1.1

History of Network Models

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Goes back to L. Euler's work on the Konigsberg Bridges Problem (1736).

1800's	Gustav Kirchoff, pioneer of electricity and mechanics constructed network flow models to analyze current flows.
1955	T E Harris & General F S Ross of US Army, to determine extent to which East European rail network can support war effort of Soviet Union if they attack West Europe, posed problem of finding maximum flow from one point to another in a capacitated network, to L R Ford, D R Fulkerson of RAND.
1956	F & F prove *max flow min cut theorem* & develop labeling algo. for max flow. Beginning of algorithmic developments.

ROUTING MODELS

An early problem considered with applications for routing postmen, street sweepers, school buses, trash collection vehicles etc. is **Edge covering route problem**.

Called **Chinese Postman problem** after **Guan Mei Go** who published it in 1962 "A mailman has to cover his beat before returning to post office. Find shortest walking route for mailman". Efficiently solvable by the **Blossom algorithm for minimum cost perfect matching** in $O(n^3)$ time.

Node covering route problem is to find shortest route that goes thro' all nodes in a given connected network. In the family of **TSP (traveling salesman problem)**. No polynomial time algo. known for these, but **branch & bound** can handle moderate sized problems reasonably efficiently.

Hamiltonian tour in a graph is a is a simple cycle that passes through each node exactly once. Check whether following **Petersen's graph** has a Hamiltonian tour.

Vehicle routing Problems: Several customers in a region need to be visited for servicing. Fleet of vehicles available.

(i) Partition customers into subsets, each subset to be serviced by one vehicle.

(ii) Develop a route for each vehicle to visit the customers in its assigned subset.

There may be vehicle capacity constraints, route length constraints etc. Objective to minimize either total no. of vehicles used, or total mileage of all vehicles. Delivery companies face this problem daily.

Network model for a typical Production-Distribution problem

Chair-maker's problem: Raw material wood.

2 suppliers S_1, S_2 who supply wood to plants

3 plants P_1, P_2, P_3 who transform wood into chairs & ship them to wholesalers

3 wholesalers W_1, W_2, W_3 who sell the chairs

Plant	Prod. cost $/chair	Prod. capacity chairs/day	L. B. for chairs made/day
1	8	1200	0
2	4	600	200
3	5	450	300

Supplier of wood	Ship. cost ($/lb) to plant			Min. to be purchased from supplier	Price ($/lb)
	1	2	3		
1	.02	.03	.04	8 tons	.10
2	.05	.03	.03	10 tons	.075

Plant	Cost of shipping ($/chair) to wholesaler		
	1	2	3
1	2	1	1
2	1.5	2	1
3	1	1.5	2
Sell. price ($/chair)	25	20	22
Max. chairs wanted/day	2100	1600	1700
Min. chairs wanted/day	500	400	300

To maximize net profit = Sales revenue from selling chairs − cost of wood.

Whole problem can be modeled as a min cost flow problem in a network. First convert and measure all flows in common units.

1 chair needs 20 lbs. wood on average. So measure wood in chair-units (1 chair-unit = 20 lbs., so 1 ton = 100 chair units) and measure all flows in chair-units.