

### PROBLEM SET 3 (DUE ON THURSDAY, SEP 29)

(All Exercises are references to the August 29, 2022 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 3.6.K (sometimes functions are determined by their values on closed points)
- Problem 2.** Use the structure sheaf  $\mathcal{O}_{\text{Spec } A}$  to show that if  $\text{Spec } A$  is disconnected, then  $A$  is isomorphic to the product of two nonzero rings.
- Problem 3.** Let  $X = \text{Spec } k[x, y, z]/(xz, yz)$  and let  $U \subset X$  be the complement of the closed point  $[(x, y, z)]$ . Compute the ring  $\mathcal{O}_X(U)$  along with the restriction map  $\text{res}_{X,U} : \mathcal{O}_X(X) \rightarrow \mathcal{O}_X(U)$ . Is  $\text{res}_{X,U}$  isomorphic to some localization map  $A \rightarrow S^{-1}A$ ?
- Problem 4.** Exercise 4.3.A (classifying isomorphisms of affine schemes)