THE ATOMIC NUCLEUS

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New York Toronto London

McGRAW-HILL BOOK COMPANY, INC.

1955

CHAPTER 20

Radiative Collisions of Electrons with Atomic Nuclei

The discovery of a continuous X-ray spectrum, or bremsstrahlung, which results from the *inelastic* collision of electrons with nuclei was the first step in the beginning of a new era in physics. Röntgen in 1895 first reported these mysterious rays, whose investigation was to lead into the field known today as "modern physics."

X rays are divided into two main types: (1) the line spectra, or *characteristic spectra*, which are electromagnetic radiations given off by an atom as it fills vacancies in a $K, L, M, \ldots$ shell; and (2) the *continuous spectra*, or bremsstrahlung, which are associated with the deflection of incident charged particles by the coulomb fields of nuclei.

1. *Theory of Bremsstrahlung*

a. Classical Considerations. According to classical theory, whenever a charge experiences an acceleration it will radiate. Therefore, whenever an incident charged particle is deflected from its path or has its velocity changed, it should emit electromagnetic radiation whose amplitude is proportional to the acceleration. The acceleration produced by a nucleus of charge Ze on a particle of charge ze and mass M is proportional to Zze^2/M. Thus the intensity, which is proportional to the square of the amplitude, will vary as Z^2z^2/M^2.

Thus the total bremsstrahlung per atom varies as the *square of the atomic number* of the absorbing material—a fact that is well confirmed by experiment. We also see that the total bremsstrahlung varies inversely with the *square of the mass of the incident particle.* Therefore protons and α particles will produce about one-millionth the bremsstrahlung of an electron of the same velocity. The μ meson, at first thought to be an electron in cosmic-ray studies, owes its discovery to the fact that its radiative losses were far too small for an electron. It was later found to have a rest mass about $212m_0$, which would mean its radiative losses are about 40,000 times smaller than the losses of an electron of the same velocity. Because of this strong mass dependence, bremsstrahlung is almost completely negligible for all swift particles other than electrons.

In an individual deflection by a nucleus, the incident particle can radiate any amount of energy from zero up to its total kinetic energy T.

Thus the maximum quantum energy $(h\nu)_{\max}$ at the short wavelength limit of the continuous X-ray spectrum is

$$(h\nu)_{\max} = T \tag{1.1}$$

This relationship was established experimentally by Duane and Hunt (D40) in 1915 and is known as Duane and Hunt's law.

b. Quantum-mechanical Theory of Bremsstrahlung. The deflection of a swift electron of velocity $V = \beta c$, rest mass m_0, by a nucleus of charge Ze falls in the domain of $Z/137\beta \ll 1$, if Z is not too large. This puts the interaction into the familiar "blackout" domain, where the true character of the interaction may differ from that which would be deduced from classical mechanics. In a quantum-mechanical treatment we have seen that the first approximation of Born's method calls for neglecting $Z/137\beta$ compared with unity. Born's first approximation is therefore applicable to the problem of bremsstrahlung, except for initial or final electrons of very low velocity.

The quantum-mechanical theory for the bremsstrahlung of relativistic electrons has been developed by Bethe and Heitler (B49, B39, H29) and others, using Dirac's relativistic theory of the electron and the first approximation of Born. Bethe and Maximon (B51) have derived the differential cross section without use of the Born approximation but under the analogous limitations of $T \gg m_0c^2$ and $(T - h\nu) \gg m_0c^2$. The nonrelativistic theory has been developed by Sommerfeld (S61), using exact wave functions, and his equations have been integrated over all angles by Weinstock (W18), for comparison with experiment (H23).

In quantum-mechanical theory, a plane wave representing the electron enters the nuclear field, is scattered, and has a small but finite chance of emitting a photon. The electron is acted on by the electromagnetic field of the emitted photon, as well as by the coulomb field of the nucleus. The intermediate states of the system involve the negative energy states which characterize the Dirac electron theory. The theory of bremsstrahlung is intimately related to the theory of electron pair production (Chap. 24, Sec. 2) by energetic photons in the field of a nucleus.

Because the radiative process involves the coupling of the electron with the electromagnetic field of the emitted photon, the cross sections for radiation are of the order of $\frac{1}{137}$ times the cross sections for elastic scattering. Most of the individual deflections of incident electrons by atomic nuclei are elastic. In only a small number of instances is a photon emitted.

Recall that radiative forces were not taken into account in Mott's theory of the elastic scattering of electrons by nuclei (Chap. 19). In that theory, the influence of energy losses by radiation cannot be taken into account, because the probability that the deflected electron will radiate is of the order of $2\pi e^2/hc = \frac{1}{137}$, and such terms are neglected in comparison with unity in the first-order perturbation theory (Born approximation) used to develop the theory of elastic scattering. The influence of radiative losses on elastic collisions is estimated to be less than 2 or 3 per cent (M66).

Comparison with Classical Theory. The classical theory of bremsstrahlung incorrectly predicted the emission of radiation in *every* collision in which an electron is deflected. Yet for the averages over all collisions, the classical (p. 176 of H29) and the quantum-mechanical cross sections are of the same order of magnitude, namely,

$$\sigma_{\text{rad}} \sim \frac{Z^2}{137}\left(\frac{e^2}{m_0c^2}\right)^2 \qquad \text{cm}^2/\text{nucleus} \tag{1.2}$$

In the quantum-mechanical model there is a small but finite probability that a photon will be emitted each time a particle suffers a deflection; however, this probability is so small that usually no photon is emitted. In the few collisions which are accompanied by photon emission, a relatively large amount of energy is radiated. In this way the quantum theory replaces the multitude of small classical energy losses by a much smaller number of larger energy losses, the averages being about the same in the two theories. Of course, the spectral distributions are very different in the two models. All experimental results are in agreement with the quantum-mechanical model.

Angular Distribution. In the radiative collision, the initial momentum of the incident electron becomes shared between the momenta of three bodies: the residual electron, the atomic nucleus, and the emitted photon. Therefore the photon can have any momentum and the corresponding energy up to $h\nu_{\max} = T$. The momentum $h\nu/c$ of a photon is generally very small compared with the momentum of an electron having the same energy. Only at extreme relativistic energies do these momenta become equal. For the radiative collisions of moderate-energy electrons, momentum is substantially conserved between the nucleus and the deflected electron. The photon carries relatively only a very small momentum and can be emitted in any direction. At extreme relativistic energies, however, *both* the photon and the residual electron tend to proceed in the same direction as the incident electron. The average angle between the direction of the incident electron and the emitted quantum is then of the order of m_0c^2/T (B39, S10). The large-angle distribution of the bremsstrahlung from very-high-energy electrons is available from the calculations by Hough (H64).

Effects of Nuclear Radius and of Screening. In general, the bulk of the radiation losses of electrons occurs at relatively large distances from the nucleus. As in the case of elastic scattering, the major contributions arise from a region which is much farther from the nucleus than would be given by classical considerations. The dominant contributions to the radiative cross section come from distances of the order of the rationalized Compton wavelength $\hbar/m_0c$ ($= 385 \times 10^{-13}$ cm) and larger (B49). For much smaller distances, the corresponding scattering volume is small; at much larger distances, the scattering is reduced by interference. At extreme relativistic energies, say, $>$ 10-Mev electrons and in heavy atoms, the radiative losses at very large distances from the nucleus are reduced by screening (B39), while the losses in very close collisions are further reduced by the effects of the finite size of the nucleus (H64).

Differential Radiative Cross Section. The principal quantitative results of the quantum-mechanical theory of radiative collisions may be stated in the following way. For nuclei of charge Ze, the differential cross section $d\sigma_{\text{rad}}$ for the emission of a photon in the energy range between $h\nu$ and $h\nu + d(h\nu)$, by incident electrons of kinetic energy T and total energy $T + m_0c^2$, can be written

$$d\sigma_{\text{rad}} = \sigma_0 B Z^2 \frac{T + m_0c^2}{T} \frac{d(h\nu)}{h\nu} \qquad \text{cm}^2/\text{nucleus} \tag{1.3}$$

where

$$\sigma_0 = \frac{1}{137}\left(\frac{e^2}{m_0c^2}\right)^2 = 0.580 \text{ millibarn/nucleus} \tag{1.4}$$

and B is a very slowly varying function of Z and T, of the order of magnitude of 10. Figure 1.1 shows the theoretical values of B, which vary by only about a factor of 5 for all values of $h\nu$, T, and Z. Of course, $h\nu \le h\nu_{\max} = T$. The electron energy T is marked in Mev on each curve. The high-energy curves ($\ge$ 10 Mev) include screening corrections for Pb. The lower curve is for 60-kev electrons in Al, according to Sommerfeld's nonrelativistic quantum-mechanical theory (p. 170 of H29). Comparison with Eq. (1.3) shows that, for each value of T, the parameter B is proportional to the *intensity per frequency interval.* The general constancy of B, for all values of $h\nu$, as found experimentally (Fig. 1.3*a*), for $T \ll m_0c^2$, is fairly well predicted by these theoretical curves.

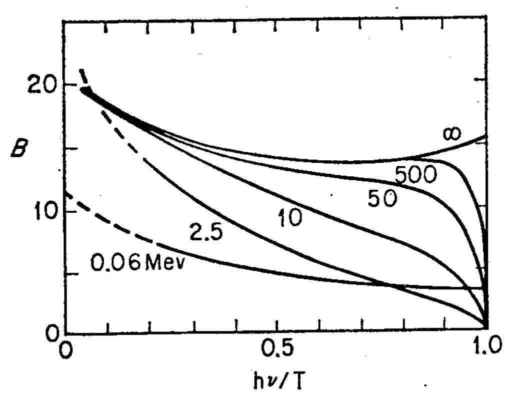

Fig. 1.1 The coefficient B of the differential cross section for bremsstrahlung, Eq. (1.3), according to the numerical evaluations by Heitler (H29).

Experimental studies using 19.5-Mev electrons (K26) suggest that the true bremsstrahlung spectrum is more nearly independent of $h\nu/T$ and is about 10 per cent more intense than the theoretical spectrum; that is, B is more nearly constant and its *average* value is about 10 per cent larger than shown here.

Bremsstrahlung from Heavy Particles. If the incident particle is not an electron but is some particle having charge ze and rest mass M, then the total energy of the particle becomes $(T + Mc^2)$ in the numerator of Eq. (1.3), while the principal effect of the larger rest mass is found in σ_0 which becomes $\frac{1}{137}(ze^2/Mc^2)^2$. As would be expected, the bremsstrahlung from $\sim$2-Mev protons is not significant in comparison with the characteristic K and L X rays produced by ionization in the target (L33, B36).