

PROBLEM SET 3 (DUE ON THURSDAY, SEP 28)

(All Exercises are references to the July 31, 2023 version of *Foundations of Algebraic Geometry* by R. Vakil.)

- Problem 1.** Exercise 3.6.K (sometimes functions are determined by their values on closed points)
- Problem 2.** Use the structure sheaf $\mathcal{O}_{\text{Spec } A}$ to show that if $\text{Spec } A$ is disconnected, then A is isomorphic to the product of two nonzero rings. (Be careful to show that the two rings are nonzero!)
- Problem 3.** Let $X = \text{Spec } k[x, y, z]/(xz, yz)$ and let $U \subset X$ be the complement of the closed point $[(x, y, z)]$. Compute the ring $\mathcal{O}_X(U)$ along with the restriction map $\text{res}_{X,U} : \mathcal{O}_X(X) \rightarrow \mathcal{O}_X(U)$. Is $\text{res}_{X,U}$ isomorphic to some localization map $A \rightarrow S^{-1}A$?
- Problem 4.** Exercise 4.3.A (classifying isomorphisms of affine schemes)