2.1

Goals for NLP algorithms?

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Can we find Global Minimum efficiently: Can check whether a QP is convex QP efficiently.

For convex QPs have very efficient algos. for finding global minima.

For convex NLPs, we have efficient algos. to compute global minima through computing stationary points.

However checking whether a general NLP model is a convex program is hard. Also, most real world NLP models tend to be nonconvex.

Finding a global minimum in a nonconvex program (even a nonconvex QP) is hard. In these, no good characterizations to check if local min is global min.

The area Global Minimization is very active. But so far no reliable and efficient methods for a nonconvex program.

For Nonconvex programs can we find a local minimum efficiently?

Even this hard. Also checking whether a given feasible solution is local minimum for nonconvex program is hard.

Murty & Kabadi ["Some NP-Complete Problems in Quadratic and Nonlinear Programming" , *Mathematical Programming*, 39(1987)117-129] show checking whether 0 is a local min for the simple QP

$$\text{Minimize} \quad x^T D x$$

$$\text{s. to} \quad x \geq 0$$

is NP-Hard when D is not a PSD matrix.

What type of solutions can we get for nonconvex programs?

NLP algorithms usually iterative methods generating sequence of points $x^0, x^1, x^2, \ldots$

Most are **Descent Algorithms**, i.e., they generate a descent sequence (either objective function, or an infeasibility measure, or

a combination of both, strictly decreases along the sequence).

At the moment, the best that we can hope for a nonconvex program is a descent sequence that is guaranteed to converge to a stationary point.