8

The hardness of the blend = weighted average of hardness of oils in the blend, must be between 3.0 and 5.8.

The company's blending plant has a monthly capacity for processing:

at most 250 vats of corn, peanut, and saflower oils combined
at most 200 vats of sesame and sunflower oils combined

At the start of the 6-month period assume that there are 500 vats of each kind of vegetable oil in storage. The plan must provide for the same quantities of each oil in storage at the end of the 6-month period. For each oil, the storage capacity is 1000 vats; the cost of keeping the oil in storage from one month to the next is $8/vat.

The amount of cooking oil produced = 0.96(sum of the amounts of vegetable oils blended).

Given the product's popularity, all the cooking oil that is produced in a month can be sold in that month itself at a price of $290/vat.

It is required to formulate a 6-month plan that gives the amount of each vegetable oil to purchase each month, the amount of cooking oil to produce each month, and the amount of each vegetable oil to hold over in storage for the succeding month. Formulate this as an LP.

Solve the LP, and write a brief report explaing the output, and any useful planning information you can get from it.

4. Forming a committee: A school committee is forming an ad hoc committee to prepare a very important report on all aspects of the school district.

About the audience: The school committee chair is Barbara Firestone, CPA. Ms. Firestone is also the Controller of a major automotive supplier, AutoParts, Inc. She knows a lot about schools, finance, and accounting, but not a lot about engineering. She is familiar with the types of problems industrial engineers can solve, but she is not at all familiar with industrial engineering problem-solving methods. Ms. Firestone needs to be able to explain her choices of citizens to appoint to this ad hoc committee. As chair, she is authorized by the bylaws of the school committee to appoint citizens to an ad hoc committee. She does not need the approval of the other members of the school committee. Since this is a very important report and it fits her management style, she plans to seek a consensus on appointments to the ad hoc committee from school-committee members before actually making the appointments.

About you: You are a Staff Engineer with a very prestigious engineering consulting firm, IOE 310, Inc. You would like to be a Senior Engineer. IOE 310, Inc. has a reputation for rapidly promoting engineers who bring business into the company. You are new to the consulting business and want to build a reputation as an outstanding engineering consultant.

About the scenario: You learned that the school committee was forming the ad hoc committee from your cousin who is a secretary with the school district. As a concerned citizen you decided to volunteer your time to help the school committee. You set up an appointment and met with Ms. Firestone, School Committee Chair. She explained her problem to you and was very pleased to have your assistance.

During your meeting you had an opportunity to talk a little about business, and you learned that AutoParts, Inc. is having some minor problems with inventory control and in other areas in which you and your colleagues at IOE 310, Inc. have considerable expertise. The management team at AutoParts, Inc. is meeting next month for a quarterly budget review. Ms. Firestone told you that she is going to recommend funding for expert consulting services. You seemed to have stumbled upon a business opportunity here.

What you have accomplished so far: Ms. Firestone gave you a list of 42 different types of expertise (numbered 0 to 41) that she feels need to be represented on the ad hoc committee. You surveyed 33 people (numbered 0 to 32) who are willing to serve on the ad hoc committee and identified their areas of expertise. The data is given in the table: E is the number of expertise type, and cit. with E. are the people with that expertise.

Your technical assignment: Ms. Firestone feels that it is very important to keep the size of the committee small. Determine the smallest committee that contains all types of expertise.

Your communication assignment: You definitely have a moderately difficult report writing situation: you need to write a recommendation and you can not make any simplifying assumptions. Your task goal is to have your recommendation on the citizens to appoint to the ad hoc committee approved by Ms. Firestone and the other members of the school committee. Your social goal is to increase your credibility with Ms. Firestone. Her perception of your abilities as an engineer are sure to be greatly influenced by the quality of your report.

E.	Cit. with E.	E.	Cit. with E.	E.	Cit. with E.
0	2, 4, 7, 18, 22	1	8, 0, 6, 19, 32	2	7, 3, 6, 13, 25
3	4, 5, 14, 29	4	10, 18, 27	5	11, 12, 20, 23
6	7, 12, 14, 22	7	1, 16, 18, 28	8	13, 16, 17, 28
9	4, 12, 17, 18, 22	10	6, 14, 18, 19, 32	11	3, 13, 17, 25, 26
12	4, 5, 14, 29	13	16, 18, 27	14	11, 20, 23, 31
15	7, 12, 14, 22	16	16, 18, 28, 32	17	13, 17, 32
18	19, 26, 28, 30	19	15, 16, 18, 32	20	17, 22, 26, 28
21	2, 8, 12, 14, 17	22	2, 6, 9, 11, 28	23	3, 5, 23, 26, 27
24	4, 9, 14, 15	25	0, 8, 17	26	1, 10, 13, 19
27	2, 7, 12, 24	28	0, 8, 16, 18	29	6, 7, 13, 28
30	2, 8, 12, 17, 24	31	9, 26, 28, 31, 32	32	6, 7, 15, 23, 31
33	5, 14, 19, 24	34	7, 21, 28	35	3, 11, 30, 31
36	4, 18, 27, 32	37	8, 17, 25, 32	38	7, 23, 32
39	3, 28, 29, 30	40	5, 13, 18, 30	41	7, 26, 31, 32

5. Setting up Telemarketing centers: A company wants to set up **3 or 4 telemarketing centers.** They identified 7 different **sites** as being suitable locations for setting up these centers.

These centers are to serve customers in eight area codes (501, 918, 316, 417, 314, 816, 502, 606). Let:

$c_{ij} =$ Communication costs (in \$/hour) to the company, if customers from area code j are allocated to be served by a center set up at site i.

$d_i =$ Cost (in \$million/year) of operating a communication center at site i

Data on these cost coefficients is given below. Formulate a model to determine the best sites to set up the centers, and the allocation of area codes to the centers set up, so as to minimize the total annual operational and communication costs for serving all the area codes given above, subject to the constraints.

Solve the model. Explain the computer output and the optimum solution obtained.

Site i	d_i	c_{ij} for area code $j =$							
		501	918	316	417	314	816	502	606
1. DA	5	14	35	29	32	25	13	14	20
2. AT	8	18	18	22	18	26	23	12	15
3. LV	4	22	25	12	19	30	17	26	25
4. DE	9	24	30	19	14	12	16	18	30
5. LR	3	19	20	23	16	23	11	28	12
6. MP	4.5	23	21	17	21	20	23	20	10
7. SL	5.5	17	18	12	10	19	22	16	22

DA = Dallas, TX; AT = Atlanta, GA; LV = Louisville, KY;
DE = Denver, CO; LR = Little Rock, AR; MP = Memphis, TN
SL = St. Louis, MO

Aim of Computer Projects in Our LP Courses

Katta G. Murty, IOE Dept. U. of Michigan, Ann Arbor.

1. To make sure that our students can model & solve at least well structured problems using software systems in common use.

2. To make sure that they derive all useful planning information from model, such as:

   - Infeasibility analysis if model infeasible, ways to make it feasible
   - Opt. Sol. & check for its reasonableness
   - Slack values at Opt., critical resources, slack resources
   - When is opt. dual sol. the marginal value (MV) vector for RHS constants
   - Derive planning info. by marginal analysis; for example:

     Making one unit of a new product needs:
     3 units resource 1 (MV of \$10 net profit/unit)
     5 units resource 2 (MV \$15/unit)
     7 units resource 3 (MV \$20/unit)
     Then this product worth making if its net profit potential $\geq 3 \times 10 + 5 \times 15 + 7 \times 20 = \$215$/unit.

3. To give them practice in preparing good reports interpreting the output from the model

An example Project Problem

Optimization of crude oil refining & blending.
3 suppliers of crude with following data:

	Availability B/day	Price \$/B
Crude 1	60,000	15.25
2	90,000	13.00
3	80,000	14.00

Distillation outputs at fractionater (IP = Interm. Product)

	Yield of IP (B/B)			
	DN	DHO	DGO	P
Crude 1	0.19	0.27	0.38	0.05
2	0.16	0.32	0.27	0.13
3	0.02	0.24	0.26	0.39

Processing cost \$0.6/B of crude
Distillation capacity = 110,000 B/day

2nd stage processing of some IPs in a CC (Catalytic cracker). Only DHO, DGO processed here. N = normal level, H = high level. Capacity: either 100,000 B of DHO/day, or 50,000 B of DGO/day or any convex combination.

	Yield at CC (B/B)		
	CN	CHO	P
DGO at N	0.48	0.52	0.15
DHO at N	0.18	0.8	0.11
DHO at H	0.32	0.69	0.1

Processing cost \$0.95/B at N, \$1.5/B at H

Blending IPs into final products

DHO	$\rightarrow$ Heating oil. Sells for \$39.75/B. Daily demand $\leq 40,000$B
DGO	$\rightarrow$ Diesel fuel. \$39/B. No limit on demand
P + CHO (17 : 2 mix)	$\rightarrow$ Heavy fuel oil. \$30/B. No limit
DN + CN (20 : α where $\alpha \geq$ 17 mix)	$\rightarrow$ Gasoline. Quality $\uparrow$ as $\alpha \uparrow$. \$43.25/B. Demand $\leq 40,000$ B

Need determine complete daily production plan to max daily net profit.
We encourage use of AMPL in our classes.
After obtaining opt. primal & dual sols. students analyze to answer following type questions.

Which crude supply to increase, & how much can we pay per B & still break even.

What is the benefit from expanding capacity at fractionater & the CC?

What is the benefit of increasing demand for gasoline & heating oil?

etc.

Many nice computer project problems with data can be found in following books:

K. G. Murty, *Operations Research: Deterministic Optimization Models*, Prentice Hall, 1995.

K. McAloon & C. Tretkoff, *Optimization and Computational Loogic*, Wiley-Interscience, 1996.

Aim of Computer Projects in Our IP & Combinatorial optimization Courses

How to model the problem so that a good solution can be obtained within reasonable time using a combination of IP & heuristic approaches.

An example Project Problem

Planning for the military training of US Army National Guard forces (ANG) using mobile combat simulators.

ANG forces reside in 368 towns & cities spread all over the country, each of these places is called an **Armory**.

ANG forces grouped into **platoons** each with 16 soldiers. Each platoon is allotted 4 tanks. Mobile trainer is a platoon level trainer, it trains a whole platoon at a time using 4 tanks.

21 mobile trainers to be fielded. Army identified 29 sites in whole country (these sites to be called **HQ sites**) which are suitable to serve as **home bases** for these mobile trainers. At each home base, ≥ 1 mobile trainers can be stationed.

ANG training takes place only during weekends. **Constraint;** Each unit must receive training at a training center ≤ 100 miles from their armory.

Home bases for mobile trainers can serve as training centers. Besides these, a set of **Secondary training sites (STS)** to be selected from among the 368 armories + HQ sites not selected as home bases, to serve as training centers. The mobile trainers will travel from their home bases to cover the STS and give training to units visiting them for training.

Each armory has 3 platoons to be trained; all of them should be allocated to the same training center. A mobile trainer can train upto 3 platoons during a weekend.

Armories classified into 3 classes. Units in class 1 need to train 4 weekends/year. Those in class 2 need to train twice/year. Those on class 3 need to train once/year. For each unit, time gap between repeat training sessions should be ≥ 8 weeks.

Other input data: The $(368 + 29) \times (368 + 29)$ Euclidean distance matrix giving the distance between every pair of locations among armories & HQ sites.

Decisions to be made:

Home base selection: Select home bases for each of the 21 mobile trainers from among the 29 HQ sites. A **Location problem**.

STS Selection: Select as many STS as necessary from armories & remaining HQ sites, so that each armory is within 100 miles of at least one home base or STS. A **Selection Problem¿**

Allocate units to home base or STS: Allocate each armory to a home base or STS within 100 miles of it, for training. An **Allocation Problem**.

Routing mobile trainers: Develop routes for mobile trainers to cover all STS. A **Partitioning** & **Routing Problem**.

Schedule training sessions: Develop a time schedule over the weekends of the year, for each unit to receive as many training sessions as they are supposed to. A **Scheduling Problem**.

Objectives to optimize:

OBJ1: Min total annual mobile trainer fleet mileage.
vspace0.2in **OBJ2:** Min platoon-bus mileage

OBJ3: Min no. of STS.

Large scale multiobjective problem involving many of the different types of combinatorial optimization problems, within itself.

You can read the report on this problem and its solution in our paper:

Katta G. Murty, Philipp A. Djang, "The US Army National Guard's Mobile Training Simulators Location and Routing Problem" *Operations Research*, 47, no. 2 (March-April 1999)175-182.