Monitoring Production Allocations to Plants

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1 Introduction

The problem addressed is the following. A company:

> Has manufacturing plants at k different sites. The index $t = 1$ to k denotes the various manufacturing plants.
>
> Manufactures n different end products called $P_1, \ldots, P_n$. The index $j = 1$ to n is used to denote the various products.
>
> Markets the products in Z different regions or zones. The index $z = 1$ to Z denotes the various zones.
>
> Uses m different input materials or resources to manufacture the products. The index $i = 1$ to m denotes these various input materials.

It is assumed that production planning is carried out on a period by period basis. A period can be any time interval like a week, month, quarter, etc., but we will describe this in terms of a "**month**" which is the most commonly used planning period.

A **production plan for a month** specifies the following: in that month, how much quantity of each product is manufactured at each plant; and how much of the output of various products manufactured at each plant is shipped to the various marketing zones. The month for which a production plan is being worked out is called the **planning month**.

Now-a-days most of the companies "**make to order**" rather than "**make to stock**"; i.e., the company receives orders for the various products from the customers (who may be individuals, wholesalers, retailers, dealers, etc.) according to an agreed upon delivery schedule, and manufactures in each month sufficient quantities to cover that month's delivery agreements. In this mode of operation, production planning is normally carried out in the following way. Several months before the planning month, a tentative production plan for that month is prepared for that month based on forecasted demands for the various products in the various zones. Then the data generated from the arriving customer orders is used to revise the production plan until the beginning of the planning month at which time the version of the production plan is finalized.

Here we describe the procedures that can be used to make the initial (tentative) production plan, and to revise it using the customer order data. We describe the data needed in Section 2. In Section 3 we descrie a mathematical model (actually a linear program) whose solution is the production plan. In Section 4 we describe the procedures that can be used to revise the input data based on customer order information, for obtaining a revised production plan.

2 The Data Needed for the Models

Plant-Product Incidence Set: This data is in the form of a set called PPI (Plant-Product Incidence Set) = $\{(t, j)$: Product j can be manufactured in the plant at site $t\}$.

BOM (Bill Of Materials) Data:

$a_{ij} =$ units of ith input material needed to make one unit of end product j; $i = 1$ to m, $j = 1$ to n.

$A = (a_{ij}) =$ the $m \times n$ input-output matrix.

$A_{.j} =$ $(a_{1j}, \ldots, a_{mj})^T$, the jth column vector of the input-output matrix A.

$A_{.j}$ is called the BOM for the jth end product, it is assumed to be the same at all the plants where end product j can be manufactured.

Production Capacity Data: For each end product, there is a finite production capacity at each plant where it can be manufactured.

$PC_{tj} =$ production capacity (in units per month) in the plant at site t to manufacture end product j, for all $(t, j) \in$ PPI.

Input Material Supply Data: Supply contracts for input materials are negotiated with suppliers, at regular prices. Later on, if quantities beyond a certain level are needed due to changes in demand, the price per unit will be higher as the supplier has to produce it in overtime, etc.

$s_{it} =$ maximum number of units of input material i available at site t at the regular price of \$ c_{it}/unit, in planning month.

$S_{it} =$ maximum additional units of input material i available at site t, beyond regular supply, at higher price of \$ C_{it}/unit, in planning month.

We assume that other production costs are the same at all the sites, so we ignore them in the model.

Plant-Zone Marketing Data: This data is a set called PZM (Plant-Zone Marketing Set) = $\{(t, z)$: Plant at site t can ship to market zone $z\}$.

Plant-zone pairs which are too far apart, or the transit between which requires high lead times due to the presence of things like customs barriers, or due to lack of good transportation channels, etc. are not included in the PZM set.

Plant-Zone Shipping Cost Data:

$\gamma_{tz,j} =$ \$/unit to ship one unit of product j from the plant at site t to zone z, for all $(t, z) \in$ PZM, and $j = 1$ to n.

Zone-Product Demand Data:

$D_{zj} =$ amount of product j to be made available in planning month for shipping to zone z, for $z = 1$ to Z, and $j = 1$ to n.

How to Handle Uncertainty in Demand, Demand Inflation Factor

Let

$d_{zj} =$ forecast of expected demand in zone z for product j during the planning month.

The actual demand is a random variable whose exact value will not be known until the planning month is over.

If we make the amount produced D_{zj} = the forecast of expected demand d_{zj}, then if the actual demand is higher, the company will incur lost sales and the consequent lost profit making opportunity. If the actual demand is less than D_{zj}, the company can carry over the excess product for sale into the next month with a slight inventory carry-over cost. That's why most of the time companies prefer to take D_{zj} higher than the forecasted demand, particularly while under a climate of growing demand.

The optimal value for D_{zj} can be determined through statistical inference if information on the variance of the demand is available. Unfortunately, in these days of ever changing markets, getting reliable demand variance estimates is very hard.

Under these circumstances many companies are leaning towards taking

$$D_{zj} = (1 + \alpha)d_{zj}$$

where α is a positive **demand inflation factor** in a growth climate.

This demand inflation factor α is an input parameter whose value has to be set according to the company's desire. A popular choice for the value of α in a growth or stable climate is between 0.25 to 0.3. In fact, a simulation can be run with different values of α to help the company determine a desirable value for it.

In the same way, under a declining demand climate, slightly negative values for α can be considered.

Estimate of Leftover Stock of Products at Plants:

$ST_{tj} =$ estimate of quantity of product j that will be in stock in the plant at site t at the beginning of the planning month, for all $(t, j) \in$ PPI.

3 Model to Determine Production Allocations to Plants

The decision variables in the model are the following.

$x_{tj} =$ units of product j to be manufactured in the plant at site t during the planning month, for $(t, j) \in$ PPI.

$y_{tz,j} =$ units of product j shipped from the plant at site t to zone z in planning month, for $(t, z) \in$ PZM, $j = 1$ to n.

Then the model for determining the optimum allocations is

Min. $\sum_{i=1}^{m} \sum_{t=1}^{k} [\sum\{(a_{ij}x_{tj} - u_{it}^{-})c_{it} + u_{it}^{-}C_{it} : \text{over } j \text{ s. th. } (t,j) \in \text{PPI}\}]$
$+ \sum_{(t,z)\in\text{PZM}} \sum_{j=1}^{n} y_{tz,j}\gamma_{tz,j}$

s. to $STtj + x_{tj} \leq \sum\{y_{tz,j} : \text{over } t \text{ s. th. } (t,z) \in \text{PZM}\}$ for $(t,j) \in \text{PPI}$

$\sum\{y_{tz,j} : \text{over } t \text{ s. th. } (t,z) \in \text{PZM}\} \geq D_{zj}$ for all z, j

$\sum\{a_{ij}x_{tj} : \text{over } j \text{ s. th. } (t,j) \in \text{PPI}\} + u_{it}^{+} - u_{it}^{-} = s_{it}$ for all i, t

$0 \leq x_{tj} \leq PC_{tj}$ for $(t,j) \in \text{PPI}$

$0 \leq u_{it}^{-} \leq S_{it}$ for all i, t

all $y_{tz,j}, u_{it}^{+} \geq 0$.

This is a standard linear programming model for production allocation. The objective function consists of the costs of input material supplies and end product shipping costs. The model can be extended to include other relevent cost terms and other constraints as the application demands.

The demand for each product j for the planning period in each marketing zone is forecasted usually at least a month or two before it. Using the forecasts as d_{zj}, the D_{zj} are obtained as discussed in Section 2 with an inflation factor, and the tentative production plan prepared by solving the model with this data.

What to Do if This Model is Infeasible

If this model is infeasible, in some zones, the demand for some end products during the planning month exceeds the production capacity of the plants who can ship those products to those zones. In this case the final Phase I solution can be used to identify those zones and the end products which will be in short supply there. It can also be used to indicate the types of minimal changes in the data for the planning month to make the model feasible. The revised model can then be solved, and its optimum solution used for making the production plans.

4 Revising the Production Plan Based on Customer Order Information

Customer orders for end products for delivery during the planning month keep arriving daily in the central office. These order quantities are tabulated for each zone separately, and for each end product.

Also arriving daily are cancellations of some orders placed earlier. These order cancellations are also tabulated for each end product and each zone separately. So, the net order quantity for delivery during the planning month, for any end product in any zone, at the end of a day is:

> (net order quantity at the end of previous day) + (quantity of orders received during the day) − (quantity of cancellation of earlier orders received during the day).

Forecasting the Final Order Quantity

The following analysis has to be carried out for each product, in each marketing zone separatey, using the customer order data pertaining to it.

We would like to forecast the final order quantity from this zone for this product for delivery in the planning month, which is in the future. Suppose there are T more days including today, on which customers can place orders for delivery during the planning month. Let

$q =$	net order quantity at the beginning of today
$\xi_{T-r} =$	unknown quantity of cstomer orders received on the rth day from today ($r = 0$ is today), $r = 0$ to $T - 1$
$\eta_{T-r} =$	unknown quantity of cancellations of earlier orders received on the rth day from today, $r = 0$ to $T - 1$.

Today, these ξ_{T-r}, η_{T-r} are unknown random variables. The final order quantity for the planning month will then be

$$q + (\xi_T + \xi_{T-1} + \ldots + \xi_1) - (\eta_T + \eta_{T-1} + \ldots + \eta_1).$$

For forecasting, we need to estimate the expected value of this quantity which is itself a random variable.

If orders for delivery during the planning month are being placed for several days already, then the data for those days constitutes past data on these ξ_ℓ, η_ℓ-variables. From an analysis of this data, the stochastic nature of these ξ_ℓ, η_ℓ variables can be deduced. The forecasting technique that is appropriate to use depends on how the expected values of these ξ_ℓ, η_ℓ variables behave (stable over time, increasing, decreasing, increasing-decreasing, etc.) as we approach the planning month. This behavior is largely determined by the type of end product, customer habits, etc.

We denote the expected values of these ξ_ℓ, η_ℓ variables by $E(\xi_\ell), E(\eta_\ell)$.

If it is Reasonable to Assme that $E(\xi_\ell), E(\eta_\ell)$ Are Independent of ℓ

In this case $E(\xi_\ell), E(\eta_\ell)$ can be estimated by something like the moving average method (take the estimates to be the avarage of the values of these variables over the most recent s days, for some $s \geq 2$). If these estimates for $E(\xi_\ell), E(\eta_\ell)$ are a, b, then

$$\text{forecast of final order quantity} = \max\{0, q + aT - bT\}$$

Using this forecasted value as the d_{zj}, the D_{zj} value is obtained as discussed in Section 2 with an inflation factor. Once all the D_{zj} values are obtained, the revised production plan can be obtained by solving the model in Section 3 with this data.

If it is Reasonable to Assume that $E(\xi_\ell), E(\eta_\ell)$ Increase as ℓ Decreases

This happens if more and more customer orders tend to be placed closer to the delivery dates. In this case $E(\xi_\ell)$ can be estimated by exponential smoothing or a method similar to that.

If $E(\xi_\ell)$ increases linearly as ℓ decreases, then it can be approximated by $a - b\ell$ where the positive constants a, b can be approximated from past estimates of $E(\xi_\ell)$. Under the assumption that a certain percentage of orders placed will be cancelled before they are delivered, $E(\eta_\ell)$ will be of the form $c - d\ell$ where approximate values of c, d can again be obtained from estimates of past values of $E(\eta_\ell)$. Once approximate values of a, b, c, d are obtained

$$\begin{aligned}\text{forecast of final order quantity} = \; & \max\{0, q + aT - bT(T+1)/2 - \\ & cT + dT(T+1)/2\}\end{aligned}$$

For other types of changes in the values of $E(\xi_\ell)$, $E(\eta_{\ell)}$ corresponding techniques can be developed.

Once the forecasts of final order quantities are obtained, the revised production plan can be obtained as discussed earlier.