Dynamical Evolution of Globular Clusters

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We investigate the dynamical evolution of Galactic Globular Cluster System in considerably greater detail than has been done hitherto, finding that destruction rates are significantly larger than given by most previous estimates. For the evolution of individual clusters we use a Fokker-Planck code including most important physical processes governing the evolution: evaporation of stars due to two-body relaxation, tidal truncation of clusters, compressive gravitational shocks while clusters pass through the Galactic disk, and tidal shocks due to passage close to the bulge. Gravitational shocks are treated comprehensively, using a recent result by Kundić & Ostriker (1995) that the $\langle \Delta E^2 \rangle$ shock-induced relaxation term, driving an additional dispersion of energies, is generally more important than the usual energy shift term $\langle \Delta E \rangle$. Various functional forms of the correction factor are adopted to allow for the adiabatic conservation of stellar actions in a presence of transient gravitational perturbation.

We find that the total destruction rate for our sample is much larger than that given by AHO with of order half of the present clusters destroyed in the next Hubble time (6 ± 1 per Gyr). Alternatively put, the typical time to destruction is comparable to the typical age, a results that would follow from (but is not required by) an initially power law distribution of destruction times. We discuss some implications for a past history of the Globular Cluster System, and the initial distribution of the destruction times raising the possibility that the current population is but a very small fraction of the initial population with the remnants of the destroyed clusters constituting presently a large fraction of the spheroid (bulge+halo) stellar population.

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1. Previous Studies

Globular clusters are thought to be the oldest stellar systems in our Galaxy, and a history of attempts by theoreticians and observers to understand the keys of their evolution is as old as the discipline of stellar dynamics, with a comprehensive review provided by Spitzer (1987). Yet, these relatively simple stellar systems are not understood well enough to predict their future with desirable accuracy, in part due to lack of accurate observational values for the current dynamical state and in part due to a residual uncertainty concerning the complete catalog of relevant physical processes operating on these systems. More than that, we have almost no clues about their past, and in particular, whether what we see now is representative of the real initial Globular Cluster System, or just a small leftover after a great destructive battle which occured earlier in the history of the Galaxy.

Some first successful theoretical considerations over the processes governing the dynamical evolution of globular clusters started in 1938, when the Soviet astronomer V. A. Ambartsumian wrote a paper describing what we now call evaporation of clusters due to two-body relaxation. Significant progress in understanding of the evolution of an isolated cluster was achieved using Monte Carlo and Fokker-Planck calculations. An excellent summary of the results for isolated clusters is given by Spitzer (1987). However clusters orbit around the Galaxy, and experience time-varying tidal gravitational forces. The average tidal field determines the tidal cutoff of the cluster density distribution; but since the tidal force is obviously changing as a cluster approaches and recedes from the Galactic center, this approximation is too simple. The time-dependent part is a sum of

the forces produced by different galactic components, for which we consider the disk and the bulge. Ostriker, Spitzer, & Chevalier (1972) noted that the variation of the gravitational acceleration as the cluster passes through the disk compresses it to the Galactic plane, and the orbits of stars forming the cluster receive a tidal kick. On average, a cluster will gain energy and the evolution accelerates. A similar effect is produced by the compact nuclear component when the cluster approaches close to the Galactic center (see Aguilar, Hut, & Ostriker 1988; hereafter AHO).

Chernoff, Kochanek, & Shapiro (1986) used a semi-analytical Monte-Carlo technique to estimate the importance of the different mechanisms acting upon the cluster. They considered two-body relaxation, tidal stripping of stars outside a Roche lobe, and the first order tidal shocking effect - due to the disk and interactions with giant molecular clouds (GMC). For each of those processes they calculated the cluster mass and energy changes associated with them to predict the evolution. They followed the cluster evolution only up to core collapse, and assumed a single-mass King model (King 1966) for the internal cluster structure. Tidal heating due to the GMC was found to be negligible compare to the disk shocks. Note however, that the Galactic model assumed in that work (Bahcall, Schmidt, & Soneira 1983; BSS) is strongly favorable to the disk shock due to the exponential increase of disk surface density as galactocentric radius decreases. An alternative dynamical model for the Galaxy (Ostriker & Caldwell 1983; OC) predicts different disk behavior, and we can expect that the disk shock importance differs for these two models. Chernoff et al. also concluded that many of the clusters located within inner 3 kpc from the Galactic center have undergone a core collapse, and many of them may already have been destroyed. A number of authors have pointed out the bulge and stellar spheroid themselves could be composed from remnants of the destroyed globular clusters, if prior destruction of globular clusters occured at high enough rate. Chernoff & Shapiro (1987) used a similar method and included a power-law initial mass function for the cluster stars. Mass loss due to stellar evolution is important for an early evolution of the cluster, but then fades away because the mass loss is large only for massive stars whose lifetime is short.

The most comprehensive study to date of a single cluster evolution was done by Chernoff & Weinberg (1990). They used a Fokker-Planck code with an extensive spectrum of stellar masses (20 species), which includes stellar evolution and relaxation processes. It was found that the core collapse for a multi-mass model occurs much faster than for any single-component cluster, and involves smaller number of stars. In a single mass case energy is transferred between the cluster core and the envelope, which requires a conduction mechanism acting on a half-mass relaxation time scale. In the multi-mass case the energy exchange occurs between different species inside the core, and thus requires only few core relaxation times. Also, mass segregation (Spitzer 1987) speeds up the collapse. Mass loss during first 5×10^9 yr is sufficiently strong to disrupt weakly concentrated clusters ($c < 0.6$). Combined with the relaxation, it destroys many low massive and low concetrated clusters within a Hubble time. Today's clusters may be leftovers from the more massive clusters surviving the initial mass loss and undergoing core collapse.

However only comparison with observations can answer which models are right and which destruction mechanisms are more important than others. The present characteristics and evolution rates of the observed globular clusters were investigated by AHO who used a sample of the 83 Galactic globular clusters with the known structural parameters and line-of-sight velocities. They considered virtually all the important physical mechanisms (but not the mass loss due to stellar evolution) to calculate the present day destruction rates for the clusters in the sample. The rates were defined as the inverse

time it takes for a given mechanism to dissolve a cluster, in units of a Hubble time which is nominally defined as 10^{10} yr. They estimated the rates for the evaporation, disk and bulge tidal shocks, and dynamical friction. The last process has not been widely investigated in its application for the clusters evolution. Its effect, the gradual spiraling of the cluster toward the Galactic center, as the cluster loses orbital energy due to continuous interactions with field stars, is a relatively unimportant one, except for unusually massive clusters. They calculated a number of orbits associated with each cluster in the sample and took a median over the whole resulting distribution for a corresponding destruction rate. They investigated two Galactic models (OC and BSS), and two kinematic models for the globular cluster system: isotropic, which corresponds to the current distribution, and radially biased, which resembles that of the halo stars and might be closer to an *initial* cluster distribution. The assumption concerning the velocity distribution is nessesary as only one velocity component is observed. The central bulge was found to be very efficient in destroying clusters on highly elongated orbits. This leads to an *isotropization* of the orbits, and even preferrential survival of tangentially biased orbits in the Hubble time. Thus, an intially radial distribution better fits the present population, than an initially isotropic one. At the current time, evaporation is the most important destruction process. The difference between the two Galactic models was found to be relatively small, except for the disk shocking that obviously depends on the surface mass profile. AHO introduced a weighting factor that accounts for the fact that some orbits are less probable because of the various destruction mechanisms acting upon the cluster. Such a weighting reduces the computed rates, but converges to a similar value of 0.05 for all Galactic models and kinematic profiles considered. This implies that at present only 4 or 5 clusters are destroyed per Hubble time.

Finally, a recent paper Hut & Djorgovski (1992) used a statistical approach and analytical estimates for the relaxation times for sample of 140 clusters. They concluded that the current evaporation rate is 5 ± 3 Gyr^{-1} (about ten times the rate found by AHO), which means that a significant fraction of the present day clusters would be destroyed in a next Hubble time.

2. Method

2.1. *Sample of Globular Clusters*

We have used the most recent compilation of globular cluster parameters (Djorgovski & Meylan, 1993). A detailed description of our sample is given in Gnedin & Ostriker (1996; hereafter GO).

The Ostriker-Caldwell (1983) model D-150 was assumed for the Galaxy. Following AHO we investigate two *initial* kinematic models for the globular cluster system. The first one has an isotropic peculiar velocity distribution with constant velocity dispersion. This is the simplest model still consistent with the observations. We used the one-dimentional dispersion of 118 km s^{-1}, resulting from our sample. The second model is anisotropic with the anisotropy radius $r_a \approx 7.6$ kpc (AHO). Cluster orbits for this distribution are nearly isotropic within r_a, and become more and more radial with increasing distance from the center. The two unknown velocity components for each cluster were drawn randomly using the rejection method (Press et al. 1992) in such a way that the total three-dimentional peculiar velocity be consistent with the assumed kinematic model.

2.2. *Evaporation*

Two-body relaxation leads to the escape of stars approaching the unbound tail of the cluster velocity distribution (Ambartsumian 1938; Spitzer 1940). Tidal truncation due

to the Galactic potential accelerates this process. Much more dramatic effects result from a gravitational instability, when the inner part of cluster contracts (core collapse), and the envelope expands. This, in turn, accelerates the rate of evaporation of stars from the cluster. A recent review of pre- and post-core-collapse evolution of a tidally truncated cluster is given by Goodman (1993).

It is conventional to express the life-time of a cluster in terms of the half-mass relaxation time (Spitzer & Hart 1971)

$$t_{\rm rh} \equiv 0.138 \frac{M^{1/2} R_h^{3/2}}{G^{1/2} m_* \ln(\Lambda)}, \tag{2.1}$$

where M is the total cluster mass, R_h is the half-mass radius, m_* - average stellar mass, and $\ln(\Lambda) = \ln(0.4N)$ - Coulomb logarithm, N being the number of stars in the cluster. We use our result to quantify the total destruction time in terms of $t_{\rm rh}$.

2.3. *Disk Shocking*

When a cluster passes through the Galactic disk, it experiences a time-varying gravitational force squashing the cluster toward the disk. Impact of the shock on individual stars depends on the ratio of the characteristic shock time to the dynamical time of stars

$$x_d \equiv \frac{2\omega H}{V_z}, \tag{2.2}$$

where ω is the stellar oscillation frequency, H is the vertical scale height of the disk, and V_z is the vertical component of the cluster orbital velocity. Ostriker, Spitzer, & Chevalier (1972) originally computed the energy change per unit mass for the cluster in the impulse approximation, where stars are assumed to move little during the shock. This is valid for outer stars in the cluster. Spitzer (1987) extended the analysis for stars in the harmonic oscillator potential. He obtained

$$\langle \Delta E \rangle_{\rm disk} = \frac{2\, g_m^2\, r^2}{3\, V_z^2}\, A_d(x_d), \tag{2.3}$$

where g_m is the maximum vertical gravitational acceleration produced by the disk, r is the distance from a star to the cluster center, and the factor A_d accounts for the adiabatic protection of star's orbit:

$$A_d(x) = \exp\left(-\frac{x^2}{2}\right). \tag{2.4}$$

Recently Weinberg (1994) showed that actual impact of the shock is less damped by the adiabatic conservation due to numerous resonances. The adiabatic factor A_d follows power-law in x_d, instead of the exponential. We take the simplest form of the correction

$$A_d(x) = \left(1 + \frac{x^2}{4}\right)^{-2}. \tag{2.5}$$

We refer to equation (2.4) as the Spitzer correction, and to equation (2.5) the Weinberg correction. Further discussion of the adiabatic protection can be found in Gnedin et al. (1996; hereafter GHJO).

Besides the shift in the energy of stars ($\langle \Delta E \rangle$), a gravitational shock induces also a quadratic term ($\langle \Delta E^2 \rangle$), which governs the dispersion over energy spectrum and thus pushes some loosely bound stars outside the tidal radius, speeding the disassociation of the cluster (Kundić & Ostriker (1995) 1995; hereafter KO). This tidal shock relaxation can be in many cases competitive with two-body relaxation in causing the evolution of a cluster. Like ordinary relaxation it causes a spread (diffusion) of initially similar orbits

and ultimately will tend to induce core collapse. The shock relaxation term is (KO eq. [25])

$$\langle \Delta E^2 \rangle_{\rm disk} = \frac{4\, g_m^2\, \omega^2\, r^4}{5\, V_z^2}\, A_d(x). \tag{2.6}$$

Disk shocks occur twice during the orbital period of the cluster, so we define the disk shock time scale at the half-mass radius as

$$t_{\rm disk} \equiv \frac{P_{\rm orb}}{2}\left(\frac{-E}{\Delta E_h}\right) = \frac{3}{8}\left(\frac{V_z}{g_m}\right)^2 P_{\rm orb}\,\omega_h^2, \tag{2.7}$$

where $P_{\rm orb}$ is the orbital period, $E \approx -0.2GM/R_h$ - cluster energy, and the ΔE and ω are evaluated at R_h.

2.4. *Bulge Shock*

Similar to the compressive disk shock, every globular cluster experiences a tidal shock during its passage close to the Galactic center. The massive compact component at the center of the Galaxy (bulge) induces a strong tidal force on the cluster near the perigalactic point of the cluster orbit. The difference between this effect and that from the smooth and steady tidal field produced by the Galaxy at every moment in a cluster's life is primarily due to the time dependence of the bulge shock. The formalism is given by GHJO.

$$\langle \Delta E \rangle_{\rm bulge} = \frac{4}{3}\left(\frac{G\, M_b}{V_p\, R_p^2}\right)^2 r^2\, A_b(x_b)\, \chi(R_p)\, \lambda(R_p, R_a). \tag{2.8}$$

Here M_b is the bulge mass, V_p is the cluster velocity at the perigalacticon R_p, and A_b is the corresponding adiabatic correction. Two new corrections $\chi(R_p)$ and $\lambda(R_p, R_a)$ take into account the extended mass distribution of the bulge (Gnedin & Hernquist, 1996), and the time variation of the tidal force along an elliptic orbit of the cluster (AHO).

The adiabatic correction $A_b(x)$ is given by Spitzer: $A_b(x) = 1/2\,[L_x(x)+L_y(x)+L_z(x)]$ in his notation (Spitzer 1958, eqs. [36-38]), where $x_b = 2\omega R_p/V_p$. We assume the same Weinberg correction as for the disk shock, equation (2.5).

As in the case of the disk shock, the bulge shock induces the second, relaxation term

$$\langle \Delta E^2 \rangle_{\rm bulge} = \frac{8}{5}\left(\frac{G\, M_b}{V_p\, R_p^2}\right)^2 \omega^2\, r^4\, A_b(x_b)\, \chi(R_p)\, \lambda(R_p, R_a). \tag{2.9}$$

Similarly we define the bulge shock time scale. However in this case, the effect occurs only once per cluster orbital period.

$$t_{\rm bulge} \equiv P_{\rm orb}\left(\frac{-E}{\Delta E_h}\right) = \frac{3}{8}\left(\frac{V_p\, R_p^2}{G\, M_b}\right)^2 P_{\rm orb}\,\omega_h^2. \tag{2.10}$$

2.5. *The Fokker-Planck code*

Our Fokker-Planck code is described in Gnedin, Lee, & Ostriker (1996), and is a modification of the code used by Lee & Ostriker (1987). We perform several sets of runs of the Fokker-Planck code, increasing number of the processes allowed to act upon the cluster. We organized the following series: 0) evaporation only; evaporation + 1) disk shock, $\langle \Delta E \rangle$ term only, no tidal shock relaxation; 2) disk shock, including the relaxation term $\langle \Delta E^2 \rangle$; 3) disk shock + bulge shock, without tidal shock relaxation; and finally 4) disk shock + bulge shock, including the relaxation. The last run includes all the processes we consider in this paper and represents the total destruction rate computed for the globular clusters.

FIGURE 1. Evaporation time in units of initial relaxation time versus cluster concentration. Dots are results of the Fokker-Planck calculations with the observational data as input. Solid line - out fit, equation (3.12).

For each set of runs we repeat the calculations four times, allowing for the different kinematic models and adiabatic conservation factors for the shocks. First three runs are for the isotropic velocity distribution. The first run includes the Spitzer correction, the second - Weinberg correction, and the third assumes the impulse approximation with no adiabatic correction applied. The last run uses the anisotropic kinematic model and the Spitzer adiabatic correction.

Given the destruction times t_d for all the clusters in the sample, we obtain the destruction rate in units of a Hubble time

$$\nu = \frac{t_{\rm Hubble}}{t_d} = \frac{10^{10}\ {\rm yr}}{t_d}. \tag{2.11}$$

This definition agrees with the destruction rates of AHO, thus allowing a direct comparison. Since the resulting distribution of the rates is broad and in general asymmetric around the mean, we choose to take a median $\tilde{\nu}$ of the sample as a characteristic value, in accordance with AHO. Detailed discussion of this procedure and error evaluation are given in GO.

3. Results

3.1. *Evaporation*

We turn first to the evaporation of stars from clusters. Although many studies have been devoted to normal two-body relaxation, we are unaware of any complete survey of a distribution of destruction times for different cluster parameters. Figure 1 shows the evaporation time in units of the initial relaxation time versus cluster structural parameter, concentration $c \equiv \log(R_t/R_c)$, where R_t and R_c are the tidal and core radii, respectively. Almost all points on this Figure lie along one curve. This fact is quite amazing, if one recalls that we have used the observational data with inherent errors of measurements. We fit the dependence by a simple formula

$$\frac{T_{rh}}{T_{ev}} = 1.290 \times 10^{-1} - 1.170 \times 10^{-1}\, c + 3.282 \times 10^{-2}\, c^2 + \frac{7.355 \times 10^{-4}}{(c - 0.55)^2}, \tag{3.12}$$

	Isotropic			Anisotropic
Process	Spitzer	Weinberg	Impulse	Spitzer
Evaporation	$0.290\ ^{+0.020}_{-0.013}$	$0.290\ ^{+0.020}_{-0.013}$	$0.290\ ^{+0.020}_{-0.013}$	$0.290\ ^{+0.020}_{-0.013}$
+ Disk 1st	$0.307\ ^{+0.073}_{-0.017}$	$0.307\ ^{+0.073}_{-0.018}$	$0.312\ ^{+0.069}_{-0.021}$	$0.307\ ^{+0.073}_{-0.018}$
+ Disk 2nd	$0.348\ ^{+0.048}_{-0.049}$	$0.350\ ^{+0.051}_{-0.050}$	$0.363\ ^{+0.038}_{-0.050}$	$0.317\ ^{+0.078}_{-0.010}$
+ Bulge 1st	$0.467\ ^{+0.073}_{-0.058}$	$0.433\ ^{+0.091}_{-0.038}$	$0.601\ ^{+0.096}_{-0.091}$	$0.398\ ^{+0.088}_{-0.085}$
+ Bulge 2nd	$0.506\ ^{+0.117}_{-0.043}$	$0.483\ ^{+0.075}_{-0.068}$	$0.787\ ^{+0.093}_{-0.099}$	$0.406\ ^{+0.082}_{-0.051}$

TABLE 1. Destruction rates for the Globular Cluster sample.

FIGURE 2. Mean probability per Hubble time for cluster destruction due to evaporation, and the evaporation plus gravitational shocks: disk shock without relaxation, disk shock including relaxation term, disk+bulge shocks, and disk+bulge shock including relaxation, as a function of present day galactocentric distance. The error bars indicate the interquartile range of the distribution. Adiabatic correction are due to Spitzer.

where T_{rh} is the half-mass relaxation time, and T_{ev} is the total evaporation time. Our fit is shown on Figure 1 as a solid line.

3.2. *Gravitational Shocks*

Table 1 summarizes the results of our simulation when all the physical processes are acting on the clusters.

We investigate the dependence of the destruction rates on the current observed cluster position. Figure 2 shows the destruction rates associated with the several combinations of the physical mechanisms vs galactocentric radius, for our sample. The results correspond to the Spitzer adiabatic correction. We present also histogram distributions of the destruction rates in Figures 3.

3.3. *Vital Diagram for Globular Clusters*

We do not know what was the initial distribution of globular clusters in our Galaxy. But we can imagine that they occupied some volume in a given parameter space. All the physical mechanisms considered above tend to destroy clusters with time, and thus to reduce the allowed volume. They superimpose particular boundaries which distinguish the present day clusters from those being already dissolved (or never formed). Ostriker (1974) and Fall & Rees (1977) considered the cluster mass versus their typical

FIGURE 3. Histogram distribution of the destruction rates from Figure 2. Spitzer adiabatic correction is used.

size (half-mass radius R_h) diagram. The authors included evaporation, disk shock and tidal shock heating (for the latter they assume the tidal interaction with the clusters themselves rather than with the bulge), plus dynamical friction. These processes cut a triangle on the $R_h - M$ plane, containing the observed clusters pretty well (see also Caputo & Castellani 1984 for an updated version).

We constructed such a vital diagram for the Galactic GCS using our sample and results of the computations of its evolution. All of the processes used in the simulations, as well as the dynamical friction, participate in the diagram. The vital boundary is defined in such a way, that the sum of all the destruction rates is equal to the inverse Hubble time:

$$\frac{1}{t_{\rm Hubble}} = \frac{1}{t_{ev}} + \frac{1}{t_{sh}} + \frac{1}{t_{df}}, \tag{3.13}$$

where t_{ev}, t_{sh}, and t_{df} are the time-scales over which a cluster would be destroyed by the given process alone, for the evaporation, disk and bulge shock combined, and dynamical friction, respectively. Note that throughout this paper we adopted for simplicity $t_{\rm Hubble} = 10^{10}$ yr.

The cluster vital diagram is shown in Figure 4. One can see that a significant fraction of the clusters lies outside of the "surviving" boundary. This supports our conclusion that many clusters will be destroyed within the next Hubble time. The allowed area shrinks with decreasing galactocentric radius and vanishes altogether for clusters on circular orbits having $R < 2$ kpc. From Figures 3 and 4, and Table 1, we see that about half the clusters will be destroyed in the next Hubble time with the current rate equal to 6 ± 1 per Gyr.

4. Application to the Destruction History of Globular Clusters

Using the Fokker-Planck simulations and the sample of the globular clusters we estimated a current destruction rate per Hubble time. These results are applicable for the present day clusters evolving forward in time for the next Hubble time. It is also of prime interest to extract any possible piece of information regarding the past evolution of the clusters from the time of their formation up to now. We try to construct a simple model for the initial distribution of the clusters and to test it against the current destruction rate.

FIGURE 4. Vital diagram for the Galactic globular clusters.

Let $t_d(t)$ be a time remaining to the total destruction of a given cluster at epoch t. We then define $f(t_d;t)\,dt_d$ to be the number of clusters with the destruction time in the interval $[t_d, t_d + dt_d]$ at time t. Thus, if we treat almost all clusters as made in a short time interval at the formation of the Galaxy (an obviously gross over-simplification), then $f(t_d;0)\,dt_d$ gives the initial distribution of cluster lifetimes. The normalization is

$$N(t) = \int_0^\infty f(t_d;t)\,dt_d, \tag{4.14}$$

where $N(t)$ is the number of clusters at epoch t. We would like to advance our function f in time starting from their formation, again without a detailed prescription for the evolution of the cluster sample. Since we assume that all the clusters were formed at the same time, and that this time $t_0 \approx 0$ is very small compare to the Hubble time, the number of clusters surviving at the time t is just the number of initial clusters with $t_d > t$. Thus we have the following relation:

$$f(t_d;t) = f(t_d + t;t_0), \tag{4.15}$$

where we have neglected the contribution of $t_0 \ll t$ in the first argument of the function f. We call $f(t_d;t_0) \equiv f_i(t_d)$ the initial ditribution of the globular clusters. Integrated over all destruction times t_d it gives the initial number of gobular clusters $N(t = t_0) \equiv N_i$ formed in our Galaxy.

Now we choose two functional forms for $f_i(t_d)$, which we test then against the distribution of the destruction rates obtained in Section 3. The simplest one is that with a constant mean destruction time for all clusters. It assumes an exponential form

$$f_{i1}(t_d) \;=\; C_1\, e^{-\alpha\, t_d}. \tag{4.16}$$

It is easy to show that the mean and median for this distribution are $\bar{t}_{d1} = 1/\alpha$, $t_{m1} = \ln 2/\alpha$. Thus the mean destruction rate does not depend on time, and the evaporation of the clusters in this case is similar to radioactive decay. Given a current vital rate we can always calculate how many clusters survived from the beginning. $N(t)$ goes exponentially to zero

$$N_1(t) = \frac{C_1}{\alpha}\, e^{-\alpha t}. \tag{4.17}$$

FIGURE 5. Distribution of the destruction times for our sample for the case when all the destruction processes are present. Solid line - normalized to the present time power-law distribution, dashes - normalized exponential model.

The other function f we consider is a scale free power-law

$$f_{i2}(t_d) = C_2\, t_d^{-q}. \tag{4.18}$$

This distribution is not strictly normalizable since the total number of clusters N_i diverges as t_0 approaches zero. However we can successfully apply it to the present time. The corresponding mean and median are $\bar{t}_{d2}(t) = t/(q-2), q > 2$; $t_{m2}(t) = t(2^{1/(q-1)} - 1)$. Note that both these quantities are proportional to the time of observation. Thus it seems impossible to determine the size of the initial cluster population for this distribution given the current rate. Fortunately, we can compare the *shape* of the distribution with the current profile to choose between the two distributions. In addition we note a very important difference consequent to the two hypothesized forms for the initial distribution. In the second (power law) case we should expect that $\bar{t}_d \sim t$; whenever we look, the time to destructionfor the clusters that remain is of the same order as the age of the existing sample. In the first case this might occur, but it would require a coinsidence between the initial typical time to destruction and the later point in time when an observer examines the system. The most important point to be made in this paper is that the numbers in the last line of Table 1 are of order unity. This is consistent with the power law assumption and allows the possibility of large fractional destruction.

On Figure 5 we plot a histogram of the destruction rates for our sample of 119 clusters. The dotted region gives the number of cluster per logarithmic bin in t_d. Two panels correspond to the results obtained for the isotropic kinematic model with the Spitzer and Weinberg adiabatic corrections, respectively. We use the medians for the two cases to determine the shape of the two models for the distruction rates. The dashed curve corresponds to the exponential model, and the solid line is for the power-law one. Both lines are normalized to the present number of clusters, $N(t_{\rm Hubble})$.

We see from this plot that both models are consistent with the actual distribution. However, in our view, the power-law case better represents the overall extended shape of the histogram. The exponential model is more concentrated around the median value, and falls off very rapidly for large t_d. It predicts also much larger number of clusters in the center of the diagram than follows from the Fokker-Planck calculations. Therefore we favor the second choice, namely, the power-law distribution of the destruction times. Based on the results obtained with the Spitzer adiabatic correction, the power-law index

is $q \approx 1.636$. For the Weinberg case we find $q \approx 1.618$. Thus, we conclude that the power-law model [eq. (4.18)] with $q = 1.6$ gives naturally the present distribution of the globular cluster destruction times.

From the last sentence it immediately follows that the initial cluster population could have been much larger than the present one. The present day characteristics do not provide a definite number, though they do select this scenario. Thus it is quite possible that a very significant fraction of the original globular clusters have been already destroyed. Their remnants might constitute now the spherical stellar component of our Galaxy. We note also from Figure 2 that clusters close to the Galactic centers are most heavily attacked by the bulge shock, and therefore most of the dissolved clusters could be in the very central region of the Galaxy. A strong bulge component present in the Ostriker & Caldwell galactic model could be enhanced by the input from the destroyed clusters. Note, that we do *not* require the existance of the bulge from the first days of the Galactic history in order to destroy the clusters. As follows from the power-law distribution, most of the initially formed clusters had relatively short life-times, and presumably were not very massive. Thus the main mechanism driving their dissolution could be the internal relaxation. Though, if present in some form, the central Galactic component would be very efficient in the cluster destruction. In any case the overall potential of the Galaxy is changed only very slightly if we were to imagine that all bulge and spheroid stars were put back into globular clusters with appropriate orbits. Thus even the bulge shock tidal evaporation processes would be essentially unchanged, if we were to put all stars in the quasispherical distribution back into clusters.

Observational data on stellar population indicate that the spheroid of the Galaxy is kinematically and chemically distinct from the disk (Norris & Ryan 1991). Halo stars are old (with an age slightly exceeding our adopted Hubble time of 10^{10} yr) and metal poor. Because the density of the protogalactic gas cloud is not high enough to form stars efficiently, the oldest stars in the Galaxy are likely to have been formed in clusters perhaps not too different from the present population of globular clusters. Now we see most of the metal poor stars in the halo, which suggest that most of them could be left over from the disrupted clusters. Harris (1991) proposed two arguments against this idea: 1) orbits of the present globular clusters are more isotropically distributed than those of spheroid stars; 2) the clusters have systematically lower metallicity than the field stars, and become more metal poor with increasing galactocentric distance R. However Lee & Goodman (1995) pointed out a possible solution for this problem. The first of these arguments can be moderated if we consider the initial cluster distribution. AHO argued that the kinematic model describing better current population is an isotropic one with the orbits getting more and more radial as R inscreases. The destruction mechanisms are the strongest in the central few kiloparsecs, especially the bulge shock. Thus the clusters in the most elongated orbits with small perigalactic distance R_p would be destroyed first. Their remnants would populate the spheroid as wee see it now, with the preferentially radial orbits of stars. Surviving clusters, on the other hand, have more isotropic orbits consistent with the observations.

To resolve the problem of metallicities, we refer to van den Bergh (1995), who found that [Fe/H] correlates somewhat more strongly with perigalacticon R_p than with their current position R. Thus the metal rich clusters with smaller R_p, which would be destroyed first, would enrich the halo stellar population. Another solution comes from the observed age difference in the cluster population. Formation period could have been extended up to $2-5$ Gyr (see references in Norris & Ryan 1991). Thus the very first low mass clusters could dissolve due to the internal relaxation, but their stars would enrich the primordial gas for the next generation of clusters. Our power-law hypothesis for the

distribution of the clusters predicts that the mean (and median) destruction time is proportional to the time elapsed since the formation, so that newly formed clusters would be again most susceptable to disruption. If they were destroyed within the remaining $7 - 10$ Gyr, their stars are the population II halo stars we see in our Galaxy.

We are greatly indebted to Hyung Mok Lee, who provided us with his Fokker-Planck code. We thank Luis Aguilar for sharing the orbit integration routine used by AHO. This project was supported in part by NSF grant AST-9424416.

REFERENCES

Aguilar, L., Hut, P., & Ostriker, J. P. 1988, *ApJ, 335, 720*

Ambartsumian, V. A. *1938, Uch. Zap. L.G.U., 22, 19. English transl. in IAU Symposium 113, Dynamics of Star Clusters, ed. J. Goodman, & P. Hut (Dordrecht: Reidel), 521*

Bahcall, J. N., Schmidt, M., & Soneira, R. M. *1983,* ApJ, 265, 730

Caputo, F., & Castellani, V. 1984, *MNRAS, 207, 185*

Chernoff, D. F., Kochanek, C. S., & Shapiro, S. L. *1986,* ApJ, 309, 183

Chernoff, D. F., & Shapiro, S. L. 1987, *ApJ, 322, 113*

Chernoff, D. F., & Weinberg, M. D. *1990,* ApJ, 351, 121

Djorgovski, S. G. & Meylan, G., eds. Structure and Dynamics of Globular Clusters, ASP Conf. Ser., vol. 50 (San Francisco: ASP)

Fall, S. M., & Rees, M. J. 1977, *MNRAS, 181, 37P*

Gnedin, O. Y., & Hernquist, L. *1996, in preparation*

Gnedin, O. Y., Hernquist, L., Johnston, K. V., & Ostriker, J. P. *1996, in preparation*

Gnedin, O. Y., Lee, H. M., & Ostriker, J. P. *1996, in preparation*

Gnedin, O. Y., & Ostriker, J. P. *1996, in preparation*

Goodman, J. *1993, in Structure and Dynamics of Globular Clusters, ASP Conf. Ser., vol. 50, ed. S. G. Djorgovski, & G. Meylan (San Francisco: ASP), 87*

Harris, W. E. *1991,* ARA&A, 29, 543

Hut, P., & Djorgovski, S. 1992, *Nature, 359, 806*

King, I. R. *1966,* AJ, 71, 64

Kundić, T., & Ostriker, J. P. 1995, *ApJ, 438, 702*

Lee, H. M., & Goodman, J. *1995,* ApJ, 443, 109

Lee, H. M., & Ostriker, J. P. 1987, *ApJ, 322, 123*

Norris, J. E., & Ryan, S. G. *1991,* ApJ, 380, 403

Ostriker, J. P. 1974; unpublished

Ostriker, J. P., & Caldwell, J. A. 1983, in Kinematics, Dynamics and Structure of the Milky Way, ed. W. L. H. Shuter (Dordrecht: Reidel), 249

Ostriker, J. P., Spitzer, L. Jr., & Chevalier, R. A. 1972, *ApJ Lett., 176, L51*

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. *1992, Numerical Recipes in C (2nd ed.; Cambridge: Cambridge Univ. Press), p. 290*

Spitzer, L. Jr. *1940,* MNRAS, 100, 397

Spitzer, L. Jr. 1958, *ApJ, 127, 17*

Spitzer, L. Jr. *1987, Dynamical Evolution of Globular Clusters (Princeton: Princeton Univ. Press)*

Spitzer, L. Jr., & Hart, M. H. *1971,* ApJ, 164, 399

van den Bergh, S. 1995, preprint

Weinberg, M. D. 1994, *AJ, 108, 1398; 1403; 1414*