

## PROBLEM SET 2 (DUE ON THURSDAY, SEP 21)

(All Exercises are references to the July 31, 2023 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Let  $\pi : \mathbb{Q}[x] \rightarrow \mathbb{C}[x]$  be the ring homomorphism sending  $x$  to  $x$ . Let  $\pi^* : \text{Spec } \mathbb{C}[x] \rightarrow \text{Spec } \mathbb{Q}[x]$  be the induced map of spectra. For each point  $p \in \text{Spec } \mathbb{Q}[x]$ , describe the fiber  $(\pi^*)^{-1}(p)$  (as a set).
- Problem 2.** Let  $n > 0$  and let  $\pi : \mathbb{Z} \rightarrow \mathbb{Z}[x_1, \dots, x_n]$  be the unique ring homomorphism. Let  $\pi^* : \text{Spec } \mathbb{Z}[x_1, \dots, x_n] \rightarrow \text{Spec } \mathbb{Z}$  be the induced map of spectra. For each point  $p \in \text{Spec } \mathbb{Z}$ , describe a bijection between the fiber  $(\pi^*)^{-1}(p)$  and  $\text{Spec } k_p[x_1, \dots, x_n]$  for some field  $k_p$ . (Essential Algebra Exercises 3.2.J and 3.2.K might be useful here - I'll talk about them a little on Tuesday, but you might want to think about them yourself anyway. Exercise 3.2.Q has some discussion and a picture that might be helpful.)
- Problem 3.** Exercise 3.5.B (covering  $\text{Spec } A$  with distinguished open sets)
- Problem 4.** Exercise 3.5.E (equivalent conditions to  $D(f) \subset D(g)$ )
- Problem 5.** Exercise 3.6.J (when are the closed points in  $\text{Spec } A$  dense? (As suggested in the hint, you will want to read the statement of Zariski's Lemma in 3.2.5.))