NERS 312

Elements of Nuclear Engineering and Radiological Sciences II

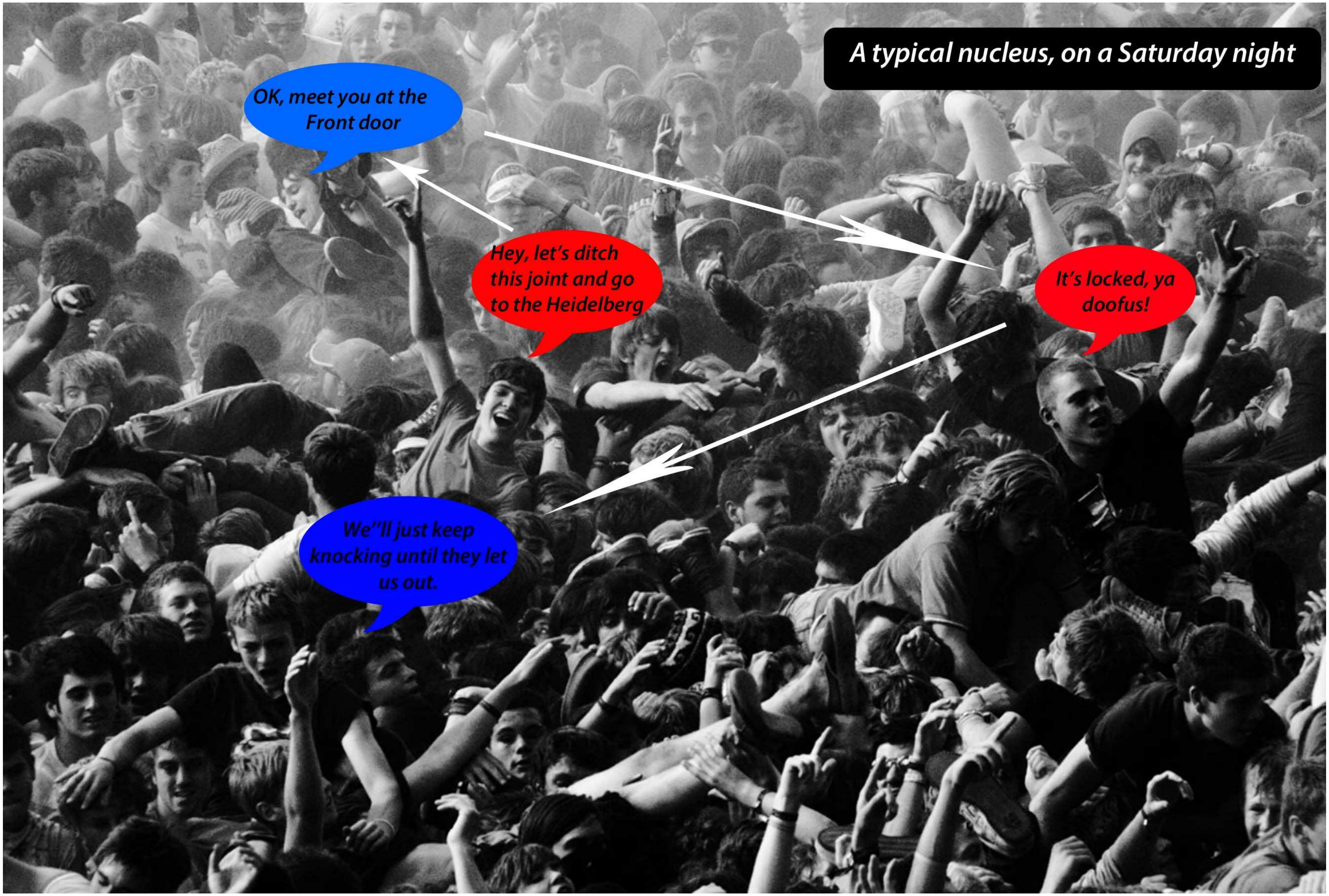
aka Nuclear Physics for Nuclear Engineers

Lecture Notes for Chapter 14: α decay

Supplement to (Krane II: Chapter 8)

The lecture number corresponds directly to the chapter number in the online book.
The section numbers, and equation numbers correspond directly to those in the online book.

How α decay works



Chapter 14: In this Chapter you will learn ......

Chapter 14.1: Why α Decay Occurs

Chapter 14:2 Basic α Decay Processes

- The energetics of α decay
- Relativistic effects?

Chapter 14.3: α-decay systematics

- As Q increases, $t_{1/2}$ decreases
- Prediction of Q from the semiempirical mass formula

Chapter 14.4: Theory of α Emission

- The simplest theory of α emission
- Gamow's theory of α decay
- Krane's treatment of α decay
- Comparison with Measurements
- Cluster decay probabilities

Chapter 14.5: Angular momentum and parity in α decay

- Angular momentum
- Conservation of angular momentum and parity
- Angular intensity of α decays for elliptic nuclei

Chapter 14.5: α-decay spectroscopy

14.1—Why α decay occurs

The mass excess (in MeV) of ^{4}He and its near neighbors, is shown below.

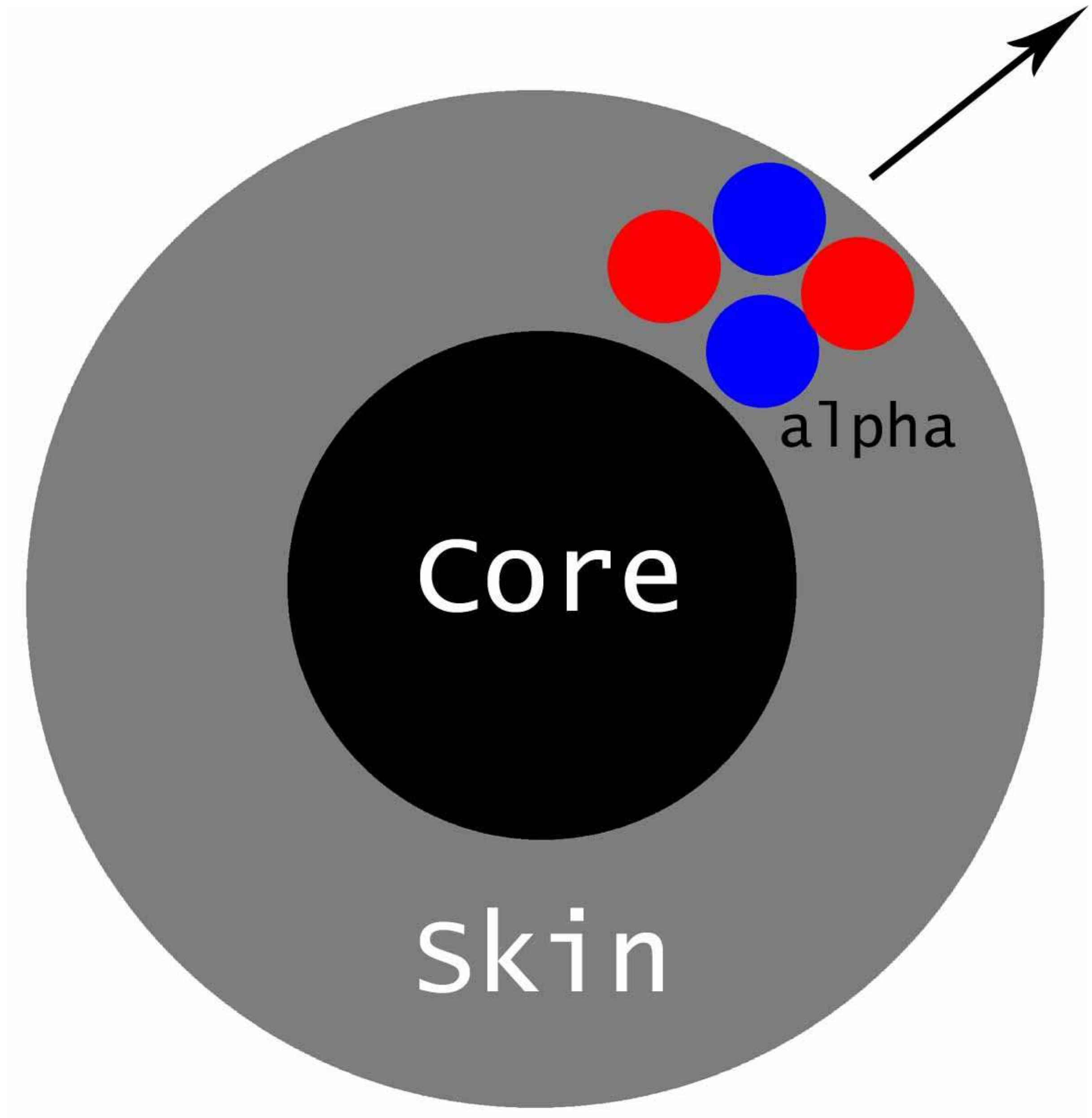
$\Downarrow N\backslash Z \Rightarrow$	0	1	2	3
0	-	7.2889705(1)	-	-
1	8.0713171(5)	13.1357216(3)	14.9312148(24)	25.32(21)
2	-	14.9498060(23)	2.4249156(1)	11.680(50)
3	-	25.90(10)	11.390(50)	14.086793(15)

Thus, we see that, compared to its low-A neighbors in the periodic table, it is bound very strongly.

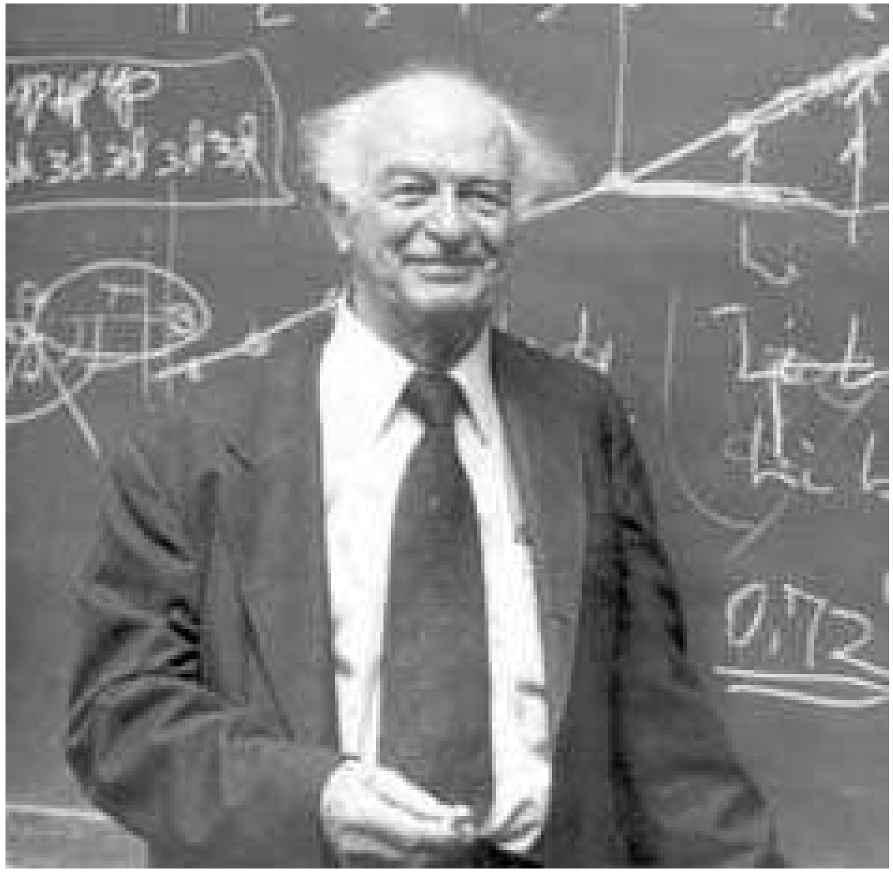
We know, from the shell model of the nucleus, that ^{4}He is a doubly-magic nucleus, and this is what we may have expected.

So, thinking classically, occasionally 2 protons and 2 neutrons appear together at the edge of a nucleus, with outward pointing momentum, and bang against the Coulomb barrier.

Every once in a while, it can tunnel through, as we saw in 311.


alpha
Core
Skin

So compelling is this concept, that the α particles can stay intact in the nucleus, prompted two-time Nobel Laureate, Linus Pauling (the only person two have won two Nobel Prizes, awarded to a single person): http://en.wikipedia.org/wiki/Linus_Pauling



to invent a "spheron model" of the nucleus.

Quoting from the SOAK-TN (Source Of All Knowledge, True or Not)

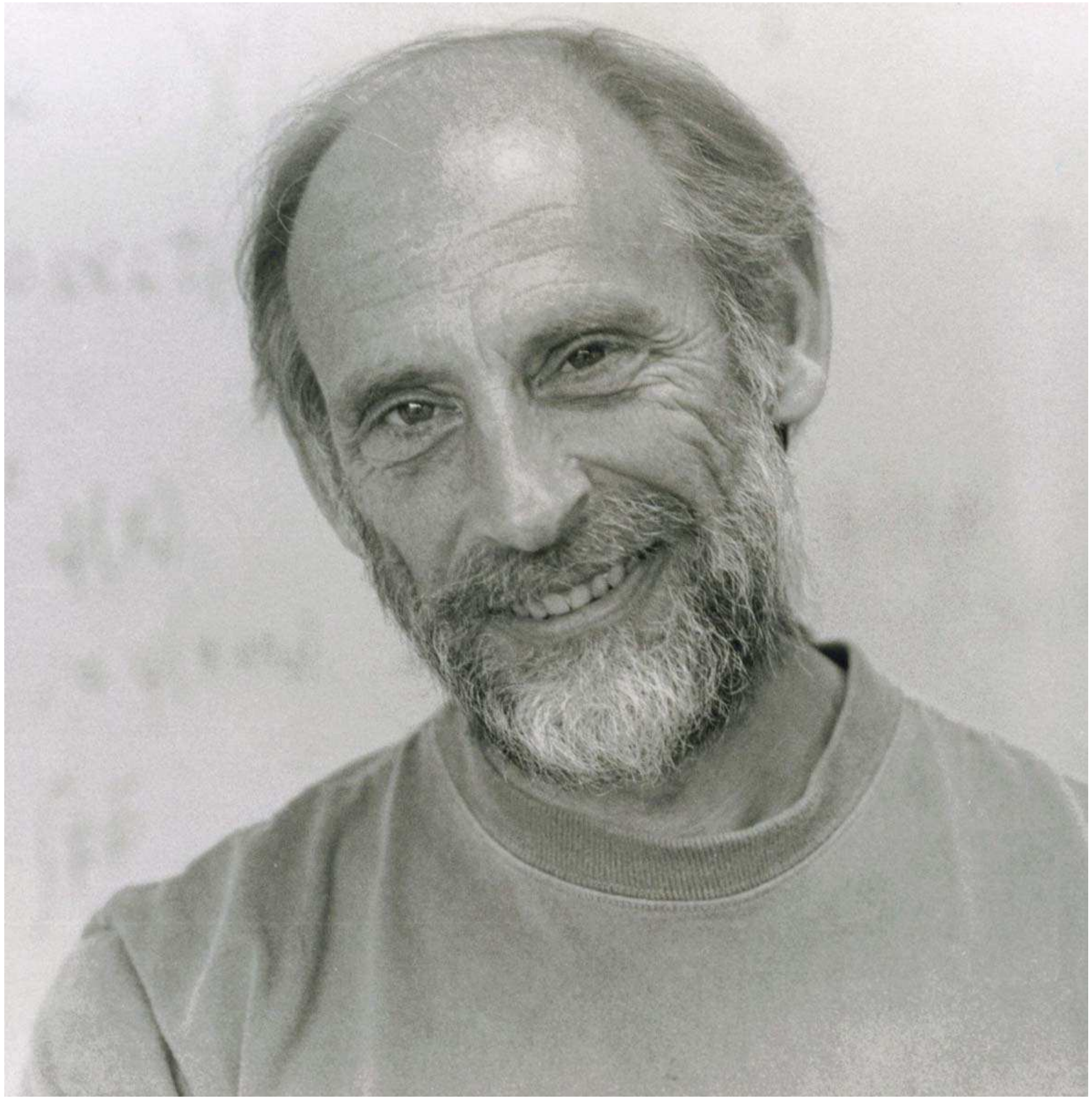
On September 16, 1952, Pauling opened a new research notebook with the words "I have decided to attack the problem of the structure of nuclei."[86] On October 15, 1965, Pauling published his Close-Packed Spheron Model of the atomic nucleus in two well respected journals, Science and the Proceedings of the National Academy of Sciences.For nearly three decades, until his death in 1994, Pauling published numerous papers on his spheron cluster model.

The basic idea behind Pauling's spheron model is that a nucleus can be viewed as a set of "clusters of nucleons". The basic nucleon clusters include the deuteron [np], helion [pnp], and triton [npn]. Even-even nuclei are described as being composed of clusters of alpha particles, as has often been done for light nuclei.

Pauling attempted to derive the shell structure of nuclei from pure geometrical considerations related to Platonic solids rather than starting from an independent particle model as in the usual shell model. In an interview given in 1990 Pauling commented on his model:

"Now recently, I have been trying to determine detailed structures of atomic nuclei by analyzing the ground state and excited state vibrational bends, as observed experimentally. From reading the physics literature, Physical Review Letters and other journals, I know that many physicists are interested in atomic nuclei, but none of them, so far as I have been able to discover, has been attacking the problem in the same way that I attack it. So I just move along at my own speed, making calculations..."

Who is this person?



http://en.wikipedia.org/wiki/Lenny_Susskind

14.2—Basic α-decay processes

An α decay is a nuclear transformation in which a nucleus reduces its energy by emitting an α-particle.

$$^{A}_{Z}X_{N} \longrightarrow {}^{A-4}_{Z-2}X'_{N-2} + {}^{4}_{2}\mathrm{He}_{2} \ ,$$

or, more compactly :

$$^{A}X \longrightarrow X' + \alpha \ .$$

The resultant nucleus, X' is usually left in an excited state, followed, possibly, by another α decay, or by any other form of radiation, eventually returning the system to the ground state.

The energetics of α decay

The α-decay process is "fueled" by the rest mass energy difference of the initial state and final state. That is, using a relativistic formalism:

$$
\begin{aligned}
E_{\rm i} &= E_{\rm f} \\
m_{_X}c^2 &= m_{_{X'}}c^2 + T_{X'} + m_\alpha c^2 + T_\alpha \\
Q &= T_{X'} + T_\alpha \quad \text{where} \\
Q &\equiv m_{_X}c^2 - m_{_{X'}}c^2 - m_\alpha c^2 \\
Q/c^2 &\approx m(^A\mathrm{X}) - m(^{A-4}\mathrm{X}') - m(^4\mathrm{He}) \ ,
\end{aligned} \tag{1}
$$

$T_{X'}$ and T_α are the kinetic energies of the nucleus and the α-particle following the decay

The atomic masses may be substituted for the nuclear masses, as shown in the last line above.

The electron masses balance in the equations, and there is negligible error in ignoring the small differences in electron binding energies.

The line of flight of the decay products are in equal and opposite directions, assuming that X was at rest. Conservation of energy and momentum apply. Thus we may solve for T_α in terms of Q (usually known). X' is usually not observed directly. Solving for T_α by eliminating X':

$$\begin{aligned}
Q &= T_\alpha + T_{X'} && (2)\\
|\vec{p}_\alpha| &= |\vec{p}_{X'}| \\
p_\alpha^2 &= p_{X'}^2 \\
2m_\alpha T_\alpha &= 2m_{X'} T_{X'} \\
(m_\alpha/m_{X'})T_\alpha &= T_{X'}\ . && (3)
\end{aligned}$$

Using (3) and (2) to eliminate $T_{X'}$ results in:

$$\begin{aligned}
Q &= T_\alpha(1 + m_\alpha/m_{X'}) \quad \text{or} && (4)\\
T_\alpha &= \frac{Q}{(1 + m_\alpha/m_{X'})} && (5)
\end{aligned}$$

$$T_\alpha \approx \frac{Q}{(1+4/A')} \quad \text{or} \tag{6}$$

$$T_\alpha \approx \frac{Q}{(1+4/A)} \quad \text{or} \tag{7}$$

$$T_\alpha \approx Q(1-4/A) \tag{8}$$

Equations (4) and (5) are exact within a non-relativistic formalism.

Equation (6) is an approximation, but a good one that is suitable for all α-decay's, including ^{8}Be$\rightarrow 2\alpha$.

Equation (7) is suitable for all the other α-emitters, while (8) is only suitable for the heavy emitters, since it assumes $A \gg 4$.

We see from (8) that the typical recoil energy, for heavy emitters is:

$$T_{X'} = Q - T_\alpha \approx (4/A)Q \ . \tag{9}$$

For a typical α-emitter, this recoil energy ($Q = 5$ MeV, $A = 200$) is 100 keV.

This is not insignificant. α-emitters are usually found in crystalline form, and that recoil energy is more than sufficient to break atomic bonds, and cause a microfracture along the track of the recoil nucleus.

Relativistic effects?

One may do a fully relativistic calculation, from which it is found that:

$$T_\alpha = \frac{Q\left(1+\frac{1}{2}\frac{Q}{m_{X'}c^2}\right)}{\left(1+\frac{m_\alpha}{m_{X'}}+\frac{Q}{m_{X'}c^2}\right)}, \tag{10}$$

$$T_{X'} = \frac{\left(\frac{m_\alpha}{m_{X'}}\right)Q\left(1+\frac{1}{2}\frac{Q}{m_\alpha c^2}\right)}{\left(1+\frac{m_\alpha}{m_{X'}}+\frac{Q}{m_{X'}c^2}\right)}. \tag{11}$$

Even in the worst-case scenario (low-A) this relativistic correction is about 2.5×10^{-4}. Thus the non-relativistic approximation is adequate for determining T_α or $T_{X'}$.

14.3—α Decay Systematics

As Q increases, $t_{1/2}$ decreases

This is, more or less, self-evident. More “fuel” implies faster decay.

The “smoothest” example of this “law” is seen in the α decay of the even-even nuclei

Shell model variation is minimized in this case, since no pair bonds are being broken.

See Figure 1, where $\log_{10}(t_{1/2})$ is plotted *vs.* Q.

Geiger and Nuttal proposed the following phenomenological fit for $\log_{10}(t_{1/2}(Q))$:

$$\log_{10}\lambda = C - DQ^{-1/2} \quad \text{or} \tag{12}$$
$$\log_{10} t_{1/2} = -C' + DQ^{-1/2} \ , \tag{13}$$

where C and D are fitting constants, and $C' = C - \log_{10}(\ln 2)$.

Odd-odd, even-odd and odd-even nuclei follow the same general systematic trend, but the data are much more scattered, and their half-lives are 2–1000 times that of their even-even counterparts.

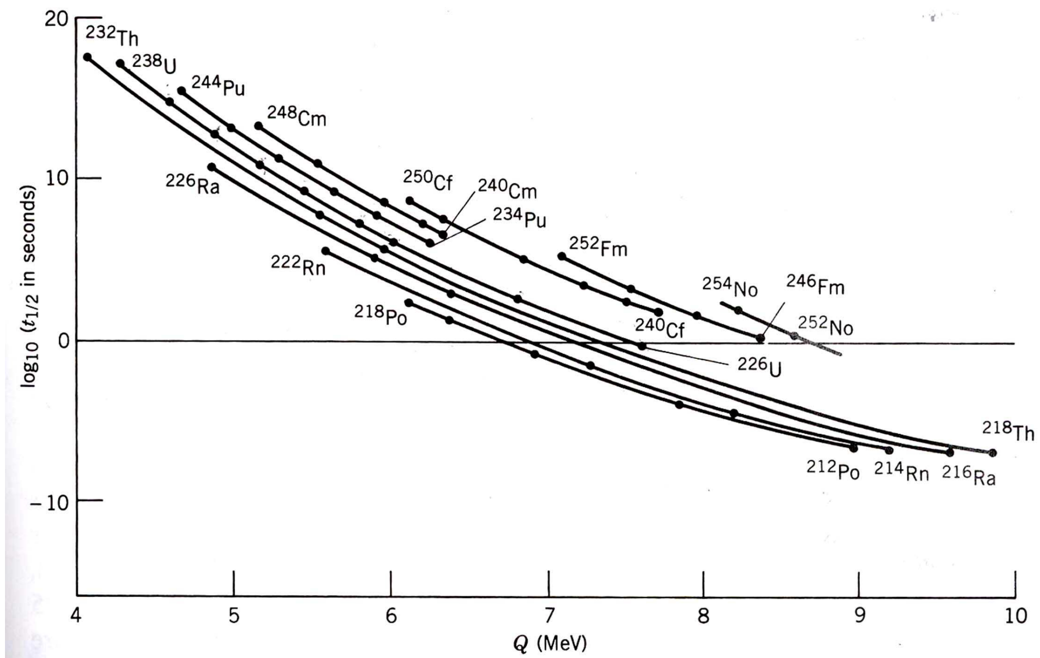


Figure 8.1 The inverse relationship between α-decay half-life and decay energy, called the Geiger-Nuttall rule. Only even-Z, even-N nuclei are shown. The solid lines connect the data points.

Figure 1: Krane's Figure 8.1.

These trends, from experimental data, are seen in Figure 2.
However, there is also evidence of the impact of the shell closing at $N = 126$, that is not seen in Figure 2.

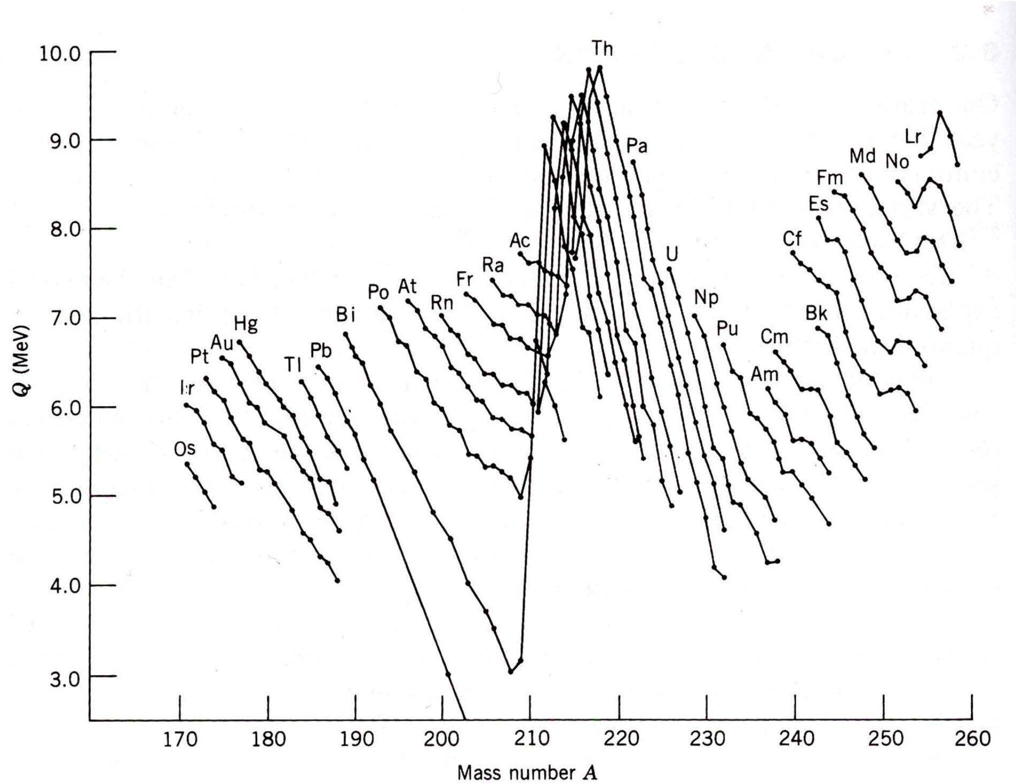


Figure 8.2 Energy released in α decay for various isotopic sequences of heavy nuclei. In contrast to Figure 8.1, both odd-A and even-A isotopes are shown, and a small amount of odd-even staggering can be seen. The effects of the shell closures at $N = 126$ (large dip in data) and $Z = 82$ (larger than average spacing between Po, Bi, and Pb sequences) are apparent.

Figure 2: Krane's Figure 8.2.

Prediction of Q from the semiempirical mass formula

The semiempirical mass formula can be employed to estimate Q_α, as a function of Z and A.

$$\begin{aligned} Q &= B(Z-2, A-4) + B(^e\mathrm{He}) - B(Z, A) \\ &\approx 28.3 - 4a_\mathrm{V} + \frac{8}{3}a_\mathrm{S}A^{-1/3} + 4a_\mathrm{C}ZA^{-1/3}(1 - Z/3A) - 4a_\mathrm{sym}(1 - 2Z/A)^2 + 3a_\mathrm{p}A^{-7/4}\ . \end{aligned} \tag{14}$$

A plot of $Q(Z, A)$ using (14) is given in Figure 3.

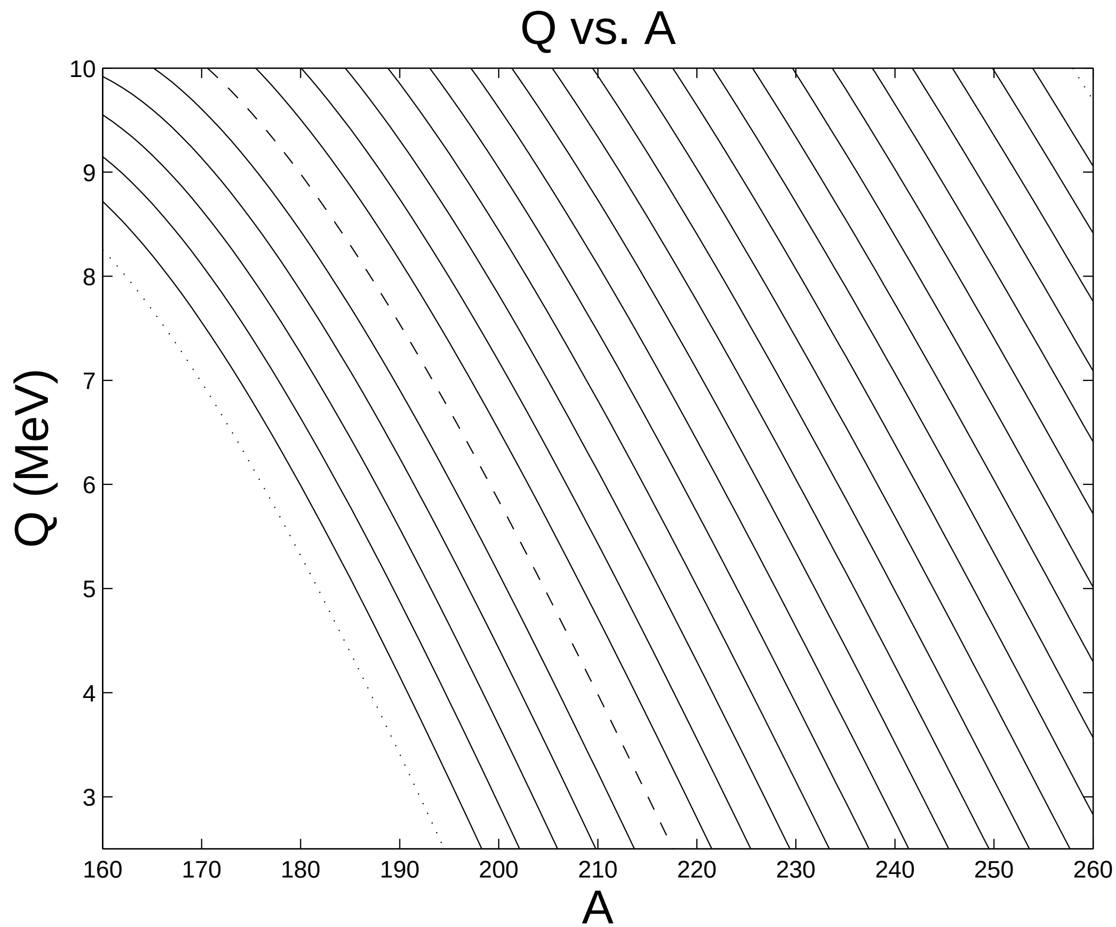


Figure 3: $Q(Z,A)$ using (14). The dashed line is for Pb $(Z = 82)$, while the lower dotted is for Os $(Z = 76)$. The upper dotted line is for Lr $(Z = 103)$. Each separate Z has its own line with higher Z's oriented to the right.

14.4—Theory of α Emission

Figure 4 shows 3 potentials that are used in the estimation of barrier penetration probabilities, for determining the halflife of an α-emitter.

The simplest potential, the rectilinear box potential, shown by the dashed line, although crude, maybe used to explain the phenomenon of α decay.

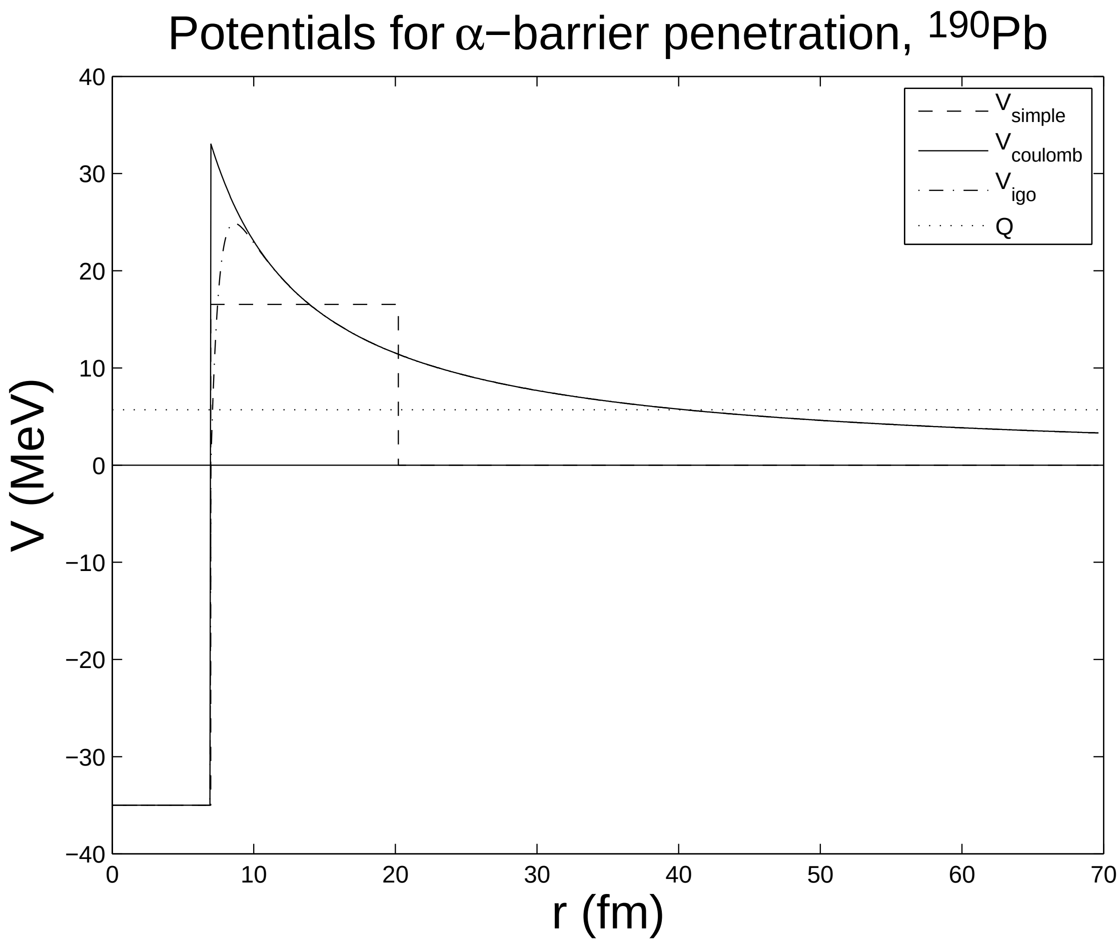


Figure 4: Potentials for α decay.

The simplest theory of α emission

In this section we solve for the decay probability using the simplest rectilinear box potential.

This potential is characterized by:

$$\begin{aligned} V_1(r < R_\mathrm{N}) &= -V_0 \\ V_2(R_\mathrm{N} < r < b) &= V_\mathrm{C} \\ V_3(r > b) &= 0 \ , \end{aligned} \tag{15}$$

where V_0 and V_C are constants.

The 3-D radial wavefunctions (assuming that the α is in an s-state), are of the form, $R_i(r) = u_i(r)/r$ take the form:

$$\begin{aligned} u_1(r) &= Ae^{ik_1 r} + Be^{-ik_1 r} \\ u_2(r) &= Ce^{k_2 r} + De^{-k_2 r} \\ u_3(r) &= Fe^{ik_3 r} \ , \end{aligned} \tag{16}$$

where

$$\begin{aligned} k_1 &= \sqrt{2m(V_0 + Q)}/\hbar \\ k_2 &= \sqrt{2m(V_\mathrm{C} - Q)}/\hbar \\ k_3 &= \sqrt{2mQ}/\hbar \ . \end{aligned} \tag{17}$$

Turning the mathematical crank, we arrive at the transmission coefficient:

$$T = \left[\frac{1}{4}\left(2 + \frac{k_1}{k_3} + \frac{k_3}{k_1}\right) + \frac{\sinh^2[2k_2(b - R_{\mathrm{N}})]}{4}\left(\frac{k_1 k_3}{k_2^2} + \frac{k_2^2}{k_1 k_3} + \frac{k_1}{k_3} + \frac{k_3}{k_1}\right)\right]^{-1} \tag{18}$$

Note: We solved a similar problem in NERS 311, but with $V_0 = 0$, that is, $k_1 = k_3$.

The factor $k_2(b - R_{\mathrm{N}}) \approx 35$ for typical α emitters. Thus, we can simplify to:

$$T = \frac{16 e^{-2k_2(b - R_{\mathrm{N}})}}{\left(\frac{k_1 k_3}{k_2^2} + \frac{k_2^2}{k_1 k_3} + \frac{k_1}{k_3} + \frac{k_3}{k_1}\right)} \tag{19}$$

Recall that the transmission coefficient is the probability of escape by a single α-particle.

To calculate the transmission rate, we estimate a "frequency factor", f, that counts the number of instances, per unit time, that a α, with velocity v_α presents itself at the barrier as an escape candidate.

There are several estimates for f:

f	Source	Estimate (s^{-1})	Remarks
v_α/R_N	Krane	$\approx 10^{21}$	Too low, by 10^1–10^2
$v_\alpha/(2R_\mathrm{N})$	Others	$\approx 5 \times 10^{20}$	Too low, by 10^1–10^2
	Fermi's Golden Rule # 2	$\approx 10^{24}$	Too high, by 10^1–10^2

The correct answer, determined by experiment, lies in between these two extremes.

However approximate our result it, it does show an extreme sensitivity to the shape of the Coulomb barrier, through the exponential factor in (19).

Gamow's theory of α decay

Gamow's theory of α decay is based on an approximate solution[1] to the Schrödinger equation. Gamow's theory gives:

$$T = \exp\left[-2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_{R_{\rm N}}^{b} {\rm d}r\,\sqrt{V(r)-Q}\right]\ , \tag{20}$$

where b is that value of that defines the r where $V(r) = Q$, on the far side of the barrier. If we apply Gamow's theory to the potential of the previous section, we obtain:

$$T_{\rm exact} = \frac{16}{\left(\frac{k_1k_3}{k_2^2} + \frac{k_2^2}{k_1k_3} + \frac{k_1}{k_3} + \frac{k_3}{k_1}\right)} T_{\rm Gamow}\ . \tag{21}$$

That factor in front is about 2–3 for most α emitters. This discrepancy is usually ignored, considering the large uncertainly in the f factor.

[1]The approximation Gamow used, is a semi-classical approximation to the Schrödinger equation, called the WKB (Wentzel-Kramers-Brillouin) method. The WKB method works best when the potential changes slowly with position, and hence the frequency of the wavefunction, $k(x)$, also changes slowly. This is not the case for the nucleus, due to its sharp nuclear edge. Consequently, it it thought that Gamow's solution can only get to within a factor or 2 or 3 of the truth. In nuclear physics, a factor of 2 or 3 is often thought of as "good agreement"!

Krane's treatment of α-decay

Krane starts out with (20), namely:

$$T = \exp\left[-2\left(\frac{2m}{\hbar^2}\right)^{1/2}\int_a^b \mathrm{d}r\,\sqrt{V(r)-Q}\right] \ ,$$

where

$$\begin{aligned} V(x) &= \frac{2(Z-2)e^2}{4\pi\epsilon_0 x} \\ V(a)\equiv B &= \frac{2(Z-2)e^2}{4\pi\epsilon_0 a} \\ a &= R_0(A-4)^{1/3} \\ V(b)\equiv Q &= \frac{2(Z-2)e^2}{4\pi\epsilon_0 b} \ . \end{aligned} \tag{22}$$

That is, the α moves in the potential of the *daughter* nucleus, B is the height of the potential at the radius of the daughter nucleus, and b is the radius where that potential is equal to Q. Therefore, $Q = B$.

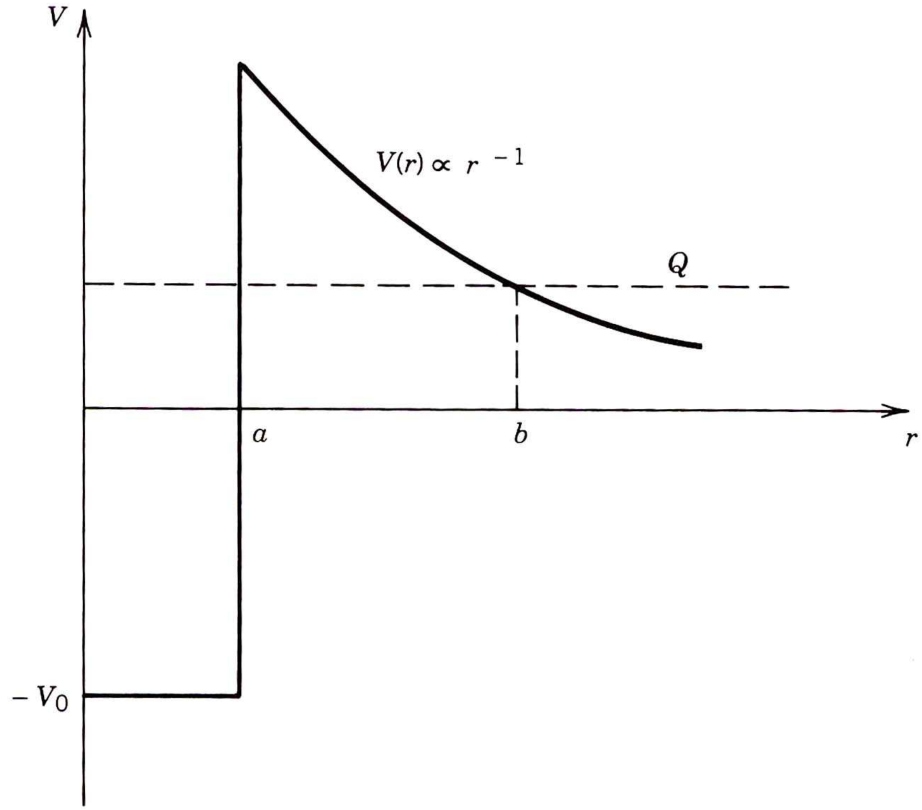


Figure 8.3 Relative potential energy of α-particle, daughter-nucleus system as a function of their separation. Inside the nuclear surface at $r = a$, the potential is represented as a square well; beyond the surface, only the Coulomb repulsion operates. The α particle tunnels through the Coulomb barrier from a to b.

Substituting the potential in (22) into (20) results in:

$$T = \exp\left\{-2\left(\frac{2m'_\alpha c^2}{Q(\hbar c)^2}\right)^{1/2}\frac{zZ'e^2}{4\pi\epsilon_0}\left[\arccos(\sqrt{x}) - \sqrt{x(1-x)}\right]\right\} \ , \tag{23}$$

where $x \equiv a/b = Q/B$. Note that the reduced mass,

$$m'_\alpha = \frac{m_\alpha m_{X'}}{m_\alpha + m_{X'}} \approx m_\alpha(1 - 4/A) \ . \tag{24}$$

has been used.

This "small" difference can result in a change in T by a factor of 2–3, even for heavy nuclei!

Krane also discusses the approximation to (23) that results in his equation (8.18). This comes from the Taylor expansion:

$$\arccos(\sqrt{x}) - \sqrt{x(1-x)} \longrightarrow \frac{\pi}{2} - 2\sqrt{x} + \mathcal{O}(x^{3/2}) \ .$$

This is only valid for small x. Typically $x \approx 0.3$, and use of Krane's (8.18) involves too much error. So, stick with the equation given on the next page, instead.

Factoring in the frequency factor, one can show that:

$$
\begin{aligned}
t_{1/2} \;=\; & \ln(2)\frac{a}{c}\sqrt{\frac{m_\alpha c^2}{2(V_0+Q)}} \times \\
& \exp\left\{2\left(\frac{2m'_\alpha c^2}{Q(\hbar c)^2}\right)^{1/2}\frac{zZ'e^2}{4\pi\epsilon_0}\left[\arccos(\sqrt{x})-\sqrt{x(1-x)}\right]\right\}\ .
\end{aligned} \tag{25}
$$

14.4.1—Comparison with Measurements

In this section we employ the simplest form of f and compute the half-life for α-decay as follows:

$$t_{1/2} = \ln(2)\frac{a}{c}\sqrt{\frac{m_\alpha c^2}{2(V_0+Q)}}\exp\left\{2\left(\frac{2m'_\alpha c^2}{Q(\hbar c)^2}\right)^{1/2}\frac{zZ'e^2}{4\pi\epsilon_0}\left[\arccos(\sqrt{x})-\sqrt{x(1-x)}\right]\right\} \quad (26)$$

The data are shown in the following table, where the half-lives of the even-even isotopes of Th $(Z=90)$ are shown.

A	Q (MeV)	$t_{1/2}$ (s) abs. meas.	$t_{1/2}$ (s) abs. calc.	$t_{1/2}$ (s) rel. meas.	$t_{1/2}$ (s) rel. calc.
220	8.95	10^{-5}	10^{-3}	5.4×10^{-9}	3.6×10^{-9}
222	8.13	2.8×10^{-3}	2.1×10^{-1}	1.5×10^{-6}	7.3×10^{-7}
224	7.31	1.04	1.1×10^{2}	5.6×10^{-4}	3.8×10^{-4}
226	6.45	1854	2.9×10^{5}	$\equiv 1$	$\equiv 1$
228	5.52	6.0×10^{7}	1.2×10^{10}	3.2×10^{4}	4.2×10^{4}
230	4.77	2.5×10^{12}	6.0×10^{14}	1.3×10^{9}	2.1×10^{9}
232	4.08	4.4×10^{17}	1.8×10^{20}	2.4×10^{14}	6.2×10^{14}

Table 1: Half-lives of Th isotopes, absolute and relative comparisons of measurement and theory.

The calculations were performed using a nuclear radius of $a = 1.25A^{1/3}$ (fm), and $V_0 = 35$ (MeV).

The absolute comparison exhibits the same trends for both experiment and calculations, with the calculations being overestimated by 2–3 orders of magnitude.

This is most likely due to a gross underestimate of f. The relative comparisons are in much better shape, showing discrepancies of about a factor of 2–3, quite a success for such a crude theory.

We note that small changes in Q result in enormous differences in the results.

In this table Q changes by about a factor of 2, while the half-lives span about 23 orders of magnitude.

The probability of escape is greatly influenced by the height and width of the Coulomb barrier.

Besides this dependence, the only other variation in the comparison relates to the nuclear radius.

This also affects the barrier since the nuclear radius is proportional to $A^{1/3}$. This hints that the remaining discrepancy, at least for the relative comparison, is related to the fine details of the shape of the barrier, perhaps mostly in the vicinity of the inner turning point.

A more refined shape of the Coulomb barrier would likely yield better results, as well would a higher-order WKB analysis that would account more precisely, for that shape variation.

Additionally, the α-particle was treated as if it were a point charge in this analysis. A refined calculation should certainly take this effect into account.

Cluster decay probabilities

If α decay can occur, surely ^{8}Be and ^{12}C decay can occur as well. It is just a matter of relative probability. For these decays, the escape probabilities are given approximately by:

$$
\begin{aligned}
T_{^8\mathrm{Be}} &= T_\alpha^2 \\
T_{^{12}\mathrm{C}} &= T_\alpha^3 \\
T_{^A\mathrm{X}} &= T_\alpha^{Z/2} \ .
\end{aligned} \tag{27}
$$

The last estimate is for a $^A\mathrm{X}$ cluster, with Z protons and an atomic mass of A.

14.5—Angular momentum and parity in α decay

Angular momentum

If the α-particle carries off angular momentum, we must add the repulsive potential associated with the centrifugal barrier to the Coulomb potential, $V_{\mathrm{C}}(r)$:

$$V(r) = V_{\mathrm{C}}(r) + \frac{l(l+1)\hbar^2}{2m'_\alpha r^2} \;, \tag{28}$$

represented by the second term on the right-hand side of (28).
The effect on ^{90}Th, with $Q = 4.5$ MeV is:

l	0	1	2	3	4	5	6
T_l/T_0	1	0.84	0.60	0.36	0.18	0.078	0.028

So, as $l \uparrow$, $T \downarrow$.

Conservation of angular momentum and parity

α decay's must satisfy the constraints given by the conservation of total angular momentum:

$$\begin{aligned}\vec{I}_i &= \vec{I}_f + \vec{I}_\alpha \\ \Pi_i &= \Pi_f \times \Pi_\alpha \ ,\end{aligned} \tag{29}$$

where i represents the parent nucleus, and f represents the daughter nucleus. Since the α-particle is a 0^+ nucleus, (29) simplifies to:

$$\begin{aligned}\vec{I}_i &= \vec{I}_f + \vec{l}_\alpha \\ \Pi_i &= \Pi_f \times (-1)^{l_\alpha} \ ,\end{aligned} \tag{30}$$

where l_α is the orbital angular momentum carried off by the α-particle.

If I_i is non-zero, the α decay is able to populate any excited state of the daughter, or go directly to the ground state.

If the initial state has total spin 0, with few exceptions it is a 0^+. In this case, (30) becomes.

$$\begin{aligned} \vec{0} &= \vec{I}_f + \vec{l}_\alpha \\ +1 &= \Pi_f \times (-1)^{l_\alpha} \ , \end{aligned} \tag{31}$$

or,

$$\begin{aligned} \vec{I}_f &= \vec{l}_\alpha \\ \Pi_f + &= (-1)^{l_\alpha} \ , \end{aligned} \tag{32}$$

Thus the only allowed daughter configurations are: $0^+, 1^-, 2^+, 3^-, 4^+, 5^-, 6^+, 7^-, 8^+, 9^- \cdots$ All other combinations are absolutely disallowed.

The α decay can show these allowed transitions quite nicely. A particularly nice example is the case where the transition is $0^+ \longrightarrow 0^+$, where the low-lying rotational band, and higher energy phonon structure are explicitly revealed through α decay.

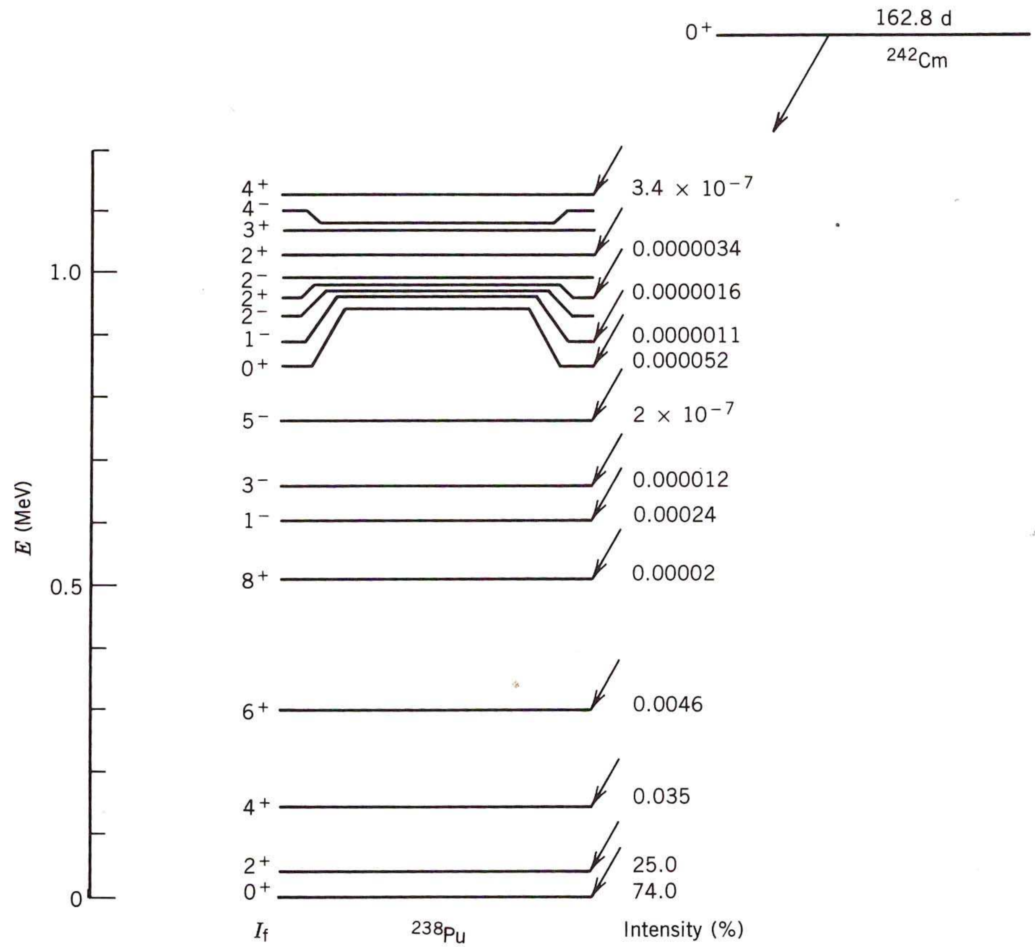


Figure 8.7 α decay of ^{242}Cm to different excited states of ^{238}Pu. The intensity of each α-decay branch is given to the right of the level.

Angular intensity of α decays for elliptic nuclei

Nuclei in the mass range $150 < A < 190$ and $A > 200$ have permanent non-spherical deformations.

These deformations can be modeled as prolate (elongated, cigar-shaped) or oblate (flat, pancake) on the azimuthal symmetry axis.

It follows that the α-decay angular distributions are anisotropic.

The α emission from the longer axes is enhanced, as seen in the following two figures.

Clearly, a more sophisticated approach is required to describe this mathematically.
This is a well-defined, do-able, but tough 499 project!

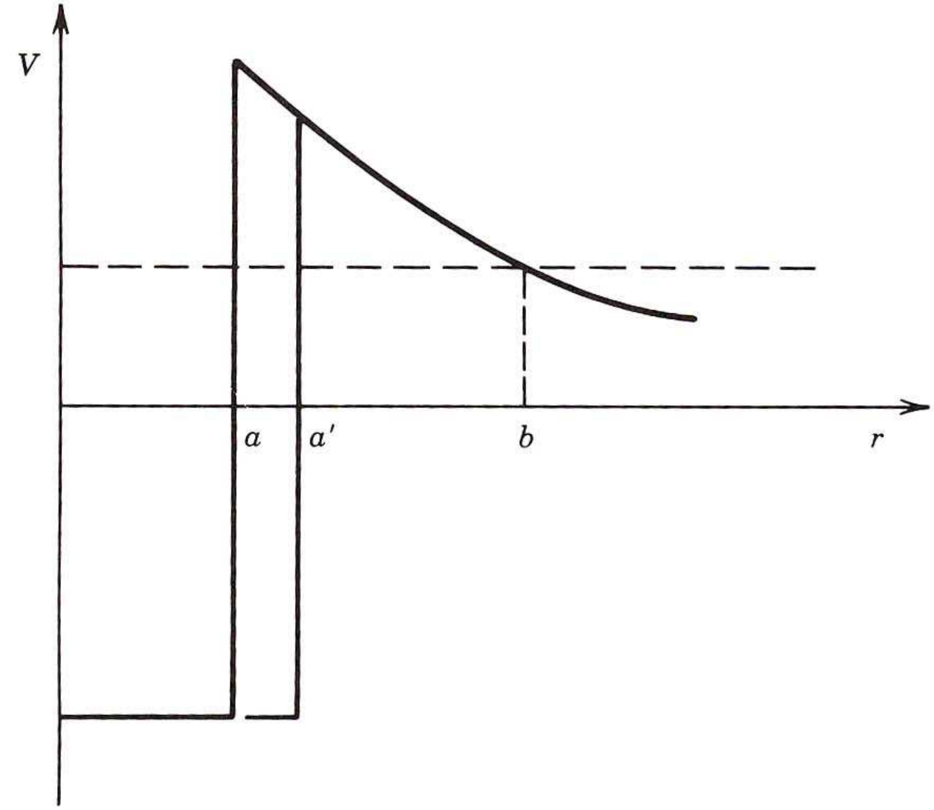


Figure 8.9 In a deformed nucleus, α particles escaping from the poles enter the Coulomb barrier at the larger separation a', and must therefore penetrate a lower, thinner barrier. It is therefore more probable to observe emission from the poles than from the equator.

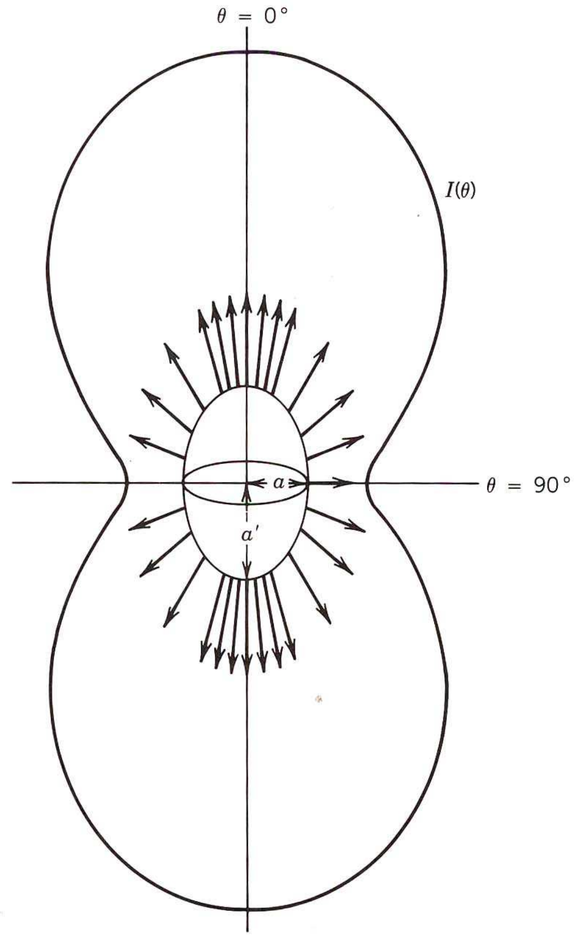


Figure 8.10 Intensity distribution of α particles emitted from the deformed nucleus at the center of the figure. The polar plot of intensity shows a pronounced angular distribution effect.