PROBLEM SET 4 (DUE ON THURSDAY, OCT 5)

(All Exercises are references to the July 31, 2023 version of *Foundations of Algebraic Geometry* by R. Vakil.)

Problem 1. Exercise 4.3.F (functions on locally ringed spaces)

Problem 2. Let $X_1 = \operatorname{Spec} k[x, y]$ and $X_2 = \operatorname{Spec} k[w, z]$ be two copies of the affine plane over a field k. Let X be the scheme formed by gluing X_1 and X_2 along the isomorphism of open subschemes $\operatorname{Spec} k[x, x^{-1}, y] \cong \operatorname{Spec} k[w, w^{-1}, z]$ induced by the ring isomorphism $k[x, x^{-1}, y] \cong k[w, w^{-1}, z]$ given by $x \mapsto w, y \mapsto w^{-1}z$. Compute the ring of global sections of the structure sheaf of X. Is X affine?

Problem 3. Exercise 7.3.M (morphisms from Spec of a local ring)

Problem 4. Describe all morphisms $\pi : \mathbb{P}^1_k \to \mathbb{P}^1_k$ that commute with the natural morphism $\mathbb{P}^1_k \to \operatorname{Spec} k$. (Note: this is saying that π is a "morphism of k-schemes". Hint: break into cases based on whether π maps the generic point to the generic point. If it does, consider the induced map of stalks there.)