6.1

Some More Properties of Convex Functions

Katta G. Murty, IOE 611 Lecture slides

THEOREM: For convex function min on convex set, every local minimum is global min.

THEOREM: For convex function min on convex set, set of alternate opt is convex set.

THEOREM: Consider: min $f(x)$ over $x \in K$ where $f(x)$ is a differentiable convex function, and K convex set.

If $\bar{x} \in K$ optimum solution of: min $\nabla f(\bar{x})x$ over $x \in K$ then $\bar{x}$ is an optimum solution of original problem.

Subgradients and Subdifferential sets: The vector d is *subgradient* for $f(x)$ at $\bar{x}$ if

$$f(x) \geq f(\bar{x}) + d^T(x - \bar{x}) \quad \forall x$$

For differentiable convex function, its gradient vector $\nabla f(\bar{x})^T$ is only subgradient at $\bar{x}$.

If convex function not differentiable at $\bar{x}$ any of its directional derivatives at $\bar{x}$ is subgradient vector for it at $\bar{x}$.

The **subdifferential set** for $f(x)$ at $\bar{x}$, $\partial f(\bar{x})$ is set of all subgradients of $f(x)$ at $\bar{x}$.

The subdifferential set of a convex function at any point is always a convex set.

THEOREM: If $f(x)$ is a differentiable convex function, $(\nabla f(x^1) - \nabla f(x^2))(x^1 - x^2) \geq 0 \ \forall x^1, x^2$.

THEOREM: Consider $\max f(x)$ over $x \in K$, $f(x)$ convex, K closed convex set. If optimum exists, a boundary point of K is optimum.

THEOREM: If $f(x)$ convex attains its max on K convex polyhedron with some extreme points, then this max attained at an extreme point of K.

Generalizations of Convexity of Functions: Quasiconvexity, Pseudoconvexity.