Glossary

$(\mathcal{N}, \mathcal{A}, \ell, k, c, \breve{s}, \breve{t}, \bar{v}$ or $V)$ denotes a network with all the data. See below for the meanings of these symbols.

Bold face capital letters such as **X, Y, A, M, E,** Γ, Δ, etc. denote sets. In citing references, we give the authors' names in capital letters, year of publication, title of the article or book, journal name or its abbreviation as defined below, volume, number in the volume if available, and inclusive pages within parentheses, in that order. Figures, equations, exercises, tableaus, arrays, comments, etc. are numbered serially within each chapter. So, for each of these, the entity $i.j$ refers to the jth entity in Chapter i.

The **size** of an optimization problem is a parameter that measures how large the problem is. Usually it is the number of digits in the data in the problem when it is encoded in binary form. When n is some measure of how large a problem is (either the size, or some quantity which determines the number of data elements in the problem), a finitely terminating algorithm for solving it is said to be of order $n^{\mathbf{r}}$ or $\mathrm{O}(n^{\mathbf{r}})$, if the worst case computational effort required by the algorithm grows as $\alpha n^{\mathbf{r}}$, where α is a number that is independent of the size and the data in the problem. The n and m that appear in the computational complexity measures for algorithms for network problems are usually the number of nodes and lines in the network. An algorithm is said to be **polynomially bounded** if the computational effort required by it to solve an instance of the problem is bounded above by a fixed polynomial in the size of the problem. **P** is the class of all problems for which there exists a polynomially bounded algorithm. **NP-complete** is a class of decision problems in discrete optimization, satisfying the property that if a polynomially bounded algorithm exists for one problem in the class, then polynomially bounded algorithms exist for every problem in the class. So far no polynomially bounded algorithm is known for any problem in the NP-complete class, and it is believed that all these problems are hard problems (in the worst case, the computational effort required for solving an instance of any of these problems by any known algorithm grows faster, asymptotically, than any polynomial in the size of the instance). See Garey and Johnson [1980 of Chapter 1] for definitions of NP-complete and NP-hard classes of problems.

wrt	With respect to.
iff	If and only if.
$\mathcal{N}$	The finite set of nodes in a network.
$\mathcal{A}$	The set of lines (arcs or edges) in a network.
$G = (\mathcal{N}, \mathcal{A})$	A network with node set $\mathcal{N}$ and line set $\mathcal{A}$.
(i, j)	An arc joining node i to node j.
$(i; j)$	An edge joining nodes i and j.
Head(e), tail(e)	The head and tail nodes on an arc e.
$(i, j)_t, (i; j)_t$	The t-th arc (edge) among a set of parallel arcs (edges) joining i to j(i and j).
e, e_t	An arc or an edge in a network.
$\mathbf{e}$	The column vector of all 1s in $\mathbb{R}^n$.
ℓ_{ij}, ℓ	The lower bound for flow on arc (i, j). ℓ is the vector of ℓ_{ij}.
k_{ij}, k	The upper bound or capacity for flow on arc (i, j). k is the vector of k_{ij}.
κ_{ij}, κ	The residual (or remaining) capacity for flow on arc (i, j). κ is the vector of κ_{ij}.
ϵ	Usually, the residual capacity of an FAP or FAC.
f_{ij}, f_t, f	The flow amount on arc (i, j) (or arc e_t) in a node-arc flow vector f.
c_{ij}, c_t, c	The unit cost coefficient or length or weight of arc (i, j) or edge $(i; j)$ (or line e_t). c is the vector of c_{ij}.

p_{ij}, p	The multiplier associated with arc (i, j) in a generalized network. p is the vector of p_{ij}.
$\breve{s}, \breve{t}$	The source and sink nodes in a network.
V_i, V	The exogenous flow amount at node i. V is the vector of V_i.
v	Value of a flow vector in Chapters 2, 5; or a vector of dual variables associated with columns of a transportation array in Chapter 3, or a node in Chapter 10.
u	A vector of dual variables associated with rows of a transportation array in Chapter 3, or a node in other chapters.
π_i, π	The dual variable or node price associated with node i, π is the vector of π_i.
μ_σ, μ	The dual variable associated with the σth blossom constraint in Chapter 10. μ is the vector of μ_σ.
$\overline{c}_{ij}, \overline{c}$	The reduced or relative cost coefficient on arc (i, j),wrt a node price vector π, it is $c_{ij} - (\pi_j - \pi_i)$. $\overline{c}$ is the vector of $\overline{c}_{ij}$.
$\mathbf{X}$	A set of nodes, e.g., the labeled nodes, or those defining the partition for a cut, etc.
$\overline{\mathbf{X}}$	$\mathcal{N} \backslash \mathbf{X}$
$[\mathbf{X}, \overline{\mathbf{X}}]$	A cut in a directed network.
$(\mathbf{X}, \overline{\mathbf{X}})$	Forward arcs of the cut $[\mathbf{X}, \overline{\mathbf{X}}]$ in a directed network.
$(\overline{\mathbf{X}}, \mathbf{X})$	Reverse arcs of the cut $[\mathbf{X}, \overline{\mathbf{X}}]$ in a directed network.
$(\mathbf{X}; \overline{\mathbf{X}})$	A cut in an undirected network.
$\mathbb{T}$	A tree.
$\mathrm{P}(i), \mathrm{S}(i)$	The predecessor and successor indices of node i.
$\mathrm{EB}(i), \mathrm{YB}(i)$	The elder brother and younger brother indices of node i.

$\mathbf{H}(\mathbb{T}, j)$	The family of a node j in the rooted tree $\mathbb{T}$, the set consisting of node j and all its descendents in $\mathbb{T}$.
$(\mathcal{N}_1, \mathcal{N}_2; \mathcal{A})$	A bipartite network with node bipartition $(\mathcal{N}_1, \mathcal{N}_2)$.
$\mathcal{P}$	A path.
$\mathcal{C}$	A chain.
$\mathbb{C}$	A cycle.
$\vec{\mathbb{C}}$	A circuit.
$\mathbf{A}_i$	After i, set of head nodes on arcs incident out of i.
$\mathbf{B}_i$	Before i, set of tail nodes on arcs incident into i.
$\mathrm{G}(f)$	Residual network of G wrt bound feasible flow f.
$\mathcal{A}(f)$	Set of residual arcs wrt a bound feasible flow f.
$\mathrm{AN}(f) = (\mathrm{N}(f), \mathrm{A}(f))$	Auxiliary network wrt feasible flow f. $\mathrm{N}(f)$, $\mathrm{A}(f)$ are nodes, arcs in it.
$\mathrm{A}^+(f), \mathrm{A}^-(f)$	The $+$, $-$ labeled arcs in $\mathrm{A}(f)$ in an auxiliary network.
$\mathcal{L}$	A layered network.
$\mathrm{A}_r^+(f), \mathrm{A}_r^-(f)$	The sets of $+$, $-$ labeled arcs in the rth layer in the layered network wrt f.
$f(\mathbf{X}, \mathbf{Y}), \ell(\mathbf{X}, \mathbf{Y}), k(\mathbf{X}, \mathbf{Y})$	Sum of f_{ij}, ℓ_{ij}, k_{ij} respectively, over arcs (i, j) with $i \in \mathbf{X}, j \in \mathbf{Y}$.
n, m	Usually, number of nodes, lines respectively in a network.
R_i, C_j	Row i, column j of an array, or the associated nodes.
$\underline{\tau}_{ij}, \overline{\tau}_{ij}$	The crash and normal durations for a job (i, j) in a project network.
$\mathbf{M}$	A matching.
$B = (\mathcal{N}_B, \mathcal{A}_B)$	A blossom defined by the subset of nodes $\mathcal{N}_B$ in Chapter 10. $\mathcal{A}_B$ is the set of edges in it.

$\mathcal{A}_*(\pi,\mu)$	Set of equality edges wrt the dual solution (π,μ) in Chapter 10.
$\mathrm{G}_*(\pi,\mu)$	The equality subnetwork wrt the dual solution (π,μ) in Chapter 10.
$\mathrm{G}^1 = (\mathcal{N}^1, \mathcal{A}^1)$	The current network in Chapter 10. $\mathcal{N}^1, \mathcal{A}^1$ are the sets of current nodes, and edges.
$\mathcal{A}^1_*(\pi,\mu)$	Set of current equality edges wrt the dual solution (π,μ) in Chapter 10.
$\mathrm{G}^1_*(\pi,\mu)$	Current equality subnetwork wrt the dual solution (π,μ) in Chapter 10.
$d_{ij}(\pi,\mu)$	A quantity used in defining the duals of matching, edge covering problems in Chapter 10. Its definition depends on the type of the primal problem.
$\mathbf{Y}^-(x), \mathbf{Y}^+(x)$	Quantities associated with a solution vector x and an odd set of nodes $\mathbf{Y}$, in matching, edge covering problems, used in defining the blossom constraint corresponding to $\mathbf{Y}$ in that problem in Chapter 10.
MB, CB	The index sets corresponding to matching and covering blossom constraints in a 1M/EC problem in Chapter 10.
$(\mathcal{N}^{\overset{\leq}{=}}, \mathcal{N}^{=}, \mathcal{N}^{\overset{\geq}{=}}, \mathcal{N}^0)$	Subsets of nodes in a partition of $\mathcal{N}$ in a 1-M/EC problem.
AP	A path in which edges are alternately matching, nonmatching edges in Chapter 10.
FAC	Flow augmenting chain.
FAP	Flow augmenting path.
ECR	Edge covering route.
LP	Linear program.
BFS	Basic feasible solution.
POS	Partially ordered set.

$\succ$	The relationship between elements in a POS.
▮	Symbol indicating end of a proof.
s. t.	Such that.
c.s. conditions	The complementary slackness optimality conditions.
Parent(e), son(e)	The parent and son nodes on an in-tree line e in a rooted tree.
$\mathbb{R}^n$	Real Euclidean n-dimentional vector space.
$\lvert\alpha\rvert$	Absolute value of real number α.
$\lvert\mathbf{F}\rvert$	Cardinality of the set $\mathbf{F}$.
$\lceil\alpha\rceil$	Ceiling of real number α, smallest integer $\geqq \alpha$. e.g., $\lceil -4.3\rceil = -4$, $\lceil 4.3\rceil = 5$.
$\lfloor\alpha\rfloor$	Floor of real number α, largest integer $\leqq \alpha$. e.g., $\lfloor -4.3\rfloor = -5$, $\lfloor 4.3\rfloor = 4$.
$n^{\mathbf{r}}$	n to the power of $\mathbf{r}$. Exponents are set in this type style to distinguish them from ordinary superscripts.
$\mathrm{O}(n^{\mathbf{r}})$	A positive valued function $g(n)$ of the nonnegative variable n is said to be $\mathrm{O}(n^{\mathbf{r}})$ if there exists a constant α s. t. $g(n) \leqq \alpha n^{\mathbf{r}}$ for all $n \geqq 0$. For meaning in the context of computational complexity see the beginning of this glossary.
$A_{i.}$	The ith row vector of the matrix A.
$A_{.j}$	The jth column vector of the matrix A.
$n!$	n factorial.
$\lvert\lvert x\rvert\rvert$	Euclidean norm of the vector x. For $x = (x_1, \ldots, x_n)$ it is $+\sqrt{x_1^{\mathbf{2}} + \ldots + x_n^{\mathbf{2}}}$.
∞	Infinity.
$\in$	Set inclusion symbol. $a \in \mathbf{D}$ means that a is an element of $\mathbf{D}$. $b \notin \mathbf{D}$ means that b is not an element of $\mathbf{D}$.
$\subset$	Subset symbol. $\mathbf{E} \subset \mathbf{F}$ means that set $\mathbf{E}$ is a subset of $\mathbf{F}$, i.e., every element of $\mathbf{E}$ is an element of $\mathbf{F}$.

$\cup$	Set union symbol.
$\cap$	Set intersection symbol.
$\emptyset$	The empty set.
$\setminus$	Set difference symbol. $\mathbf{D} \setminus \mathbf{H}$ is the set of all elements of $\mathbf{D}$ that are not in $\mathbf{H}$.
$\geqq$	Greater than or equal to.
$\leqq$	Less than or equal to.
$\sum$	Summation symbol.
$\sum(x_j : \text{over } j \in \mathbf{J})$	Sum of x_j over j from the set $\mathbf{J}$.

Abbreviations for Journal Names

AOR	*Annals of Operations Research.*
BAMS	*Bulletin of the American Mathematical Society.*
CACM	*Communications of the Association for Computing Machinery.*
COR	*Computers and Operations Research.*
DAM	*Discrete Applied Mathematics.*
EJOR	*European Journal of Operations Research.*
IPL	*Information Processing Letters.*
JACM	*Journal of the Association for Computing Machinery.*
JORS	*Journal of the Operational Research Society.*
MOR	*Mathematics of Operations Research.*
MP	*Mathematical Programming .*
MPS	*Mathematical Programming Study.*
MS	*Management Science.*
NRLQ	*Naval Research Logistics Quarterly.*
OR	*Operations Research.*
ORQ	*Operations Research Quarterly.*
QAM	*Quarterly of Applied Mathematics.*
TS	*Transportation Science.*
OR Letters	*Operations Research Letters.*