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Safety Guaranteed Connected Cruise Control

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Abstract— **In this paper, we design a connected cruise controller with safety guarantees. In particular, we utilize a control safety function in order to guarantee the safety of a given control law. We establish the notion of safety chart to graphically represent the safe combinations of control parameters. Moreover, we establish an intervention algorithm that maintains safety when parameters are chosen outside the safe parameter regime. The results are also demonstrated with the help of numerical simulations.**

I. Introduction

Recent results in utilizing vehicle-to-vehicle (V2V) connectivity in vehicle automation has been showing significant improvements in congestion mitigation, fuel economy and vehicle safety [1]–[4]. However, so far there exist no guarantees that can keep the V2V-based controllers collision free. In this paper we target this important aspect and design connected cruise controllers with guaranteed safety performance. In order to keep the scope of the paper manageable we focus on the simple scenario where the follower monitors the motion of its predecessor via V2V communication.

In order to integrate safety considerations into feedback control design, one may calculate sets in state space that remain invariant under certain feedback laws [5], [6]. Such approach is usually quite challenging in practice. However, recently the notion of control safety function has been introduced [7], [8] that allows one to certify the invariance of a chosen set in state space. In this paper, we utilize these techniques for the safety enhancement of connected cruise control. In particular, we establish the notion of safety charts that allow one to select safe parameters for a given control law. Moreover, in order to handle the case with unsafe parameters, an intervention scheme is developed.

The reminder of the paper is organized as follows. Section II establishes the connected cruise control design and defines the safety requirements. Section III introduces the method of safety verification for given feedback law. This is applied to guarantee the safety of a connected cruise controller in Section IV. Finally, we conclude the paper in Section V.

II. Connected Cruise Control Design

In this section we explain the connected cruise control design using a simple predecessor-follower setup shown in Fig. 1(a). More complicated scenarios are discussed in [4], [9], [10]. Here we emphasize the role of actuation constraints and safety requirements.

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A. Modeling and feedback law

To model the longitudinal dynamics of the vehicles, we use

$$\begin{bmatrix}\dot{h}\\ \dot{v}\\ \dot{v}_1\end{bmatrix} = \begin{bmatrix}0 & -1 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}h\\ v\\ v_1\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} a_1 + \begin{bmatrix}0\\1\\0\end{bmatrix} a, \quad (1)$$

where h is the distance headway, that is, the bumper-to-bumper distance between the vehicles, v is the speed of the following vehicle, and v_1 is the speed of the preceding vehicle; see Fig. 1(a). Moreover, a_1 denotes the acceleration of the preceding vehicles which serves as a disturbance while a denotes the acceleration of the following vehicle that will be assigned by the designed connected cruise controller.

It is assumed that a and a_1 are bounded due to the limited driving and braking torques available, that is,

$$a \in [-\underline{a}, \overline{a}], \quad a_1 \in [-\underline{a}_1, \overline{a}_1]. \quad (2)$$

Moreover, we also restrict speed of both vehicles to the domain

$$v, v_1 \in [0, \overline{v}]. \quad (3)$$

We remark that one may model the longitudinal dynamics by incorporating dissipations like rolling resistance and air drag [11]. However, for safety considerations it make sense to omit these as the simple dynamics (1) will lead to slightly more conservative results.

We refer to (1) as the open-loop system. The goal of connected cruise control design is to generate a feedback law $u(h, v, v_1, a_1)$. The design criteria typically include attenuation of velocity fluctuations (often referred as string stability) as well as minimizing energy consumption [12]. According to (2) the feedback law enters (1) via $a = \mathrm{sat}(u)$ where the saturation function is represented graphically in Fig. 1(b). This leads to the closed-loop system

$$\begin{bmatrix}\dot{h}\\ \dot{v}\\ \dot{v}_1\end{bmatrix} = \begin{bmatrix}v_1 - v\\ \mathrm{sat}\big(u(h, v, v_1, a_1)\big)\\ 0\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix} a_1. \quad (4)$$

The main question we try to answer here is that given a feedback law, how to make sure that a vehicle stays safe (i.e., avoids collision). This may be achieved by choosing the control parameters (e.g., feedback gains) appropriately which will lead to the concept of safety chart. Choosing parameters from the safe regimes of the chart can ensure collision free motion. On the other hand, if the designer wishes to select parameters outside the safe parameter regime (due to other design considerations) safety may still be maintained by intervening once the given feedback law would render the vehicle unsafe.

We remark that the structure presented above differs from typical adaptive cruise control design since the acceleration of the proceeding vehicle a_1 is utilized. While one may try to extract this information by taking derivatives of h and v, this requires heavy filtering, leading to significant inaccuracies and time lags. On the other hand, acceleration information can be sent via V2V communication with low latency and thus, it can be readily used in connected cruise control design [9].

Designers may come up with a plethora of feedback laws, but we require $u(h, v, v_1, a_1)$ to be continuous in its variables with piecewise continuous derivatives; see [13]. As an example we will consider the simple controller

$$u = \alpha\big(V(h) - v\big) + \beta\big(W(v_1) - v\big), \tag{5}$$

which is widely used in practice [4]. The first term aims to maintain the velocity dependent distance given by the range policy

$$V(h) = \begin{cases} 0 & \text{if} \quad h < h_{\rm st}, \\ \kappa(h - h_{\rm st}) & \text{if} \quad h_{\rm st} \le h \le h_{\rm go}, \\ \overline{v} & \text{if} \quad h > h_{\rm go}, \end{cases} \tag{6}$$

shown in Fig. 1(c) where $h_{\rm st}$ is the desired the stopping distance, while $h_{\rm go} = \overline{v}/\kappa + h_{\rm st}$. The second term in (5) aims to match the speed of the follower with that of the predecessor while the function

$$W(v_1) = \begin{cases} v_1 & \text{if} \quad v_1 \le \overline{v}, \\ \overline{v} & \text{if} \quad v_1 > \overline{v}, \end{cases} \tag{7}$$

is introduced to avoid following a speeding predecessor.

We remark that with (5) one can ensure that $v \in [0, \overline{v}]$. On one hand, when $v = \overline{v}$, for any given h and v_1, one has

$$\begin{aligned} &\dot{v} = u \le \alpha(\overline{v} - \overline{v}) + \beta(\overline{v} - v) = 0 \\ &\Leftrightarrow \quad \dot{v} \le 0 \quad \text{if } v = \overline{v}. \end{aligned} \tag{8}$$

One the other hand, when $v = 0$, for any given h and $v_1 \ge 0$, one has

$$\begin{aligned} &\dot{v} = u \ge \alpha(0 - 0) + \beta(v_1 - v) \ge 0 \\ &\Leftrightarrow \quad \dot{v} \ge 0 \quad \text{if } v = 0. \end{aligned} \tag{9}$$

Thus, to simplify the matter, we will focus on the controller

$$u = \alpha\big(\kappa(h - h_{\rm st}) - v\big) + \beta(v_1 - v), \tag{10}$$

and draw safety charts in the parameter space spanned by the desired stopping distance $h_{\rm st}$, the range policy derivative κ, and the feedback gains α and β.

B. *Safety requirement*

A natural safety requirement would be having no collision, that is, $h > 0$ for all $t \ge 0$. In order to be more practical, here we set the requirement $h - v\tau \ge 0$ for all $t \ge 0$ where $\tau > 0$ is the minimum time headway allowed. Corresponding to this we define the target set

$$\mathcal{T} = \{(h, v) | \, h - v\tau \ge 0\}, \tag{11}$$

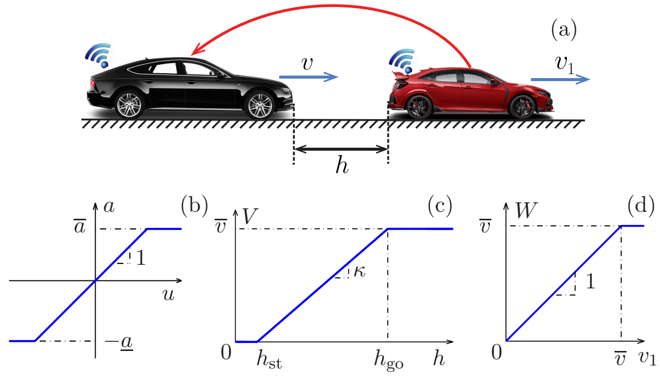

Fig. 1: (a) Two vehicles following each other on a single lane. (b) Saturation function in (4). (c) Range policy function (6). (d) Speed saturation function (7).

in state space. Thus, the safety requirement would correspond to the target set to be invariant: if the system starts in $\mathcal{T}$ it should stay in $\mathcal{T}$. However, it is easy to think of a scenario when this does not hold. Due to the limited deceleration capability of the follower (cf. (2)), for velocity v the distance needed to stop is $v^2/2\underline{a}$. When having a stationary vehicle within this distance, a collision is inevitable. Thus, the invariance of $\mathcal{T}$ can only be guaranteed if $v\tau \ge v^2/2\underline{a}$ for all v_1. This may require τ to be set to a few seconds which would result in unrealistically large distance headway.

In order to find a set in state space whose invariance can be ensured, we consider the scenario when the leader, initially traveling at speed v_1, applies its maximum deceleration $\underline{a}_1$. In the mean time, the follower, initially traveling at speed v, applies $\underline{a}$. Then, we investigate how the distance headway evolves between the vehicles before the follower comes to a halt and calculate how large the initial distance needs to be to avoid collision. Taking the maximum of this distance and $v\tau$ we obtain the set

$$\mathcal{C} = \{(h, v, v_1) | \, h - \hat{b}(v, v_1) \ge 0\}, \tag{12}$$

where $\hat{b}$ is defined as follows. If $\underline{a} \le \underline{a}_1$ then we have

$$\hat{b}(v, v_1) = \begin{cases} v\tau, & \text{if} \quad v_1 \ge f_1(v), \\ v\tau + \frac{(v - \underline{a}\tau)^2}{2\underline{a}} - \frac{v_1^2}{2\underline{a}_1}, & \text{if} \quad v_1 < f_1(v), \end{cases} \tag{13}$$

where

$$f_1(v) = \sqrt{\frac{\underline{a}_1}{\underline{a}}}(v - \underline{a}\tau). \tag{14}$$

On the other hand, if $\underline{a} > \underline{a}_1$ then

$$\hat{b}(v, v_1) = \begin{cases} v\tau, & \text{if} \quad v_1 \ge f_2(v), \\ v\tau + \frac{(v - \underline{a}\tau - v_1)^2}{2(\underline{a} - \underline{a}_1)}, & \text{if} \quad f_2(v) < v_1 < f_3(v), \\ v\tau + \frac{(v - \underline{a}\tau)^2}{2\underline{a}} - \frac{v_1^2}{2\underline{a}_1}, & \text{if} \quad v_1 \ge f_3(v), \end{cases} \tag{15}$$

where

$$f_2(v) = v - \underline{a}\tau, \quad f_3(v) = \frac{\underline{a}_1}{\underline{a}}(v - \underline{a}\tau). \tag{16}$$

A detailed derivation may be found in [8]. Having different cases in (13) and (15) corresponds to the fact that the minimal

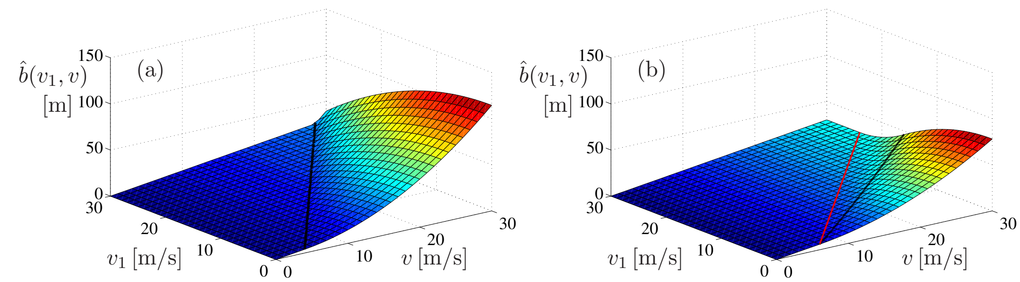

Fig. 2: Surface of $\hat{b}(v_1, v)$ defining the safe distance for minimum time headway $\tau = 1\,[\mathrm{s}]$. (a) The case given by (13) for $\underline{a} = 4\,[\mathrm{m/s^2}]$ and $\underline{a}_1 = 6\,[\mathrm{m/s^2}]$ with the black line indicating the switch (14). (b) The case given by (15) for $\underline{a} = 6\,[\mathrm{m/s^2}]$ and $\underline{a}_1 = 4\,[\mathrm{m/s^2}]$ with the black and red lines indicating the switches (16).

distance headway may appear at different phases of the braking event: right at the moment when both vehicles launch the emergency brake, during the deceleration phase of the following vehicle, and at the end when the following vehicle stops.

Indeed, $\mathcal{C} \subseteq \mathcal{T}$ as can be observed in Fig. 2 where we set the time headway $\tau = 1\,[\mathrm{s}]$ and consider $\overline{v} = 30\,[\mathrm{m/s}]$. Fig. 2(a) corresponds to $\underline{a} \leq \underline{a}_1$, in particular, $\underline{a} = 4\,[\mathrm{m/s^2}]$ and $\underline{a}_1 = 6\,[\mathrm{m/s^2}]$. That is, the surface is given by (13) where (14) is indicated by a black line. On the other hand, Fig. 2(b) corresponds to $\underline{a} > \underline{a}_1$, in particular, $\underline{a} = 6\,[\mathrm{m/s^2}]$ and $\underline{a}_1 = 4\,[\mathrm{m/s^2}]$. That is, the surface is given by (15) where (16) are indicated by red and a black lines.

At this point, our goal is to find conditions the feedback law $u(h, v, v_1, a_1)$ has to satisfy in order to ensure the invariance of $\mathcal{C}$ under the closed-loop dynamics (4). For this cause we utilize the concept of safety function described in the next section.

III. Safety Functions for Set Invariance

In this section, we depart from the specific example of connected cruise control and propose a general theoretical framework for ensuring safety for a given feedback law. The logic flow is summarized in Fig. 3.

Consider the affine control system

$$\dot{x} = f(x) + d + g(x)u, \tag{17}$$

where $x \in \mathbb{R}^n$ is the state, $d \in \mathcal{D} \subset \mathbb{R}^q$ is the disturbance and $u \in \mathcal{U} \subset \mathbb{R}^m$ is the control input, while f and g are locally Lipschitz continuous functions. Given the Lipschitz continuous feedback law $u(x; p)$ where $p \in \mathbb{R}^r$ represents the control parameters, we obtain the closed loop system

$$\dot{x} = F(x; p) + d, \tag{18}$$

where $F(x; p) = f(x) + g(x)u(x; p)$.

Our goal is to find domain $\mathcal{P} \subset \mathbb{R}^r$ in the parameter space so that parameters $p \in \mathcal{P}$ guarantee the invariance of the closed set $\mathcal{C} \subset \mathbb{R}^n$ in state space under the dynamics (18). We refer to $\mathcal{C}$ as the safety set and refer to $\mathcal{P}$ as the safe parameter domain. The graphical representations of $\mathcal{P}$ are called safety charts. Moreover, we are also interested in designing controllers that can render the system safe even when $p \in \mathbb{R}^r \setminus \mathcal{P}$.

The safety set can be defined through the super-level set of the function $b : \mathbb{R}^n \mapsto \mathbb{R}$ such that

$$\begin{aligned} \mathcal{C} &= \{x \in \mathbb{R}^n : b(x) \geq 0\}, \\ \partial\mathcal{C} &= \{x \in \mathbb{R}^n : b(x) = 0\}, \\ \mathrm{Int}\mathcal{C} &= \{x \in \mathbb{R}^n : b(x) > 0\}, \end{aligned} \tag{19}$$

where b is a continuously differentiable function or a continuous function constructed from finite number of continuously differentiable functions [8]. We recall the following definition from [7].

Definition 1: Given a set $\mathcal{C} \in \mathbb{R}^n$ defined by (19), the continuously differentiable function $b : \mathbb{R}^n \mapsto \mathbb{R}$ is a *safety function* if there exists an extended class $\mathcal{K}$ function π such that,

$$\dot{b}(x) = L_{F+d}b(x) \geq -\pi(b(x)), \quad \forall\, x \in \mathbb{R}^n, \tag{20}$$

where L_{F+d} denotes the Lie derivative.

We recall that, a continuous function $\pi : [0, a) \mapsto [0, \infty)$ is of *class* $\mathcal{K}$ for some $a > 0$ if it is strictly monotonously increasing and $\pi(0) = 0$. Moreover, a continuous function $\pi : (-b, a) \mapsto (-\infty, \infty)$ is of *extended class* $\mathcal{K}$ for some $a, b > 0$ if it is strictly monotonously increasing and $\pi(0) = 0$.

Seeking a safety function is usually challenging, particularly for a controller with nontrivial structure. Numerical techniques, such as sum of squares (SOS) programming, are usually used for complex cases. A safety function acquired this way may also lead to very conservative performance. This motivates us to go back to system (17) to find safety functions.

Definition 2: Given a set $\mathcal{C} \subset \mathbb{R}^n$ defined by (19) the continuously differentiable function $b : \mathbb{R}^n \mapsto \mathbb{R}$ is a *control*

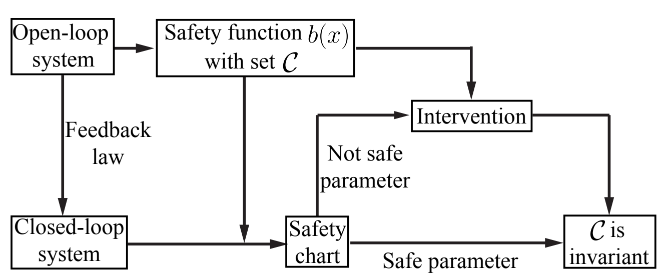

Fig. 3: Ensuring safety of a given feedback law.

safety function if there exists an extended class $\mathcal{K}$ function π such that

$$\sup_{u\in\mathcal{U}} \big[L_{f+d}b(x) + L_g b(x)u + \pi(b(x))\big] \geq 0, \quad \forall\, x \in \mathbb{R}^n. \tag{21}$$

The existence of a control safety function implies that there exist Lipschitz continuous controller $u : \mathbb{R}^n \mapsto \mathcal{U}$ such that the set $\mathcal{C}$ is invariant. The intuition behind Definition 2 is that $\forall\, x \in \mathcal{C}$, there always exist a controller such that

$$\dot{b}(x) \geq -\pi\big(b(x)\big) \;\Rightarrow\; \dot{b}(x) \geq 0 \text{ if } b(x) = 0. \tag{22}$$

Compared to the safety function, the control safety function describes the full capability of the system from safety perspective. Indeed, if one finds a control safety function for the open-loop systems it may be used as a safety function for the closed-loop systems as shown in Fig. 3. In particular, we will use this function to determine the safe control parameters for the closed-loop systems and draw safety charts in parameter space. Using safe parameters $p \in \mathcal{P}$ in control law will result in the invariance of $\mathcal{C}$ in state space. Moreover, we will also utilize the control safety function to intervene in cases when one chooses control parameters outside the safe parameter regime. We remark that in the literature, safety functions and control safety functions are also referred as barrier functions and control barrier functions, respectively [7], [8].

IV. Connected Cruise Control with Safety Guarantee

In this section, we apply the method proposed in the previous section in order to ensure the safety of a given feedback law. In particular, we will provide a constructive method to derive the safe control parameters and represent these graphically using safety charts. We also present an algorithm that allows us to use parameters outside the safe parameter domain by intervening when the system approaches the boundary of the safety set.

Based on the definition of the set $\mathcal{C}$ in (12) we define the safety function candidate

$$b(h, v, v_1) = h - \hat{b}(v, v_1). \tag{23}$$

Then, by construction, the following conditions make (23) a control safety function according to Definition 2:

$$\mathcal{C} = \{(h, v, v_1) | b(h, v, v_1) \geq 0\} \neq \emptyset, \tag{24}$$

$$\sup_{u\in\mathcal{U}} \big[L_{f+d}b(h, v, v_1) + L_g b(h, v, v_1)u + \pi\big(b(h, v, v_1)\big)\big] \geq 0, \tag{25}$$

for $v, v_1 \in [0, \overline{v}], a_1 \in [-\underline{a}_1, \overline{a}_1]$ (cf. (2,3)) where π is a function of extended class $\mathcal{K}$.

Note that, by construction, (24,25) hold for $\pi(b(h, v, v_1))$=0, and therefore they hold for any function of extended class $\mathcal{K}$. Consequently, $b(h, v, v_1)$ is a control safety function and $\mathcal{C}$ is invariant under the dynamics (1) given $a \in [-\underline{a}, \overline{a}]$ (cf. (2)). In the reminder of this section we assume that the system parameters $\underline{a}, \overline{a}, \underline{a}_1, \overline{a}_1, \tau$ are given and apply (25) in order to derive conditions for the feedback law $u(h, v, v_1, a_1)$ that ensures the invariance of $\mathcal{C}$.

A. Safety charts

With the control safety function (23), and given a feedback law $u(h, v, v_1, a_1)$, we present the main theorem that certifies the safety of the connected cruise controller.

Theorem 1: Given a continuous and piecewise continuously differentiable feedback law $u(h, v, v_1, a_1)$ such that

$$U \geq \frac{\partial u}{\partial h} \geq 0, \tag{26}$$

for some $U > 0$, a piecewise continuously differentiable safety function $b(h, v, v_1) = h - \hat{b}(v, v_1)$ such that

$$\frac{\partial \hat{b}}{\partial v} > 0, \tag{27}$$

for $v \neq 0$, and

$$v_1 - v - \frac{\partial \hat{b}}{\partial v_1} a_1 - \frac{\partial \hat{b}}{\partial v} \mathrm{sat}\Big(u\big(\hat{b}(v, v_1), v, v_1, a_1\big)\Big) \geq 0, \tag{28}$$

there exists an extended class $\mathcal{K}$ function π such that

$$L_{F+d}b(h, v, v_1) \geq -\pi(b(h, v, v_1)) \tag{29}$$

holds, and thus, $\mathcal{C}$ is invariant under (4).

The proof of Theorem 1 is given in Appendix A where we use a linear function for π.

One may show that in fact

$$u\big(\hat{b}(v, v_1), v, v_1, a_1\big) \leq -\underline{a}, \tag{30}$$

implies $L_{F+d}b(h, v, v_1) \geq 0$ for the safety function (23) with definition (13) and also with definition (15). Consequently, this is a sufficient condition for (29) and implies the invariance of $\mathcal{C}$. Based on this we will use

$$\max_{v,v_1} \Big[u\big(\hat{b}(v, v_1), v, v_1, a_1\big)\Big] \leq -\underline{a}, \tag{31}$$

rather than (28) to find control parameters that ensure safety.

For example, given the controller (10) we are searching for the parameters $p = [\alpha, \beta, \kappa, h_{\mathrm{st}}]$ such that

$$\max_{v,v_1} \Big[\alpha\Big(\kappa\big(\hat{b}(v, v_1) - h_{\mathrm{st}}\big) - v\Big) + \beta(v_1 - v)\Big] \leq -\underline{a}. \tag{32}$$

For the sake of presentation, we assume α, β and τ are given and draw safety charts in the $(h_{\mathrm{st}}, \kappa)$-plane. Before providing the corresponding detailed conditions, we remark that one needs

$$\kappa < 1/\tau, \tag{33}$$

in order to obtain a meaningful controller as κ determines the time headway in equilibrium.

We recall that $\hat{b}(v, v_1)$ is given by different analytical formulae in different domains of the (v, v_1) space; cf. (13,15). The maximum of (32) may occur either in the interior of these domains or at the boundaries of these domains given by (3,14,16).

As an example, we pick a set of control parameter $\alpha = 0.4$ [1/s], $\beta = 0.5$ [1/s], $\tau = 1$ [s], $\kappa = 0.6$ [1/s], $h_{\mathrm{st}} = 5$ [m] that were used in a real experiment [2]. The acceleration limits are given as, $\overline{a} = \overline{a}_1 = 2\,[\mathrm{m/s^2}]$, $\underline{a} = 4\,[\mathrm{m/s^2}]$, $\underline{a}_1 = 6\,[\mathrm{m/s^2}]$ which correspond to $a \leq a_1$. The conditions are summarized in Tables I, II, III. In particular, Table I is for

since α, β, τ	$1-(\beta+\alpha)\tau \geq 0$
then $h_{\rm st}, \kappa$	$\alpha\kappa(h_{\rm st} - \underline{a}\tau^2) \geq \underline{a}\big(1-\beta\tau-\alpha\tau\big)$ (red dash)

TABLE I: Safety conditions when $\hat{b}(v, v_1) = v\tau$.

if κ (cyan)	then $h_{\rm st}$
$\kappa \geq \frac{\beta \underline{a}_1}{\alpha \overline{v}}$	$\alpha\kappa\left(\overline{v}\tau + \frac{(\overline{v}-\underline{a}\tau)^2}{2\underline{a}} - h_{\rm st}\right)$ (between green $+\frac{\beta^2 \underline{a}_1}{2\alpha\kappa} - (\alpha+\beta)\overline{v} \leq -\underline{a}$ and brown solid)
$\kappa < \frac{\beta \underline{a}_1}{\alpha \overline{v}}$	$\alpha\kappa\left(\overline{v}\tau + \frac{(\overline{v}-\underline{a}\tau)^2}{2\underline{a}} - \frac{\overline{v}^2}{2\underline{a}_1} - h_{\rm st}\right) \leq \alpha\overline{v} - \underline{a}$ (magenta)

TABLE II: Safety conditions when $\underline{a} \leq \underline{a}_1$ and $\hat{b}(v, v_1) = v\tau + \frac{(v-\underline{a}\tau)^2}{2\underline{a}} - \frac{v_1^2}{2\underline{a}_1}$ along the boundary $v = \overline{v}$.

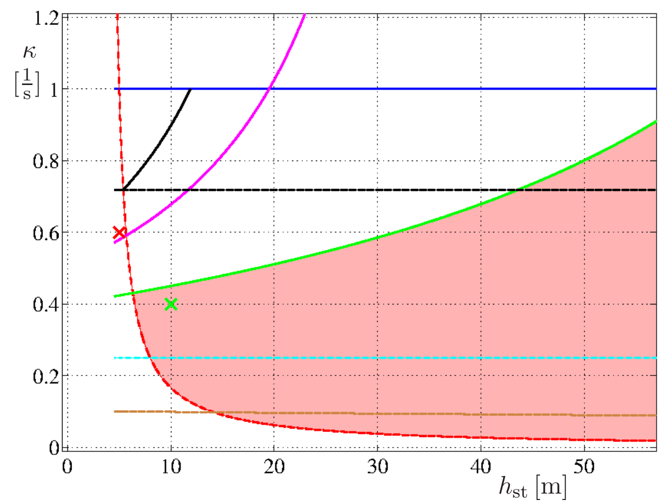

Fig. 4: Safety chart for $\underline{a}_1 = 6\,[{\rm m/s^2}]$, $\underline{a} = 4\,[{\rm m/s^2}]$, $\tau = 1\,[{\rm s}]$, $\alpha = 0.4\,[1/{\rm s}]$, and $\beta = 0.5\,[1/{\rm s}]$. The red shaded region corresponds safe $(h_{\rm st}, \kappa)$ combinations. The blue solid line refer to (33), while other color codes refer to those in Table I,II,III.

$\hat{b}(v, v_1) = v\tau$ while Tables II and III are for $\hat{b}(v, v_1) = v\tau + \frac{(v-\underline{a}\tau)^2}{2\underline{a}} - \frac{v_1^2}{2\underline{a}_1}$. The safety chart is plotted in the $(h_{\rm st}, \kappa)$-plane in Fig. 4, where the color codes of diffierent curves are given in the tables and shading indicates the region of safe parameters. We remark that the conditions summarized in the above tables give non-empty sets of $\alpha, \beta, \tau, \kappa, h_{\rm st}$. In particular, choosing sufficiently small κ and sufficiently large $h_{\rm st}$ ensures safety.

In order to show the difference between the safe and unsafe parameter combinations we mark some points inside and outside the safe parameter region (green cross at $\kappa = 0.4\,[1/{\rm s}]$, $h_{\rm st} = 10\,[{\rm m}]$ and red cross at $\kappa = 0.6\,[1/{\rm s}]$, $h_{\rm st} = 5\,[{\rm m}]$) in Fig. 4. We plot the corresponding simulation results in Fig. 5 as red solid and green dashed curves, respectively. The predecessor's velocity profile v_1 is shown in panel (a) as a black curve. After a constant-speed plateau at 30 [m/s] the predecessor applies maximum braking with $-6\,[{\rm m/s^2}]$. Initially the following vehicle is traveling with speed $v = 30\,[{\rm m/s}]$ and the initial distance between the two vehicles are given by $h = V(v)$, which equals to 80 [m] and 50 [m] for the chosen $\kappa, h_{\rm st}$ combinations; cf. (6). Panels (b,c,d) show the time profiles for v, $h - v\tau$, h. While the

since α, β	$\alpha + \beta > \beta\sqrt{\frac{\underline{a}_1}{\underline{a}}}$
then $h_{\rm st}, \kappa$	either $\alpha\kappa\tau \leq \alpha + \beta - \beta\sqrt{\frac{\underline{a}_1}{\underline{a}}}$ (black dashed) and $\alpha\kappa(h_{\rm st} - \underline{a}\tau^2) \geq \underline{a}\big(1-\beta\tau-\alpha\tau\big)$ (red dashed)
	or $\alpha\kappa\tau > \alpha + \beta - \beta\sqrt{\frac{\underline{a}_1}{\underline{a}}}$ (black dashed) and $\alpha\kappa\left(h_{\rm st} - \overline{v}\sqrt{\frac{\underline{a}}{\underline{a}_1}} - \underline{a}\tau^2\right) \geq$ (black solid) $\left(\beta\sqrt{\frac{\underline{a}_1}{\underline{a}}} - (\alpha+\beta)\right)\overline{v}\sqrt{\frac{\underline{a}}{\underline{a}_1}} + \underline{a}(1-\beta_1\tau-\alpha\tau)$

TABLE III: Safety conditions when $\underline{a} \leq \underline{a}_1$ and $\hat{b}(v, v_1) = v\tau + \frac{(v-\underline{a}\tau)^2}{2\underline{a}} - \frac{v_1^2}{2\underline{a}_1}$ along the boundary $v_1 = f_1(v)$.

unsafe controller leads to collision, the safe controller is able to bring the vehicle slowly to a stop.

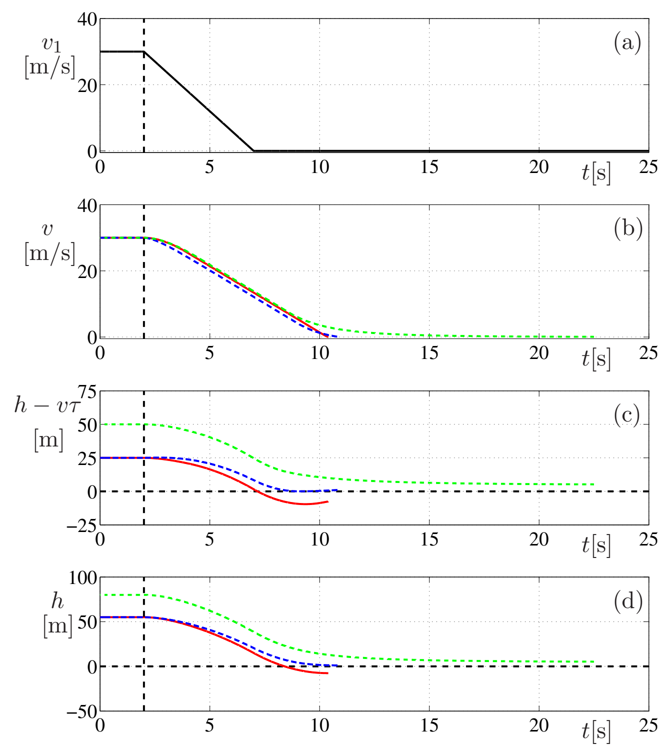

Fig. 5: Simulation results when the predecessor applies heavy braking as shown by the speed profile in panel (a). The dashed green and the solid red curves in panels (b,c,d) correspond to the safe and unsafe parameter combinations marked by a green and red crosses in Fig. 4. The blue profiles also correspond to the unsafe combination but with the intervening controller (36).

B. Intervening controller

Although the safety charts allowed us to select safe control parameters, the corresponding connected cruise control algorithm tends to keep large distances. This may not be feasible heavy traffic scenarios as it may "invite" other road users to cut in front of the vehicle. In order to resolve this issue one may prefer to choose parameter combinations outside the safe parameter region. In this section we propose an intervention that overrides the feedback law once non-safe situations are detected.

Recall that the invariance of the safety set $\mathcal{C}$ is guaranteed once (29) holds which is ensured by (28). Similar to the proof of Theorem 1 shown in Appendix A, we choose the extended class $\mathcal{K}$ function to be linear $\pi(y) = \gamma y$ and propose the control law

$$\hat{u}(h, v, v_1, a_1) = \left(\frac{\partial \hat{b}}{\partial v}\right)^{-1} \left(v_1 - v - \frac{\partial \hat{b}}{\partial v_1} a_1 + \gamma\big(h - \hat{b}(v, v_1)\big)\right) \tag{34}$$

that is applied when

$$L_{F+d} b(h, v, v_1) + \gamma\, b(h, v, v_1) \leq 0. \tag{35}$$

Then, the enhanced controller is given by

$$\min\big\{u(h, v, v_1, a_1), \hat{u}(h, v, v_1, a_1)\big\}. \tag{36}$$

We show the result of this enhanced controller while choosing u according to the feedback law (10) and setting

$$\gamma \geq \alpha\kappa \max\left\{\tau, \frac{\overline{v}}{\underline{a}}\right\}, \tag{37}$$

according to (39) in the proof of Theorem 1. We present the corresponding simulation results as blue dashed curves in Fig. 5 for the unsafe parameters ($\kappa = 0.6$ [1/s], $h_{\rm st} = 5$ [m]) marked by the red cross in Fig. 4 while using $\gamma = 1.8$ that satisfies (37). Apart from being able to maintain safety, the enhanced controller also provides a faster response compared to the one with safe parameters (green dashed curves).

V. CONCLUSION

In this study, we investigated the safety of connected cruise control and applied to a predecessor-follower system. We proposed a theoretical framework to evaluate the safety of a given control law using the notion of control safety function. The concept of safety chart was established in order to help the selection of control parameters that guarantee collision-free motion and an intervening scheme was establish to handle unsafe parameters. In the future we plan to consider other design criteria (e.g., string stability) and test the proposed connected cruise control design experimentally.

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REFERENCES

[1] C. R. He, J. I. Ge, and G. Orosz, "Data-based fuel-economy optimization of connected automated trucks in traffic," in *Proceedings of the American Control Conference*, 2018, pp. 5576–5581.

[2] J. I. Ge, S. S. Avedisov, C. R. He, W. B. Qin, M. Sadeghpour, and G. Orosz, "Experimental validation of connected automated vehicle design among human-driven vehicles impacts on traffic safety and effciency," *Transportation Research Part C*, vol. 91, pp. 335–352, 2018.

[3] L. Zhang and G. Orosz, "Consensus and disturbance attenuation in multi-agent chains with nonlinear control and time delays," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 5, pp. 781–803, 2017.

[4] G. Orosz, "Connected cruise control: modelling, delay effects, and nonlinear behaviour," *Vehicle System Dynamics*, vol. 54, no. 8, pp. 1147–1176, 2016.

[5] J. Mareczek, M. Buss, and M. W. Spong, "Invariance control for a class of cascade nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 4, pp. 636–640, 2002.

[6] E. Garone, S. Di Cairano, and I. Kolmanovsky, "Reference and command governors for systems with constraints: A survey on theory and applications," *Automatica*, no. 75, pp. 306–328, 2017.

[7] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, "Control barrier function based quadratic programs for safety critical systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, 2017.

[8] X. Xu, J. W. Grizzle, P. Tabuada, and A. D. Ames, "Correctness guarantees for the composition of lane keeping and adaptive cruise control," *IEEE Transactions on Automation Science and Engineering*, p. published online, 2017.

[9] J. I. Ge and G. Orosz, "Dynamics of connected vehicle systems with delayed acceleration feedback," *Transportation Research Part C*, vol. 46, pp. 46–64, 2014.

[10] L. Zhang and G. Orosz, "Motif-based design for forward-looking delayed networks with applications to connected vehicle systems," *IEEE Transactions on Intelligent Transportation Systems*, vol. 17, no. 6, pp. 1638–1651, 2016.

[11] C. R. He and G. Orosz, "Saving fuel using wireless vehicle-to-vehicle communication," in *Proceedings of the American Control Conference*, 2017, pp. 4946–4951.

[12] N. I. Li, C. R. He, and G. Orosz, "Sequential parametric optimization for connected cruise control with application to fuel economy optimization," in *Proceedings of the IEEE Conference on Decision and Control*. IEEE, 2016, pp. 227–232.

[13] G. Orosz, R. E. Wilson, and G. Stépán, "Traffic jams: dynamics and control," *Philosophical Transactions of the Royal Society A*, vol. 368, no. 1928, pp. 4455–4479, 2010.

APPENDIX

A. Proof of Theorem 1

Proof: Both $\hat{b}(v, v_1)$ and $u(h, v, v_1, a_1)$ are piecewise continuously differentiable. They take value from a compact set $v,\ v_1 \in [0, \overline{v}]$. Thus, $\frac{\partial \hat{b}}{\partial v}$ is bounded; see (27). Since $\frac{\partial u}{\partial h}$ has an upper bound U (see (26)), then for $b(h, v, v_1) = h - \hat{b}(v, v_1) \geq 0$, considering the sat$(\cdot)$ function, we have

$$U b(h, v, v_1) \geq \operatorname{sat}\Big(u\big(h, v, v_1, a_1\big)\Big) - \operatorname{sat}\Big(u\big(\hat{b}(v, v_1), v, v_1, a_1\big)\Big), \tag{38}$$

where the equality holds only when $h - \hat{b}(v, v_1) = 0$.

By selecting the linear extended class $\mathcal{K}$ function $\pi(y) = \gamma y$ such that

$$\gamma \geq U \max_{v, v_1 \in [0, \overline{v}]} \frac{\partial \hat{b}}{\partial v}, \tag{39}$$

we have $L_{F+d} b(h, v, v_1) + \gamma\, b(h, v, v_1)$

$$\begin{aligned}
&= v_1 - v + \underbrace{\left(\gamma - U\frac{\partial \hat{b}}{\partial v}\right)}_{\geq 0} b(x) - \frac{\partial \hat{b}}{\partial v_1} a_1 \\
&\quad - \frac{\partial \hat{b}}{\partial v}\Bigg[\operatorname{sat}\Big(u\big(\hat{b}(v, v_1), v, v_1, a_1\big)\Big) \underbrace{-U b(h, v, v_1) \\
&\quad \operatorname{sat}\Big(u\big(h, v, v_1, a_1\big)\Big) - \operatorname{sat}\Big(u\big(\hat{b}(v, v_1), v, v_1, a_1\big)\Big)}_{\leq 0}\Bigg] \\
&\geq v_1 - v - \frac{\partial \hat{b}}{\partial v_1} a_1 - \frac{\partial \hat{b}}{\partial v} \operatorname{sat}\Big(u\big(\hat{b}(v, v_1), v, v_1, a_1\big)\Big),
\end{aligned} \tag{40}$$

where the equality at last step holds only when $b(h, v, v_1) = h - \hat{b}(v_1, v) = 0$. Note that (40) gives condition (29) at $b(h, v, v_1) = 0$. ■