Math 454-001 Winter 2014 Homework 1 due : Thursday, Jan. 16

review

1. Solve the following differential equations for $y(t)$. If initial conditions are given, find the exact solution; otherwise, find the general solution.

   (a) $y' = y^2, \quad y(0) = 1$

   (b) $y' = 2y$

   (c) $y'' + y' + y = 0$

   (d) $y'' + 5y' + 6y = 0, \quad y(0) = 2, \quad y'(0) = 3$

   (e) $ty' + 2y = 4t^2, \quad y(1) = 2$ (hint : use an integrating factor)

   (f) $16y'' - 8y' + 145y = 0, \quad y(0) = -2, \quad y'(0) = 1$

2. This exercise reviews the hyperbolic functions. They are defined as

$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}.$$

   (a) Show that $\cosh x$ is an even function and that $\sinh x$ is an odd function.

   (b) Sketch $y = \cosh x$ and $y = \sinh x$ on the same set of axes for $-\infty < x < \infty$.

   (c) Show that $\cosh^2 x - \sinh^2 x = 1$.

   Note : this result implies that the point $(X, Y) = (\cosh x, \sinh x)$ lies on the hyperbola $X^2 - Y^2 = 1$ in the XY-plane. This is why $\cosh x$ and $\sinh x$ are called hyperbolic trigonometric functions. Recall that the functions $\cos x$ and $\sin x$ satisfy $\cos^2 x + \sin^2 x = 1$, which implies that the point $(X, Y) = (\cos x, \sin x)$ lies on the circle $X^2 + Y^2 = 1$; these are the usual circular trigonometric functions.

   (d) Use Euler's formula, $e^{ix} = \cos x + i \sin x$, to show that

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} = \cosh ix \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i} = -i \sinh ix.$$

   (e) Find $\frac{d}{dx} \cosh x$ and $\frac{d}{dx} \sinh x$.

   (f) Define $\tanh x = \frac{\sinh x}{\cosh x}$. Show that $\tanh x$ is an odd function and find $\frac{d}{dx} \tanh x$.

   (g) Evaluate $\lim_{x \to \pm\infty} \tanh x$ and sketch the graph of $y = \tanh x$ for $-\infty < x < \infty$.

heat eqn., equilibrium solutions

3. page 18, problem 1.4.1 b, c, f, h

4. page 18, problem 1.4.6

5. page 18, problem 1.4.7 b