How to Build a Mathematical Model?

Katta G. Murty Lecture slides

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1. Identify All Decision Variables

Controllable parameters whose values can be controlled by decision maker, which affect functioning of system. Denote them by $x_1, \ldots, x_n$.

$$x = \quad \text{Decision vector} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \quad (x_1, \ldots, x_n)^T$$

2. Identify Objective Function and All Constraints on Decision Variables

$$g_i(x) = b_i \quad \text{An Equality Constraint}$$

$$\left. \begin{array}{l} g_i(x) \geq b_i \\ g_i(x) \leq b_i \end{array} \right\} \quad \text{Inequality Constraints}$$

Constraint Functions, Right Hand Side (RHS) Constants.

$$x_j \geq b_j \text{ or } x_j \leq b_j \quad \text{Lower or Upper bound constraints on individual variables.}$$

$$x_j \geq 0 \quad \text{Lower bound constraint called } \textit{Non-negativity restriction.}$$

$$\text{Objective function called} \begin{cases} \textit{COST FUNCTION} & \text{if to be min.} \\ \textit{PROFIT FUNCTION} & \text{if to be max.} \end{cases}$$

Some Definitions

LINEAR FUNCTION: One of form $\quad c_1 x_1 + \ldots + c_n x_n$
where $\quad c = (c_1, \ldots, c_n) \quad$ is *coefficient vector* of variables in it.
Example: $x \in R^4$. $\quad 3x_2 - 7x_4$ is a linear function with coefficient vector $\quad (0, 3, 0, -7)$.

AFFINE FUNCTION: A linear function + a constant, i.e., One of form $\quad c_0 + cx$.

FEASIBLE SOLUTION: A vector x that satisfies all the constraints.

OPTIMUM SOLUTION: A feasible solution that gives the best value for objective function among all feasible solutions.

LINEAR PROGRAM: Optimization problem in which objective
function and all constraint functions are linear.

Steps in Modeling A Linear Program

1. LIST ALL DECISION VARIABLES: Each decision variable is the level at

   which an *ACTIVITY* is carried out.

2. VERIFY LINEARITY ASSUMPTIONS: *Proportionality Assumption*

   and *Additivity Assumption.* Must hold for

   objective function and all constraint functions.

3. VERIFY ALL VARIABLES ARE CONTINUOUS VARI-ABLES:

4. CONSTRUCT OBJECTIVE FUNCTION:

5. IDENTIFY ALL CONSTRAINTS & BOUNDS ON INDI-VIDUAL VARIABLES: Each

   constraint is *Material Balance Equation* or *Inequality*

   of an *ITEM.*